Operational Equivalence and Causal Structure

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Operational theory

$$\{p(X|M \wedge P) : \text{ for all } M, P\}$$

Operational equivalence

$$M_1 \sim M_2 : \quad p(X|M_1 \wedge P) = p(X|M_2 \wedge P) \quad \text{for all } P$$

Quantum mechanics

$$
\begin{tikzcd}[row sep=large]
\textbf{Operator:} & & \mathbf{O} \arrow[dl] \arrow[dr] & \\
\textbf{Measurement:} & M_1 & \sim & M_2
\end{tikzcd}
$$

Main message

Ontological models for quantum mechanics can be different with respect to causal structure, contextuality, fine-tuning, etc. when operators are realized by operationally equivalent but different measurements.

Observables

When do two measurements measure the same observable?

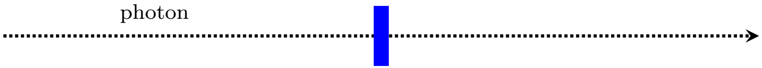
Observables

	Strict operationalism:	**Quantum mechanics:**
Operator:	**O**	**O**
	$\swarrow \qquad \searrow$	$\downarrow$
Observable:	$\mathcal{O}_1 \qquad \mathcal{O}_2$	$\mathcal{O}$
	$\downarrow \qquad \downarrow$	$\swarrow \qquad \searrow$
Measurement:	$M_1 \quad \sim \quad M_2$	$M_1 \quad \sim \quad M_2$

Observables

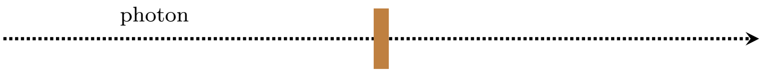
1. If both measurements can be performed **simultaneously** on each system; or
2. each preparation is an **eigenstate** for both measurements.

Polarization



M_1: polaroid

Polarization



M_2: birefringent crystal

Ontological models

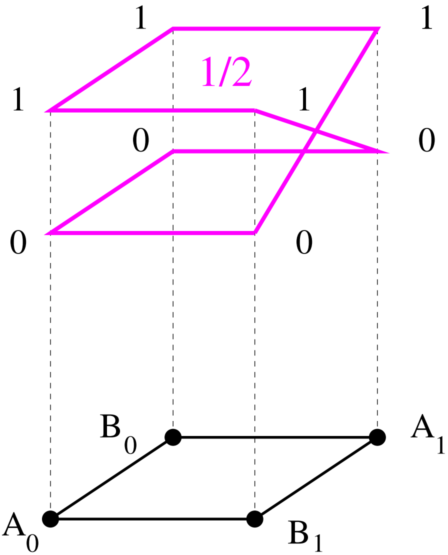
$$\{p(\lambda|P): \quad \text{for all } P\}$$

$$\{p(X|M \wedge \lambda): \quad \text{for all } M, \lambda\}$$

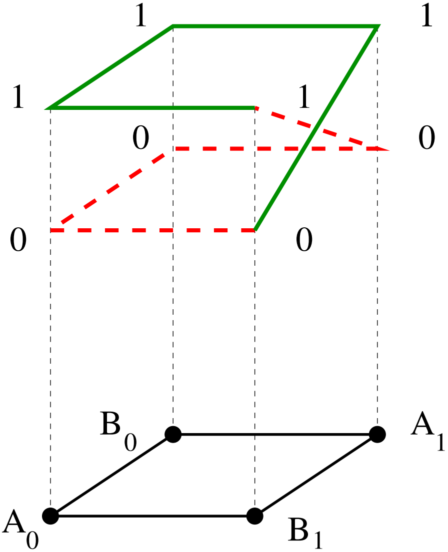
Ontological models

$$p(X|M \wedge P) = \sum_{\lambda} p(X|M \wedge \lambda)\, p(\lambda|P)$$

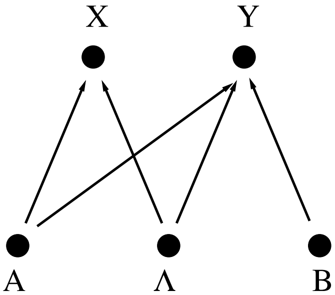
An operational theory: the PR box



An ontological model: the PR box

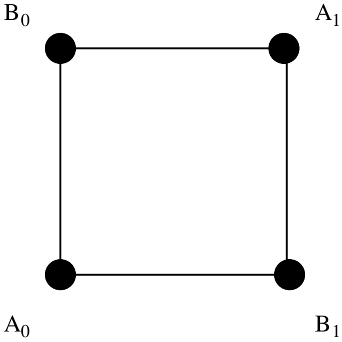


Causal structure of the PR box

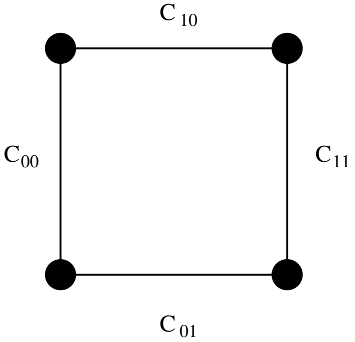


- Bell's inequality
- Contextuality
- Fine-tuning

Old measurements



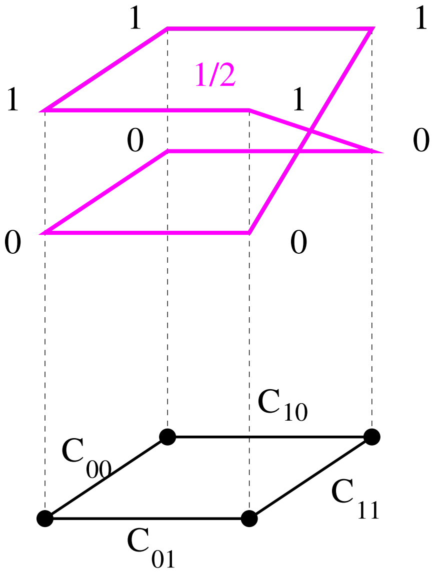
New measurements



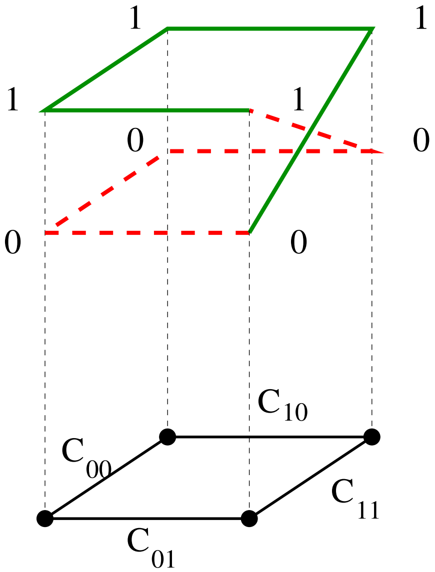
$$C_{00}^{(1)} \sim C_{01}^{(1)} \sim A_0 \qquad C_{10}^{(1)} \sim C_{11}^{(1)} \sim A_1$$

$$C_{00}^{(2)} \sim C_{10}^{(2)} \sim B_0 \qquad C_{01}^{(2)} \sim C_{11}^{(2)} \sim B_1$$

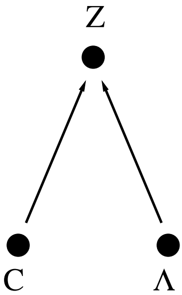
An operational theory: the PR box



An ontological model: the PR box



Causal structure of the models



- No Bell's inequality
- No contextuality
- No fine-tuning

Quantum mechanics

$\boldsymbol{\sigma}_z \otimes \mathbf{1}$	$\mathbf{1} \otimes \boldsymbol{\sigma}_z$	$\boldsymbol{\sigma}_z \otimes \boldsymbol{\sigma}_z$
$\mathbf{1} \otimes \boldsymbol{\sigma}_x$	$\boldsymbol{\sigma}_x \otimes \mathbf{1}$	$\boldsymbol{\sigma}_x \otimes \boldsymbol{\sigma}_x$
$\boldsymbol{\sigma}_z \otimes \boldsymbol{\sigma}_x$	$\boldsymbol{\sigma}_x \otimes \boldsymbol{\sigma}_z$	$\boldsymbol{\sigma}_y \otimes \boldsymbol{\sigma}_y$

Quantum mechanics

$$\begin{array}{lccc}
\textbf{Operator:} & & \boldsymbol{\sigma}_y \otimes \boldsymbol{\sigma}_y & \\
& \swarrow & & \searrow \\
\textbf{Measurement:} & A_y \wedge B_y & \sim & C^{(3)} \sim D^{(3)}
\end{array}$$

Quantum mechanics

$A_y \wedge B_y$: Measure the linear polarization of the left photon in direction y

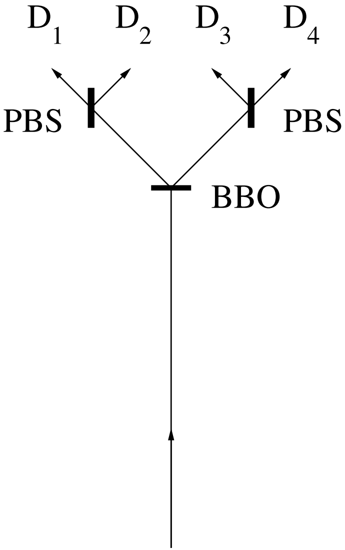
and

measure the linear polarization of the right photon in direction y

and

registers the product of the two outcomes

Quantum mechanics



$C^{(3)}$: Consider the operators in the third column

$$\boldsymbol{\sigma}_z \otimes \boldsymbol{\sigma}_z \qquad \boldsymbol{\sigma}_x \otimes \boldsymbol{\sigma}_x \qquad \boldsymbol{\sigma}_y \otimes \boldsymbol{\sigma}_y$$

with four common eigenvectors

$$|\Psi^{---}\rangle \qquad |\Psi^{++-}\rangle \qquad |\Psi^{+-+}\rangle \qquad |\Psi^{-++}\rangle$$

Perform a Bell state measurement on the photon pair corresponding to these eigenvectors

and

consider only the third index

Quantum mechanics

$D^{(3)}$: Consider the operators in the third row

$$\boldsymbol{\sigma}_z \otimes \boldsymbol{\sigma}_x \qquad \boldsymbol{\sigma}_x \otimes \boldsymbol{\sigma}_z \qquad \boldsymbol{\sigma}_y \otimes \boldsymbol{\sigma}_y$$

with four common eigenvectors

$$|\Phi^{+++}\rangle \qquad |\Phi^{--+}\rangle \qquad |\Phi^{-+-}\rangle \qquad |\Phi^{+--}\rangle$$

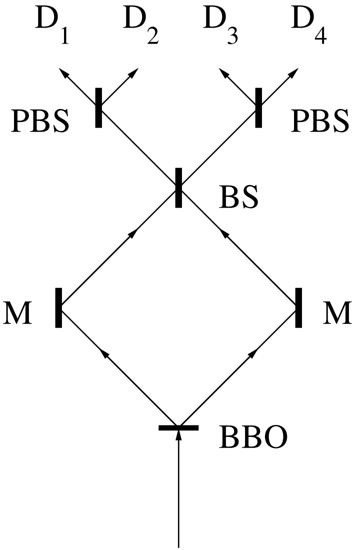
Perform a Bell state measurement on the

photon pair corresponding to these eigenvectors

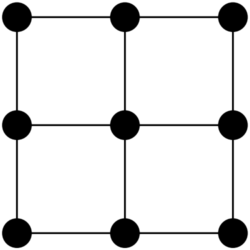
and

consider only the third index

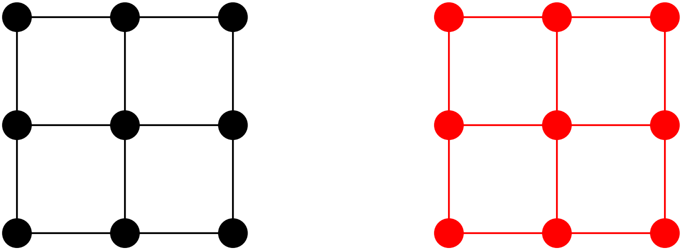
Quantum mechanics



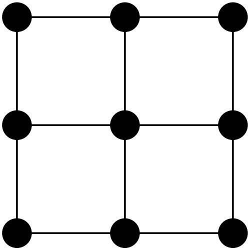
Quantum mechanics

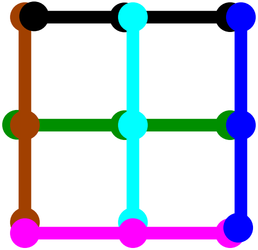


Quantum mechanics



Quantum mechanics





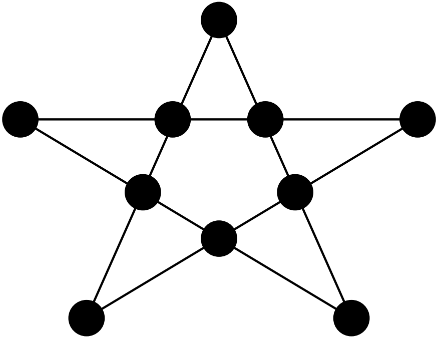
Conclusions

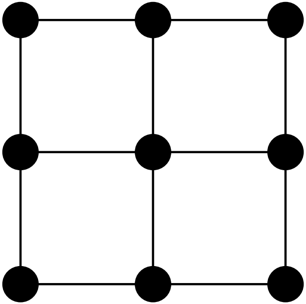
The realization of an operator in quantum mechanics by different measurements can give rise to different ontological models with respect to contextuality, causal structure, fine-tuning, etc.

More on that . . .

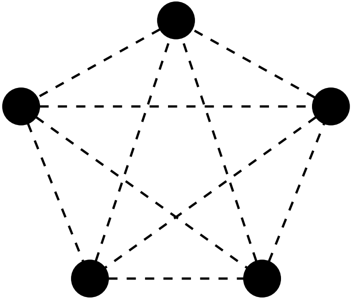
- Gábor Hofer-Szabó, "Operational equivalence and causal structure" (submitted).

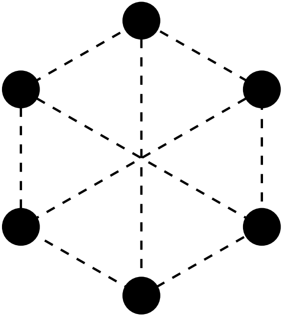
The GHZ and the Peres-Mermin graph





The GHZ and the Peres-Mermin line graph





Noncontextuality

An ontological model for QM is **noncontextual** if

- every ontic state determines the probability distribution of outcomes of every measurement independently of what other measurements are simultaneously performed **(simultaneous noncontextuality)**
- any two measurements which are represented by the same self-adjoint operator have the same probability distribution of outcomes in every ontic state **(measurement noncontextuality)**

Noncontextuality

- **Simultaneous noncontextuality:**

$$p(X|M \wedge \Lambda) = p(X|M \wedge M' \wedge \Lambda) \qquad \text{for all } \Lambda$$

- **Measurement noncontextuality:**

$$\text{If} \quad p(X|M \wedge P) = p(X'|M' \wedge P) \qquad \text{for all } P$$

$$\text{then} \quad p(X|M \wedge \Lambda) = p(X'|M' \wedge \Lambda) \qquad \text{for all } \Lambda$$