# Operational Equivalence and Causal Structure

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# Main message

Ontological models for quantum mechanics can be different with respect to causal structure, contextuality, fine-tuning, etc. when operators are realized by different measurements.

# Operational theory

$$\{p(X|M \wedge P) : \text{ for all } M, P\}$$

# Operational equivalence

$$M_1 \sim M_2 : \quad p(X|M_1 \wedge P) = p(X|M_2 \wedge P) \quad \text{for all } P$$

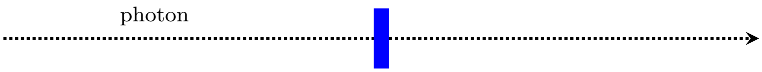
# Quantum mechanics

$$\begin{array}{rccc} \textbf{Operator:} & & \mathbf{O} & \\ & \swarrow & & \searrow \\ \textbf{Measurement:} & M_1 & \sim & M_2 \end{array}$$

# Observables

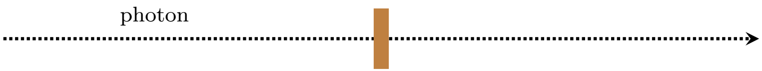
When do two measurements measure the same observable?

# Polarization




$M_1$: polaroid

# Polarization




$M_2$: birefringent crystal

# Observables

$$
\begin{tikzcd}
 & & \textbf{Strict operationalism:} & & & \textbf{Quantum mechanics:} & \\
\textbf{Operator:} & & \mathbf{O} \arrow[dl] \arrow[dr] & & & \mathbf{O} \arrow[d] & \\
\textbf{Observable:} & \mathcal{O}_1 \arrow[d] & & \mathcal{O}_2 \arrow[d] & & \mathcal{O} \arrow[dl] \arrow[dr] & \\
\textbf{Measurement:} & M_1 & \sim & M_2 & M_1 & \sim & M_2
\end{tikzcd}
$$

# Observables

When do two measurements measure the same observable?

# Observables

1. If both measurements can be performed **simultaneously** on each system; or
2. each preparation is an **eigenstate** for both measurements.

## Ontological models

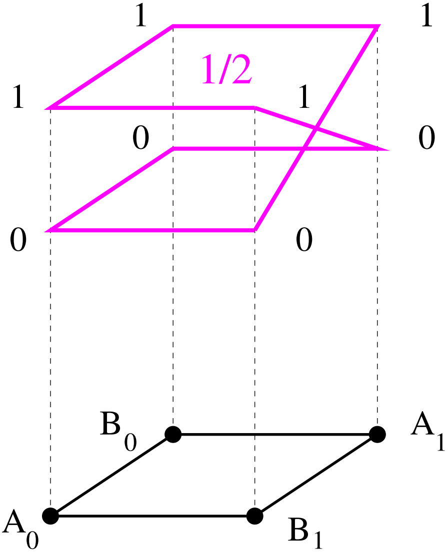
$$\{p(\lambda|P): \quad \text{for all } P\}$$

$$\{p(X|M \wedge \lambda): \quad \text{for all } M, \lambda\}$$

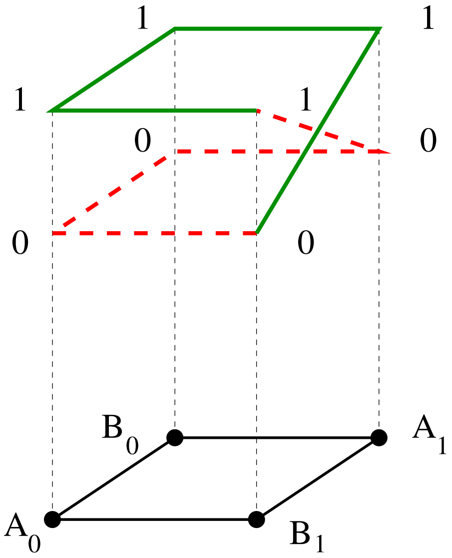
# Ontological models

$$p(X|M \wedge P) = \sum_{\lambda} p(X|M \wedge \lambda)\, p(\lambda|P)$$

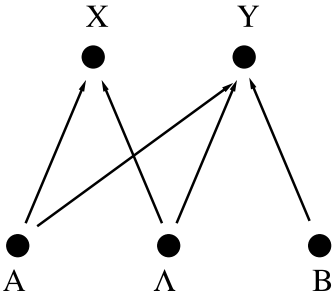
# An operational theory: the PR box



# An ontological model: the PR box

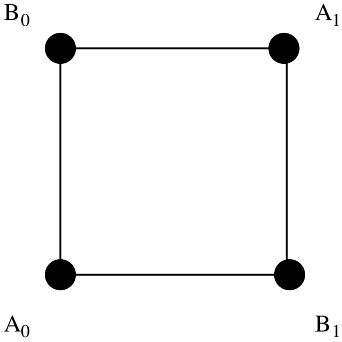


# Causal structure of the PR box

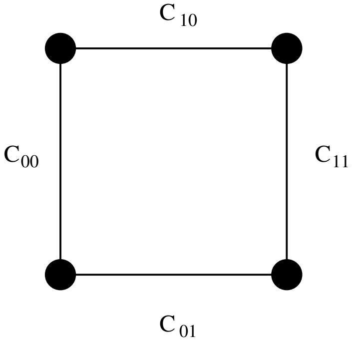



- Bell's inequality
- Contextuality
- Fine-tuning

# Old measurements



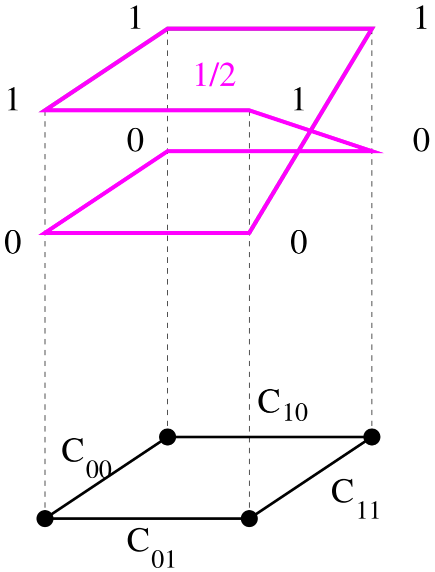
# New measurements



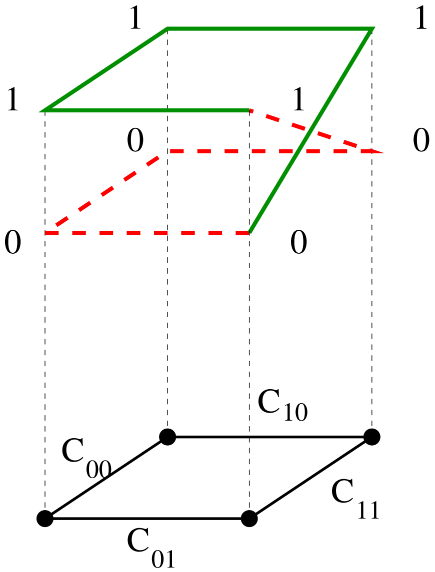

$$C_{00}^{(1)} \sim C_{01}^{(1)} \sim A_0 \qquad C_{10}^{(1)} \sim C_{11}^{(1)} \sim A_1$$

$$C_{00}^{(2)} \sim C_{10}^{(2)} \sim B_0 \qquad C_{01}^{(2)} \sim C_{11}^{(2)} \sim B_1$$

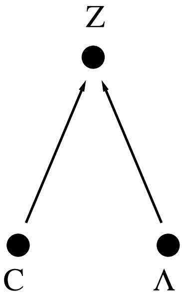
# An operational theory: the PR box



# An ontological model: the PR box



# Causal structure of the models




- No Bell's inequality
- No contextuality
- No fine-tuning

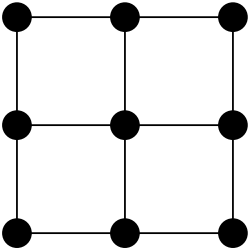
# Quantum mechanics

$$\boldsymbol{\sigma}_z \otimes \mathbf{1} \qquad \mathbf{1} \otimes \boldsymbol{\sigma}_z \qquad \boldsymbol{\sigma}_z \otimes \boldsymbol{\sigma}_z$$

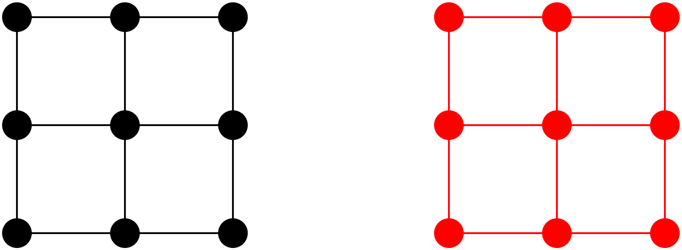
$$\mathbf{1} \otimes \boldsymbol{\sigma}_x \qquad \boldsymbol{\sigma}_x \otimes \mathbf{1} \qquad \boldsymbol{\sigma}_x \otimes \boldsymbol{\sigma}_x$$

$$\boldsymbol{\sigma}_z \otimes \boldsymbol{\sigma}_x \qquad \boldsymbol{\sigma}_x \otimes \boldsymbol{\sigma}_z \qquad \boldsymbol{\sigma}_y \otimes \boldsymbol{\sigma}_y$$

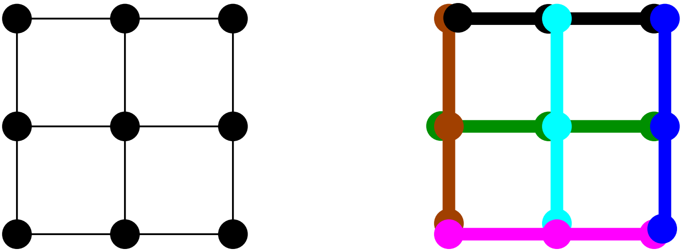
# Quantum mechanics



# Quantum mechanics



# Quantum mechanics



# Quantum mechanics

**Operator:** $\boldsymbol{\sigma}_y \otimes \boldsymbol{\sigma}_y$

$$\swarrow \qquad \searrow$$

**Measurement:** $A_y \wedge B_y \quad \sim \quad C^{(3)} \sim D^{(3)}$

# Quantum mechanics

$A_y \wedge B_y$: Measure the linear polarization of the left photon along a transverse axis $y$

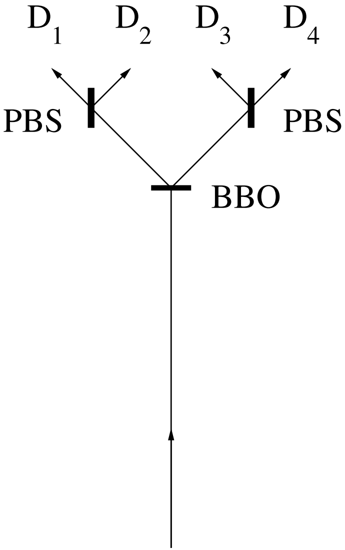
*and*

measure the linear polarization of the right photon along the same transverse axis $y$

*and*

registers the product of the two outcomes

# Quantum mechanics



# Quantum mechanics

$C^{(3)}$: Consider the operators in the third column

$$\boldsymbol{\sigma}_z \otimes \boldsymbol{\sigma}_z \qquad \boldsymbol{\sigma}_x \otimes \boldsymbol{\sigma}_x \qquad \boldsymbol{\sigma}_y \otimes \boldsymbol{\sigma}_y$$

with four common eigenvectors

$$|\Psi^{---}\rangle \qquad |\Psi^{++-}\rangle \qquad |\Psi^{+-+}\rangle \qquad |\Psi^{-++}\rangle$$

Perform a Bell state measurement on the photon pair corresponding to these eigenvectors

*and*

consider only the third index

# Quantum mechanics

$D^{(3)}$: Consider the operators in the third row

$$\boldsymbol{\sigma}_z \otimes \boldsymbol{\sigma}_x \qquad \boldsymbol{\sigma}_x \otimes \boldsymbol{\sigma}_z \qquad \boldsymbol{\sigma}_y \otimes \boldsymbol{\sigma}_y$$

with four common eigenvectors

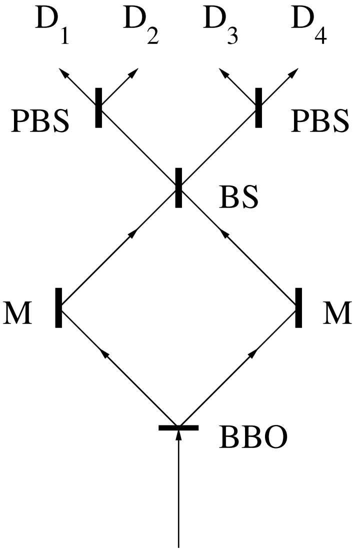
$$|\Phi^{+++}\rangle \qquad |\Phi^{--+}\rangle \qquad |\Phi^{-+-}\rangle \qquad |\Phi^{+--}\rangle$$

Perform a Bell state measurement on the photon pair corresponding to these eigenvectors

*and*

consider only the third index

# Quantum mechanics



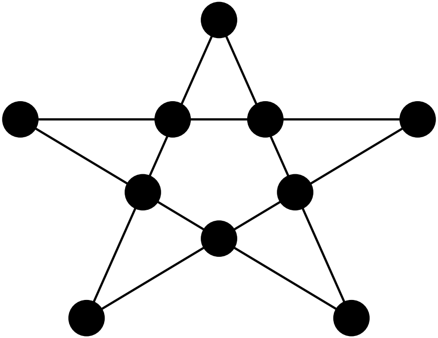
# Conclusions

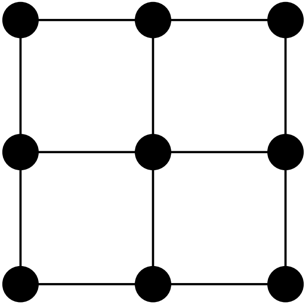
The realization of an operator in quantum mechanics by different measurements can give rise to different ontological models with respect to contextuality, causal structure, fine-tuning, etc.

## More on that . . .

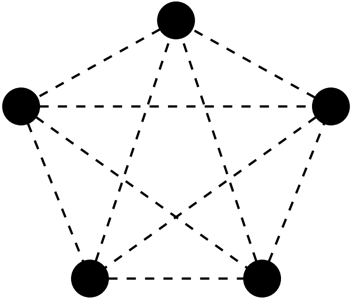
- Gábor Hofer-Szabó, ”Quantum mechanics without operational equivalence” (submitted).

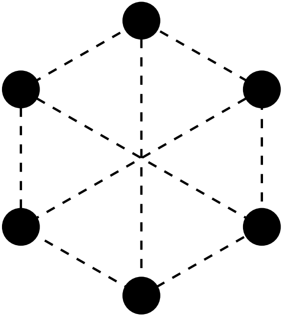
# The GHZ and the Peres-Mermin graph





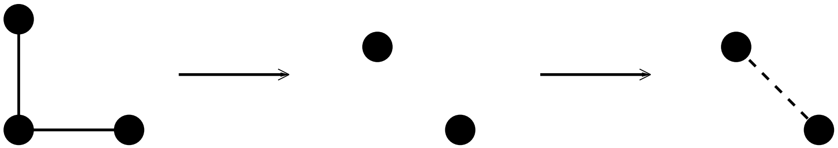
# The GHZ and the Peres-Mermin line graph





# Trivialization of a simple theory

$$\begin{array}{rcl} \textbf{Non-trivial theory} & \longrightarrow & \textbf{Trivial theory} \\ \{M_1, M_2, M_3, M_1 \wedge M_2, M_1 \wedge M_3\} & \longrightarrow & \{M_{12}, M_{13}\} \\ & & M_{12} \sim M_1 \wedge M_2 \\ & & M_{13} \sim M_1 \wedge M_3 \\ & & M_{12}^{(1)} \sim M_{13}^{(1)} \sim M_1 \end{array}$$



# Noncontextuality

An ontological model for QM is **noncontextual** if

- every ontic state determines the probability distribution of outcomes of every measurement independently of what other measurements are simultaneously performed **(simultaneous noncontextuality)**
- any two measurements which are represented by the same self-adjoint operator have the same probability distribution of outcomes in every ontic state **(measurement noncontextuality)**

# Noncontextuality

- **Simultaneous noncontextuality:**

$$p(X|M \wedge \Lambda) = p(X|M \wedge M' \wedge \Lambda) \qquad \text{for all } \Lambda$$

- **Measurement noncontextuality:**

$$\text{If} \quad p(X|M \wedge P) = p(X'|M' \wedge P) \qquad \text{for all } P$$

$$\text{then} \quad p(X|M \wedge \Lambda) = p(X'|M' \wedge \Lambda) \qquad \text{for all } \Lambda$$