Is the quantum state real?

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Is it real?

$$|\Psi\rangle$$

Is it real?

$$\langle \Psi, \mathbf{P}_x \Psi \rangle = p(X|M \wedge P)$$

$$\begin{aligned} P &: \quad \text{Preparation} \\ M &: \quad \text{Measurement} \\ X &: \quad \text{Outcome} \end{aligned}$$

Is it real?

$$\text{preparation} \quad \longrightarrow \quad |\Psi\rangle \quad \longrightarrow \quad \text{measurement}$$

Questions

- Individual system or ensemble?
- Complete or incomplete?
- Local or nonlocal?
- Ontic or epistemic?

Interpretations

Is $|\Psi\rangle$ real, as a property of an individual system?

Yes	**No**
Schrödinger, de Broglie	Copenhagen
Bohmian	QBism
Collapse	Statistical
Many-world	Relational
Modal	

Operational theory

$$\{p(X|M \wedge P) : \text{ for all } M, P\}$$

Operational equivalence

$$P_1 \sim P_2 : \quad p(X|M \wedge P_1) = p(X|M \wedge P_2) \quad \text{for all } M$$

$$M_1 \sim M_2 : \quad p(X|M_1 \wedge P) = p(X|M_2 \wedge P) \quad \text{for all } P$$

Quantum mechanics

$$\begin{array}{ccc} & |\Psi\rangle & \\ \swarrow & & \searrow \\ P_1 & \sim & P_2 \end{array}$$

$$\begin{array}{ccc} & \mathbf{O} & \\ \swarrow & & \searrow \\ M_1 & \sim & M_2 \end{array}$$

Is it real?

$$\{\text{preparation}\} \quad \longrightarrow \quad |\Psi\rangle \quad \longrightarrow \quad \{\text{measurement}\}$$

Ontological models

$$\{p(\lambda|P): \quad \text{for all } P\}$$

$$\{p(X|M \wedge \lambda): \quad \text{for all } M, \lambda\}$$

Ontological models

$$p(X|M \wedge P) = \sum_{\lambda} p(X|M \wedge \lambda)\, p(\lambda|P) \quad \text{for all } M, P$$

Contextuality

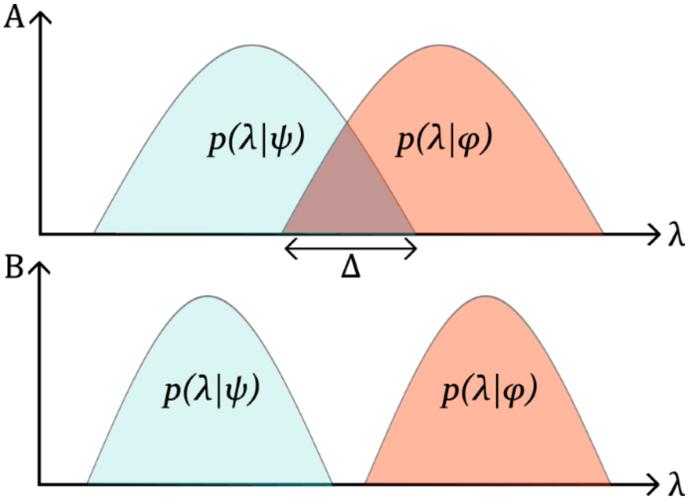
Preparation contextuality:

$$p(\lambda|P_1) \neq p(\lambda|P_2) \quad \text{for some } P_1 \sim P_2$$

Measurement contextuality:

$$p(X|M_1 \wedge \lambda)) \neq p(X|M_2 \wedge \lambda) \quad \text{for some } M_1 \sim M_2 \text{ and } \lambda$$

Ψ-ontic/Ψ-epistemic



Ψ-ontic/Ψ-epistemic

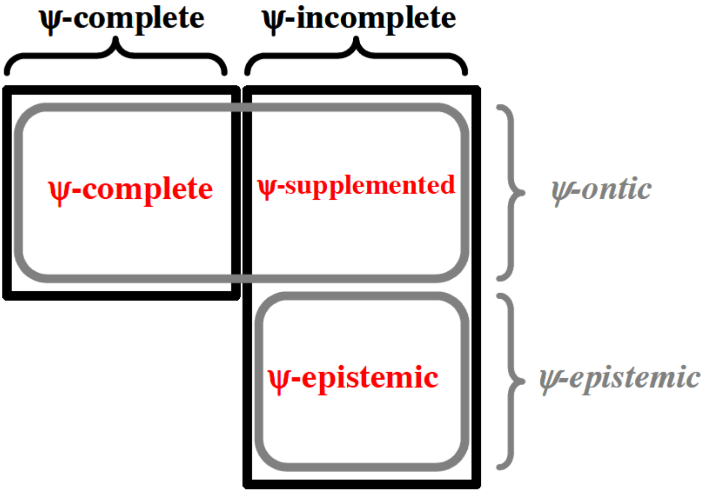
Ψ-ontic:

$$\mathrm{Supp}\big(p(\lambda|P_\Psi)\big) \cap \mathrm{Supp}\big(p(\lambda|P_\Phi)\big) = \emptyset \quad \text{for all } \Psi \neq \Phi$$

Ψ-epistemic:

$$\mathrm{Supp}\big(p(\lambda|P_\Psi)\big) \cap \mathrm{Supp}\big(p(\lambda|P_\Phi)\big) \neq \emptyset \quad \text{for all } \Psi \neq \Phi$$

Ψ-complete/Ψ-incomplete



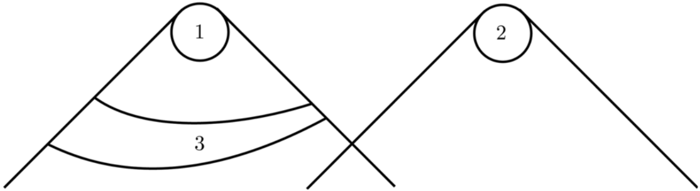
Classification

- **Ψ-complete:** $\Lambda = \{|\Psi\rangle\}$
  - Copenhagen (Heisenberg), collapse theories (GRW), many-worlds
- **Ψ-incomplete and Ψ-ontic:** $\Lambda = \{|\Psi\rangle, \lambda\}$
  - Bohmian, modal (Dieks)
- **Ψ-incomplete and Ψ-epistemic:**
  - $\Lambda = \emptyset$: Copenhagen (Bohr), Neo-Copenhagen: Qbism (Fuchs), relational (Rovelli)
  - $\Lambda = \{\lambda\}$: Statistical (Ballentine)

Some connections

- Ψ-ontic models are preparation contextual
- Ψ-complete models are outcome indeterministic
- Ψ-ontic models are nonlocal

Local causality



To rule out local Ψ-ontic models, Bell's inequalities are not needed

History

- **EPR paper, 1935:** the Copenhagen interpretation is incomplete (if local + reality criterion)
- **Letter to Schrödinger, 1935:** No Ψ-ontic, hence no Ψ-complete models (if local)
- **Bell, 1962:** neither Ψ-ontic nor Ψ-epistemic model (if local)
- **Spekkens, QBists, $\sim$2010:** » Ψ-epistemic models, again
- **PBR, 2012:** no Ψ-epistemic model

PBR theorem

Consider two ensembles prepared in two different quantum states. There are no two individual systems in the two ensembles which have the same ontic state λ. (The wave function is Ψ-ontic.)

Proof

- We prove only the special case
- Prepare two beams of electrons, one in state $P_{|0\rangle}$, the other in $P_{|+\rangle}$.
- There are no two electrons in the two beams which are in the same ontic state λ.

Proof

- Suppose there are two electrons both in ontic state λ
- Then, there is such a pair of electrons in two beams prepared in any of the four states:

$$P_{|00\rangle} \quad P_{|0+\rangle} \quad P_{|+0\rangle} \quad P_{|++\rangle}$$

- Perfom the measurement corresponding to the entangled basis

$$\frac{1}{\sqrt{2}}\big(\,|01\rangle + |10\rangle\,\big) \qquad \frac{1}{\sqrt{2}}\big(\,|0-\rangle + |1+\rangle\,\big)$$
$$\frac{1}{\sqrt{2}}\big(\,|+1\rangle + |-0\rangle\,\big) \qquad \frac{1}{\sqrt{2}}\big(\,|+-\rangle + |-+\rangle\,\big)$$

on any of the four beam-pairs

Proof

- Since

$$\frac{1}{\sqrt{2}}\big(\,|01\rangle + |10\rangle\,\big) \perp |00\rangle \qquad \frac{1}{\sqrt{2}}\big(\,|0-\rangle + |1+\rangle\,\big) \perp |0+\rangle$$
$$\frac{1}{\sqrt{2}}\big(\,|+1\rangle + |-0\rangle\,\big) \perp |+0\rangle \qquad \frac{1}{\sqrt{2}}\big(\,|+-\rangle + |-+\rangle\,\big) \perp |++\rangle$$

  in each beam-pair one outcome will be impossible

- Our pair of electrons cannot yield the first outcome in beam-pair $P_{|00\rangle}$, the second outcome in beam-pair $P_{|0+\rangle}$, etc.

Proof

- However, the outcome of the measurement on our pair of electrons cannot depend on which beam-pair it belongs to, it can depend only on λ
- But this means that each outcome of the measurement on this electron pair is impossible
- The measurement has no outcome $\lightning$
- For the general proof with $P_{|\Psi\rangle}$ and $P_{|\Phi\rangle}$, one needs 2^n beams!

Conclusion

The wavefunction represents a real physical aspect of an individual system.

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Operational theory

$$
\begin{aligned}
\sigma_z |0\rangle &= +1 |0\rangle \\
\sigma_z |1\rangle &= -1 |1\rangle \\
\sigma_x |+\rangle &= +1 |+\rangle \\
\sigma_x |-\rangle &= -1 |-\rangle
\end{aligned}
$$

Quantum mechanics

$$
\begin{array}{rccc}
\textbf{Operator:} & & \mathbf{O} & \\
 & \swarrow & & \searrow \\
\textbf{Measurement:} & M_1 & \sim & M_2
\end{array}
$$

Statistical or individual

H, T, T, H, T, H, H, H, T, T, H, T, T, H ...

Is it real?

$$\langle \Psi, \mathbf{P}_x \Psi \rangle \quad = \quad p(X | M \wedge P)$$

$$\begin{aligned} |\Psi\rangle : \text{state} \quad &\longrightarrow \quad P : \text{preparation} \\ \mathbf{O} : \text{operator} \quad &\longrightarrow \quad M : \text{measurement} \\ \mathbf{P}_x : \text{spectral projection} \quad &\longrightarrow \quad X : \text{outcome} \end{aligned}$$

Features favouring Ψ-epistemic models

1. Collapse
2. Non-orthogonal pure states cannot be perfectly distinguished by a measurement
3. No-cloning theorem
4. Non-unique decomposability of mixed states into pure states

Features favouring Ψ-ontic models

- Inference
- Eigenstate-eigenvalue link
- Existing models (Bohmian, many-worlds, modal)

No locally causal Ψ-ontic models

Alice and Bob share a singlet state

By measuring M_z or M_x, Alice prepares Bob's state:

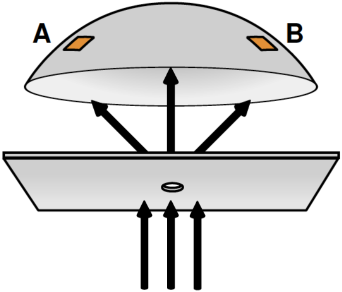
$$
\begin{aligned}
p(\lambda|P_z) &= \frac{1}{2}p(\lambda|P_{|0\rangle}) + \frac{1}{2}p(\lambda|P_{|1\rangle}) \\
p(\lambda|P_x) &= \frac{1}{2}p(\lambda|P_{|+\rangle}) + \frac{1}{2}p(\lambda|P_{|-\rangle})
\end{aligned}
$$

Due to locality: $p(\lambda|P_z) = p(\lambda|P_x)$

Taking the product:

$$
\begin{aligned}
0 \leqslant (\lambda|P_z)^2 &= p(\lambda|P_{|0\rangle})p(\lambda|P_{|+\rangle}) + p(\lambda|P_{|0\rangle})p(\lambda|P_{|-\rangle}) \\
&\quad + p(\lambda|P_{|1\rangle})p(\lambda|P_{|+\rangle}) + p(\lambda|P_{|1\rangle})p(\lambda|P_{|-\rangle})
\end{aligned}
$$

Einstein, 1927



$$p(X_A \wedge X_B | M_A \wedge M_B \wedge |\Psi\rangle) \neq p(X_A | M_A \wedge |\Psi\rangle)\, p(X_B | M_y \wedge |\Psi\rangle)$$