On the localization of the common cause

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Project

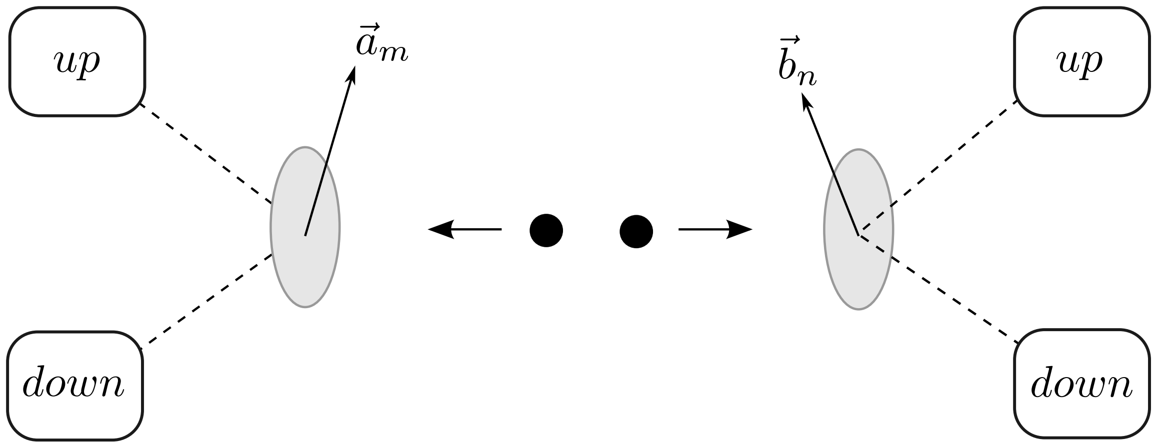
- **Question:** How the *probabilistic* and *spatiotemporal* characterizations of the *common cause* relate to one another?

Project

I. Probabilistic common causal explanation
II. What is a local physical theory?
III. Bell's local causality
IV. Localization of the common cause in a local physical theory

I. Probabilistic common causal explanation

Probabilistic common causal explanation



- **Classical probability measure space:** (Σ, p)
- **Measurement choices:** $a_m,\, b_n \in \Sigma$
- **Measurement outcomes:** $A_m,\, B_n \in \Sigma$
- **Conditional correlations:**

$$p(A_m B_n | a_m b_n) \neq p(A_m | a_m)\, p(B_n | b_n)$$

Probabilistic common causal explanation

- **Common causal explanation**: a partition $\{C_k\}$ in Σ

$$p(A_m B_n | a_m b_n C_k) = p(A_m | a_m b_n C_k)\, p(B_n | a_m b_n C_k) \qquad \text{(screening-off)}$$

$$p(A_m | a_m b_n C_k) = p(A_m | a_m C_k) \qquad \text{(locality)}$$

$$p(B_n | a_m b_n C_k) = p(B_n | b_n C_k) \qquad \text{(locality)}$$

$$p(a_m b_n C_k) = p(a_m b_n)\, p(C_k) \qquad \text{(no-conspiracy)}$$

Probabilistic common causal explanation

- **Common causal explanation**: a partition $\{C_k\}$ in Σ

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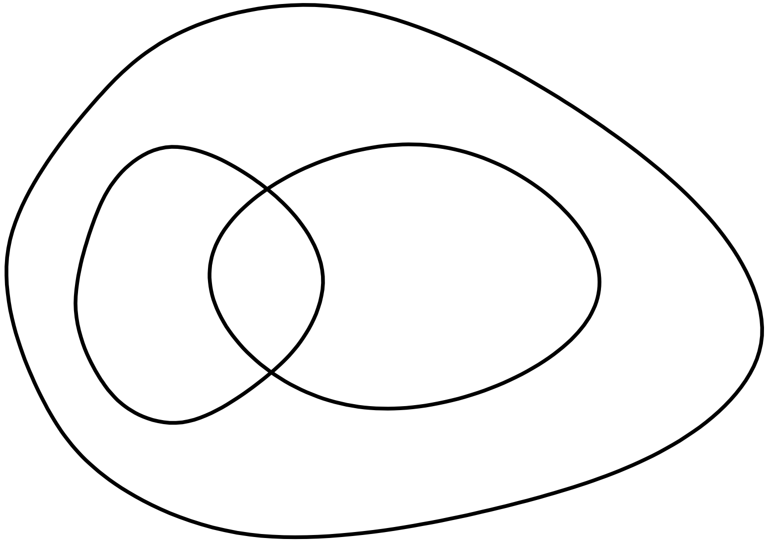
$$p(a_m b_n C_k) = p(a_m b_n)\, p(C_k) \qquad \text{(no-conspiracy)}$$

- Common causal explanation $\Longrightarrow$ Bell's inequality
- Bell's inequality is violated $\Longrightarrow$ No common causal explanation of EPR.

II. What is a local physical theory?

Local physical theory

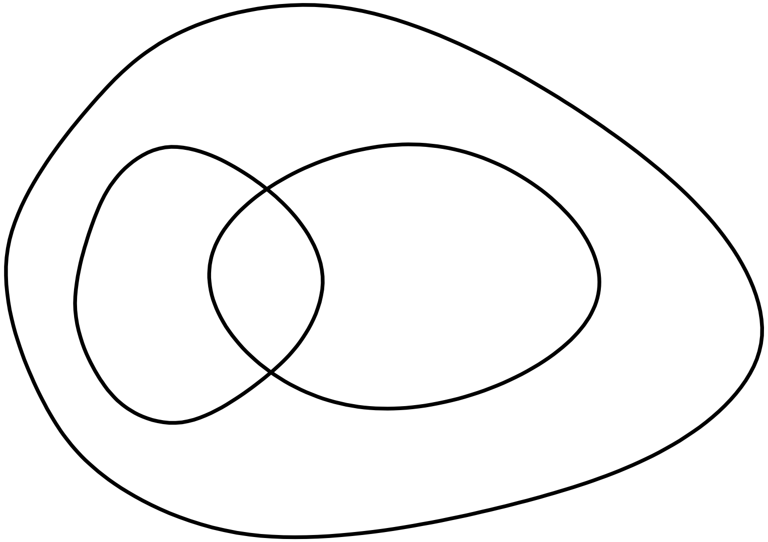
Minkowski spacetime:
Directed poset: $(\mathcal{K}, \subseteq)$



Local physical theory

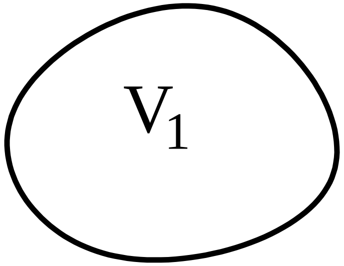
Minkowski spacetime:

Net: $\{\mathcal{N}(V), V \in \mathcal{K}\}$



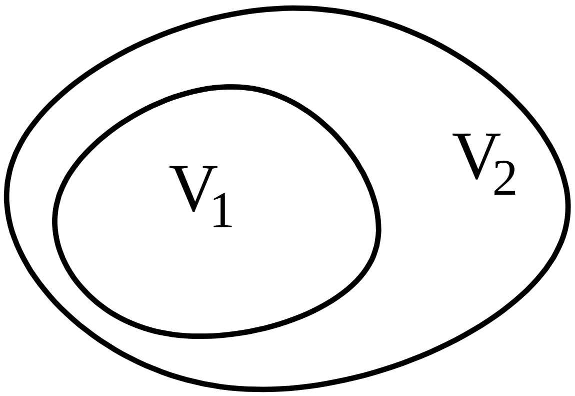
Local physical theory

Isotony:



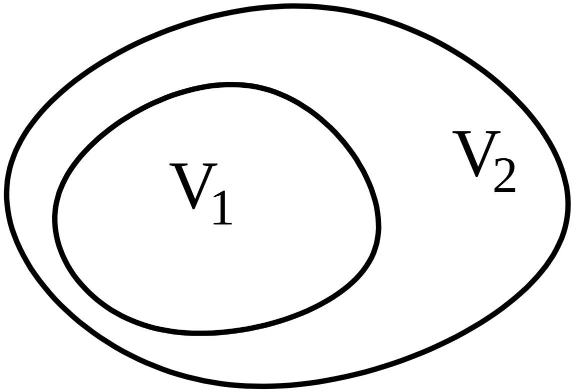
Local physical theory

Isotony: if $V_1 \subset V_2$



Local physical theory

Isotony: if $V_1 \subset V_2$, then $\mathcal{N}(V_1)$ is a subalgebra of $\mathcal{N}(V_2)$



Local physical theory

Microcausality (Einstein causality):

Local physical theory

Microcausality (Einstein causality): $[\mathcal{N}(V_1), \mathcal{N}(V_2)] = 0$

Local physical theory

Microcausality $\Longleftrightarrow$ No-signalling, parameter independence

Local physical theory

Covariance: spacetime symmetries are represented on $\mathcal{N}$

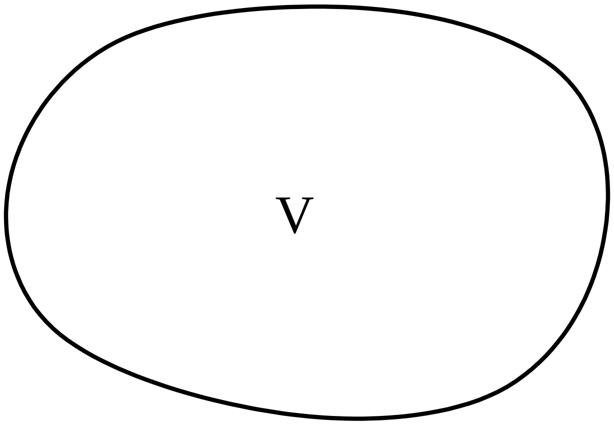
Local physical theory

Local physical theory: an isotone, microcausal and covariant net

- It embraces local classical and quantum theories

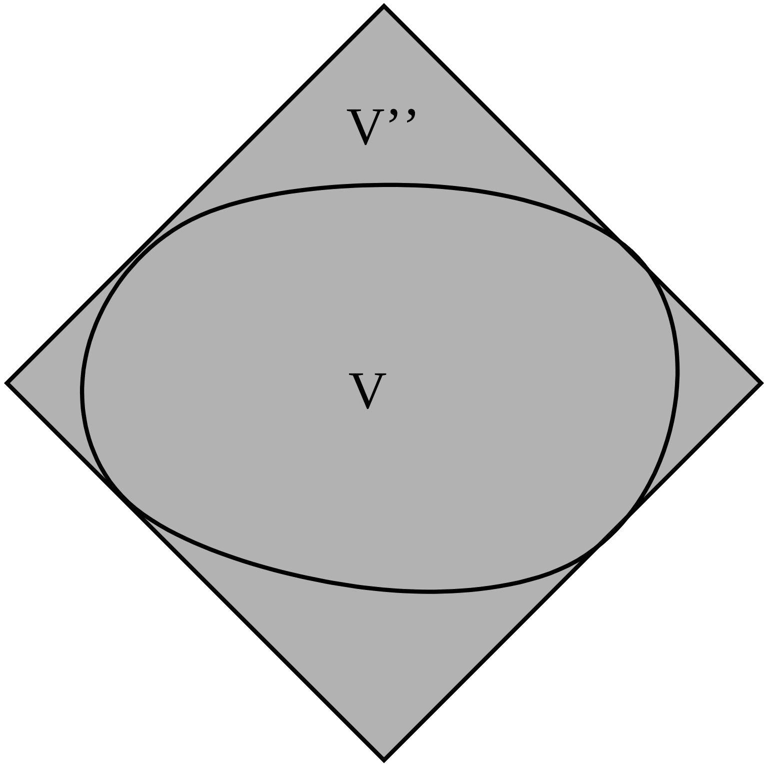
Local physical theory

Local primitive causality:



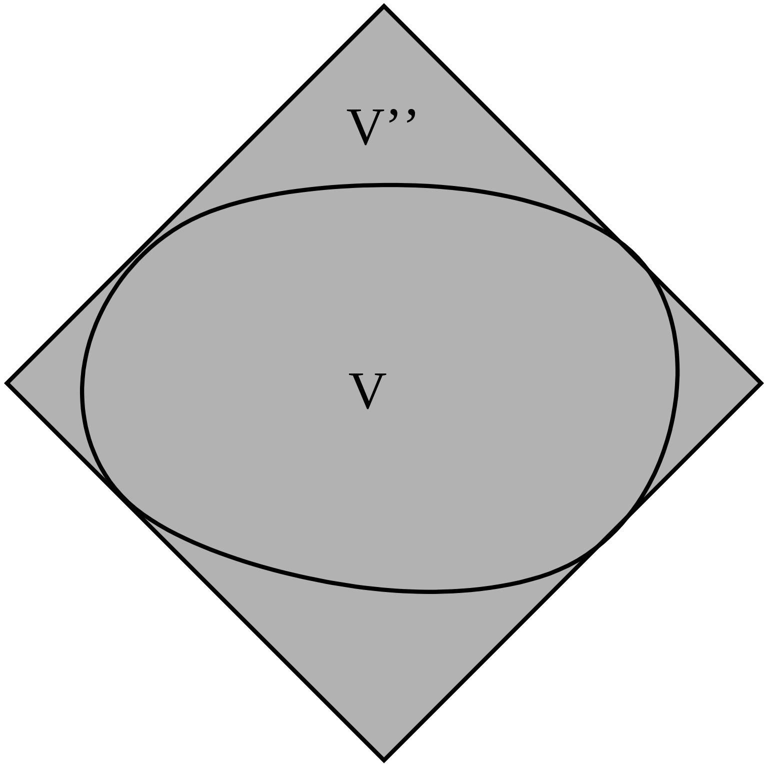
Local physical theory

Local primitive causality:



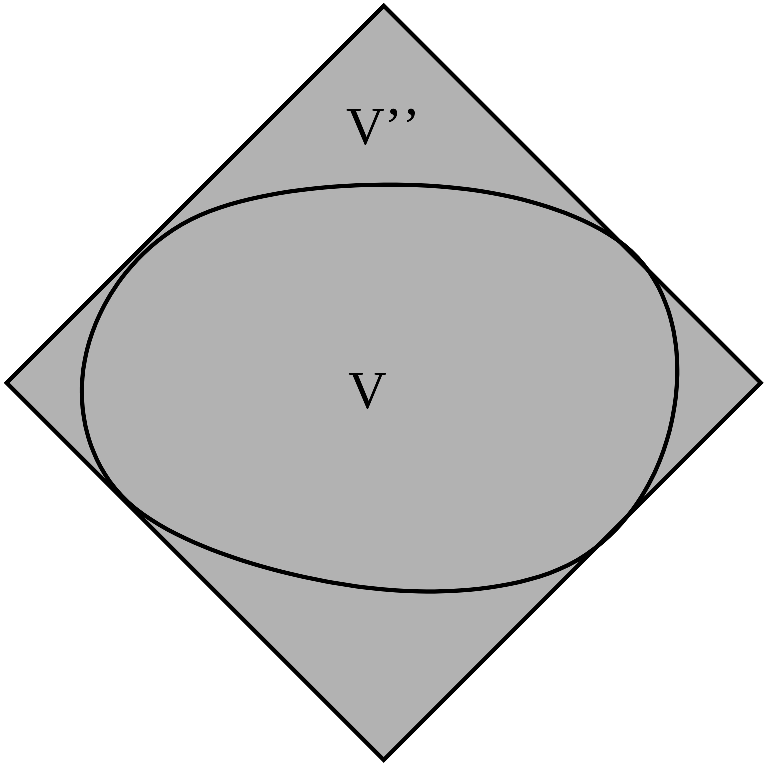
Local physical theory

Local primitive causality: $\mathcal{N}(V) = \mathcal{N}(V'')$



Local physical theory

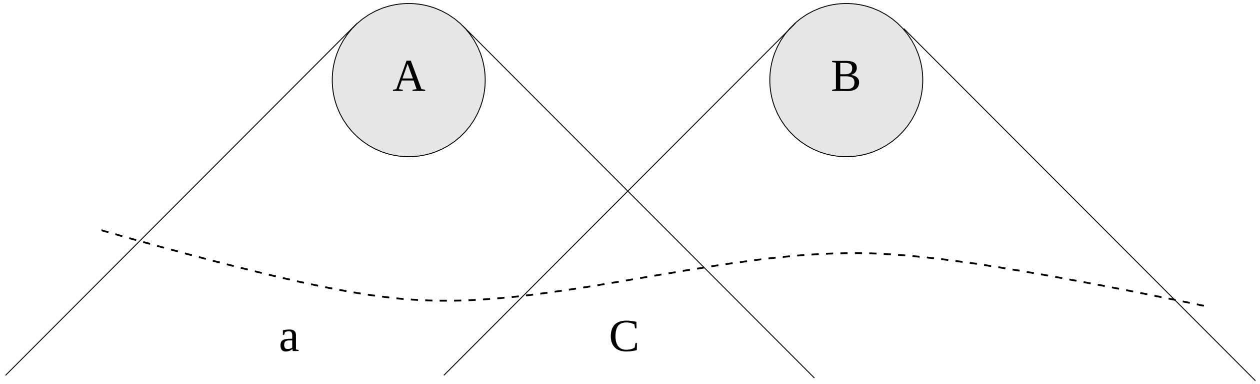
Local primitive causality $\not\Leftrightarrow$ Microcausality



III. Bell’s local causality

Bell's local causality

"Let C denote a specification of *all* beables, of some theory, belonging to the overlap of the backward light cones of spacelike regions A and B.



Let a be a specification of some beables from the remainder of the backward light cone of A, and B of some beables in the region B. Then in a *locally causal theory*

$$p(A|a, C, B) = p(A|a, C) \tag{1}$$

whenever both probabilities are given by the theory." (Bell, 1987, p. 54)

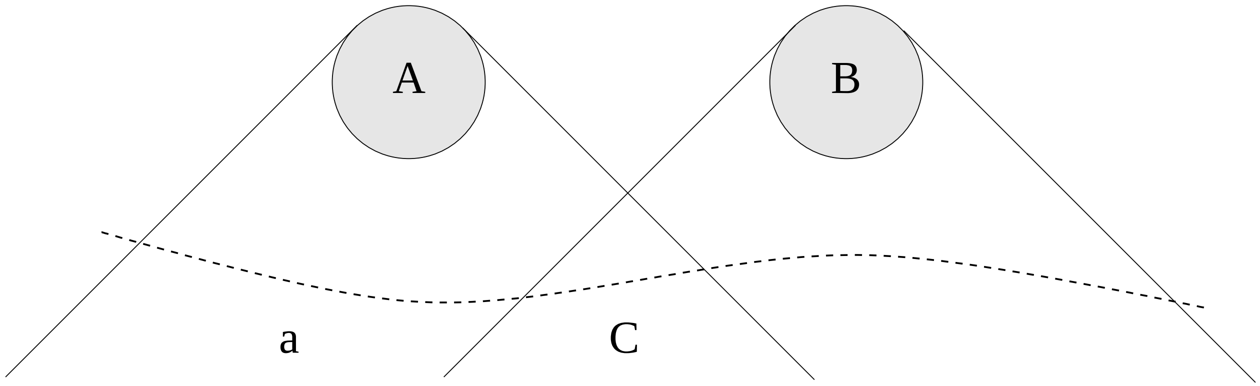
Bell's local causality

Remark:

- Local primitive causality is a dependence relation; local causality is an independence relation.
- Local primitive causality does not rely on the notion of state, it is a property of the net exclusively; local causality does depend on the state.

Bell's local causality

"Let C denote a specification of *all* beables, of some theory, belonging to the overlap of the backward light cones of spacelike regions A and B.



Let a be a specification of some beables from the remainder of the backward light cone of A, and B of some beables in the region B. Then in a *locally causal theory*

$$p(A|a, C, B) = p(A|a, C) \tag{2}$$

whenever both probabilities are given by the theory." (Bell, 1987, p. 54)

Bell's local causality

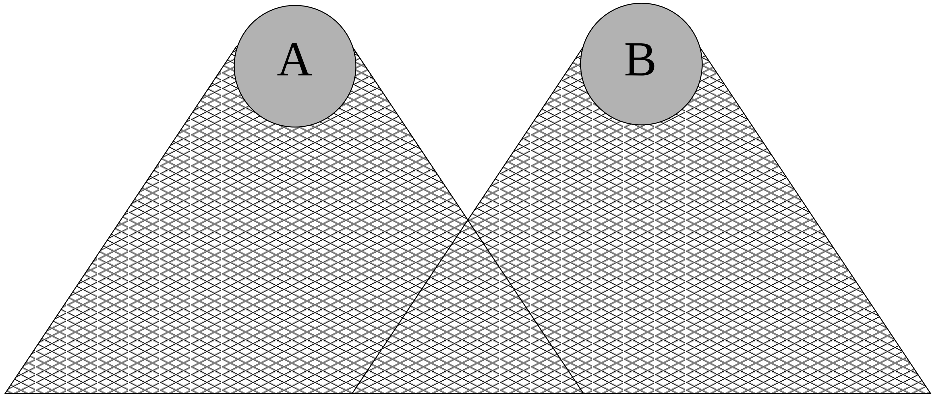
Two assumptions: the common cause is

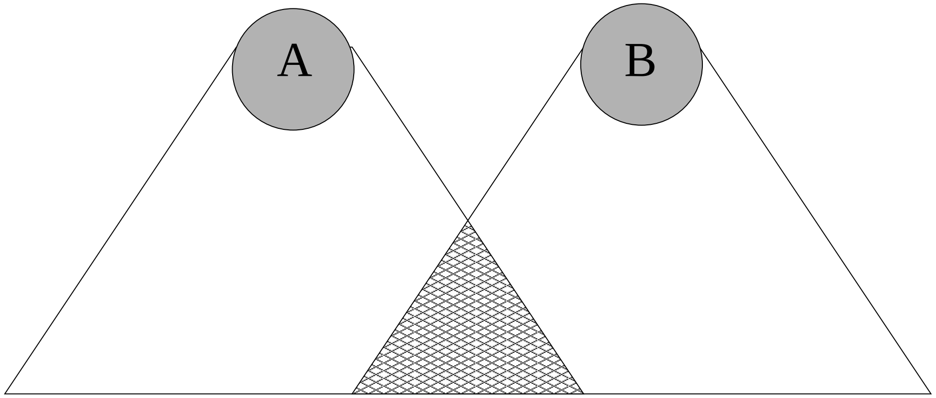
(i) "the specification of *all* beables of the theory", and

(ii) it is located in the "overlap of the backward light cones".

Bell's local causality

Two pasts:





Weak past: $I_-(V_A) \cup I_-(V_B)$

Strong past: $I_-(V_A) \cap I_-(V_B)$

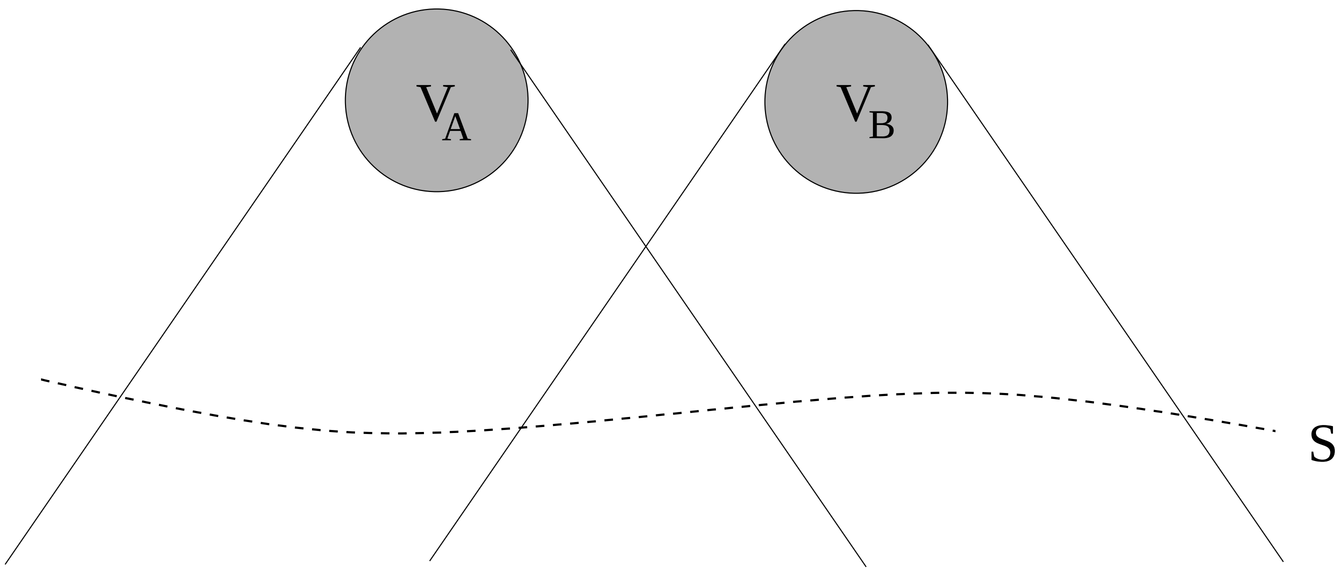
Bell's local causality

Two assumptions: C_k is

(i) is an atom of the appropriate local algebra,

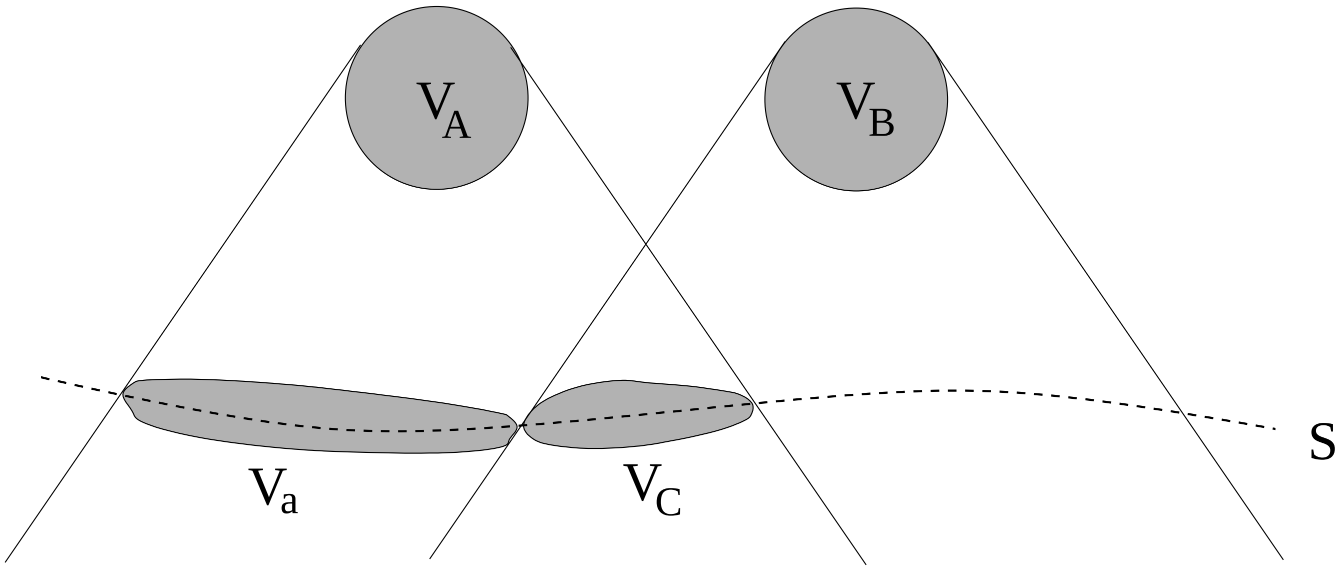
(ii) and it is located in the strong past.

Bell's local causality



Definition. A local physical theory represented by a net $\{\mathcal{N}(V), V \in \mathcal{K}\}$ is called *locally causal*, if for any pair $A \in \mathcal{N}(V_A)$ and $B \in \mathcal{N}(V_B)$ of projections supported in spacelike separated regions $V_A, V_B \in \mathcal{K}$ and for every locally faithful state ϕ establishing a correlation between A and B and for every Cauchy surface $\mathcal{S}$ (lying past to V_A and V_B), the following is true:

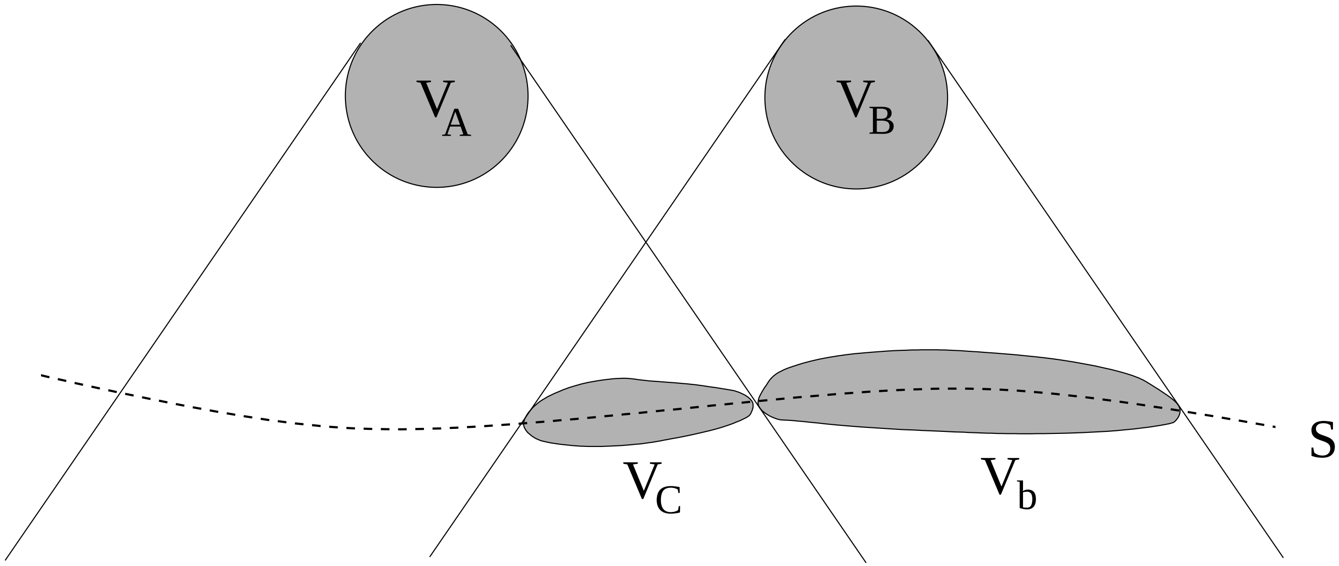
Bell's local causality



For any $a_m \in \mathcal{N}(V_a)$ and *atomic* event $C_k \in \mathcal{N}(V_C)$

$$p(A_m|a_m C_k B_n) \;=\; p(A_m|a_m C_k)$$

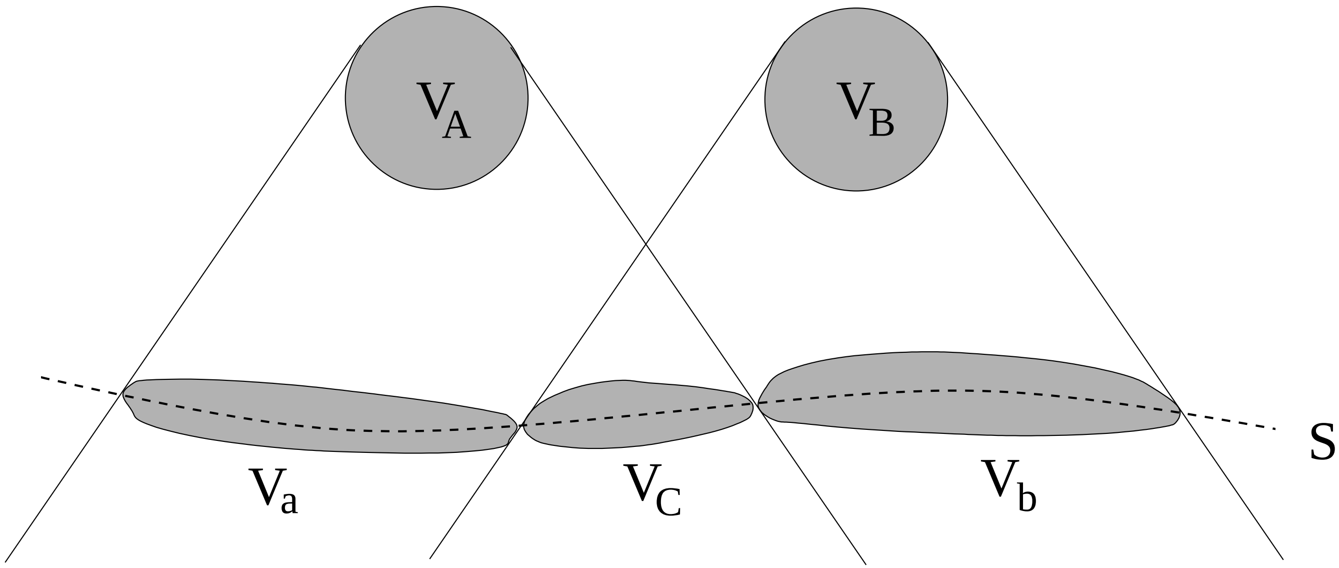
Bell's local causality



For any $b_n \in \mathcal{N}(V_b)$ and *atomic* event $C_k \in \mathcal{N}(V_C)$

$$p(B_n|A_m C_k b_n) \;=\; p(B_n|b_n C_k)$$

Bell's local causality



For any $a_m \in \mathcal{N}(V_a)$, $b_n \in \mathcal{N}(V_b)$ and *atomic* event $C_k \in \mathcal{N}(V_C)$

$$p(A_m|a_m C_k B_n) = p(A_m|a_m C_k) \qquad (3)$$

$$p(B_n|A_m C_k b_n) = p(B_n|b_n C_k) \qquad (4)$$

$$p(A_m|a_m C_k b_n) = p(A_m|a_m C_k) \qquad (5)$$

$$p(B_n|a_m C_k b_n) = p(B_n|b_n C_k) \qquad (6)$$

Bell's local causality

- (3)-(6) are just screening-off and locality!

$$
\begin{aligned}
p(A_m B_n | a_m b_n C_k) &= p(A_m | a_m b_n C_k)\, p(B_n | a_m b_n C_k) \\
p(A_m | a_m b_n C_k) &= p(A_m | a_m C_k) \\
p(B_n | a_m b_n C_k) &= p(B_n | b_n C_k)
\end{aligned}
$$

- **But.** No-conspiracy *cannot* be 'derived' from Bell's notion of local causality, it is an independent assumption!

$$
p(a_m b_n C_k) = p(a_m b_n)\, p(C_k)
$$

Bell's local causality

Two assumptions: C_k is

(i) is an atom of the appropriate local algebra,

(ii) and it is located in the strong past.

Bell's local causality

Two assumptions: C_k is

(i) is an atom of the appropriate local algebra,

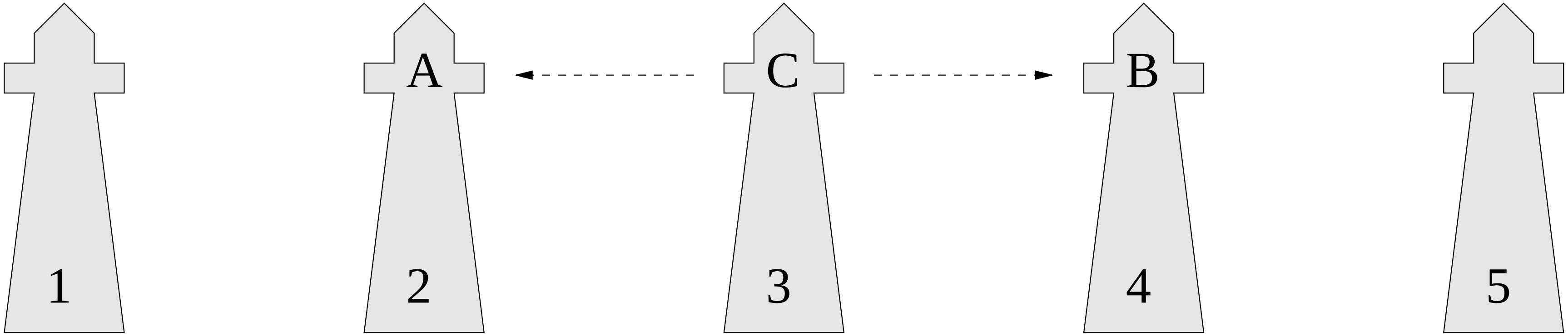
(ii) and it is located in the strong past.

Question: How non-atomic or weak common causes relate to Bell's notion of local causality?

IV. Localization of the common cause

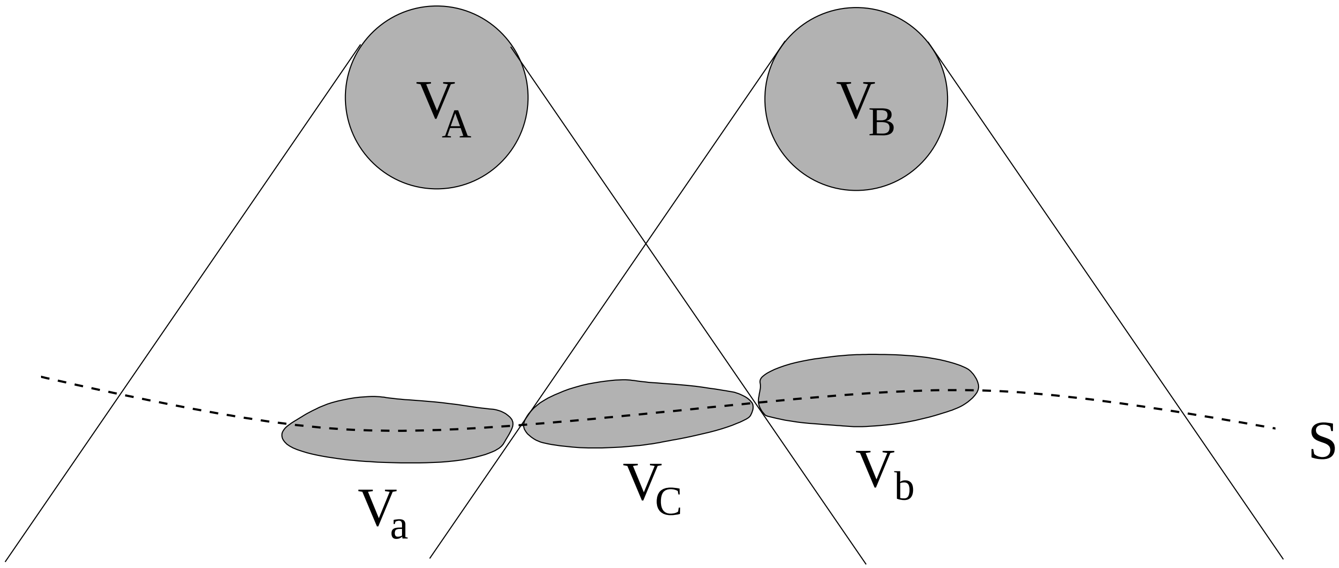
Localization of the common cause

Strong common cause: $\{C_k^S\}$



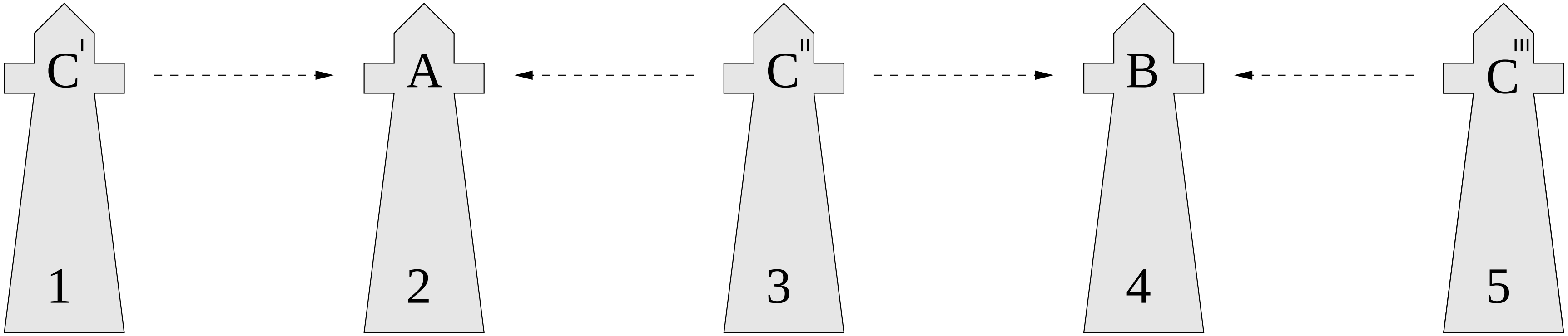
Localization of the common cause

Strong common cause: $\{C_k^S\}$



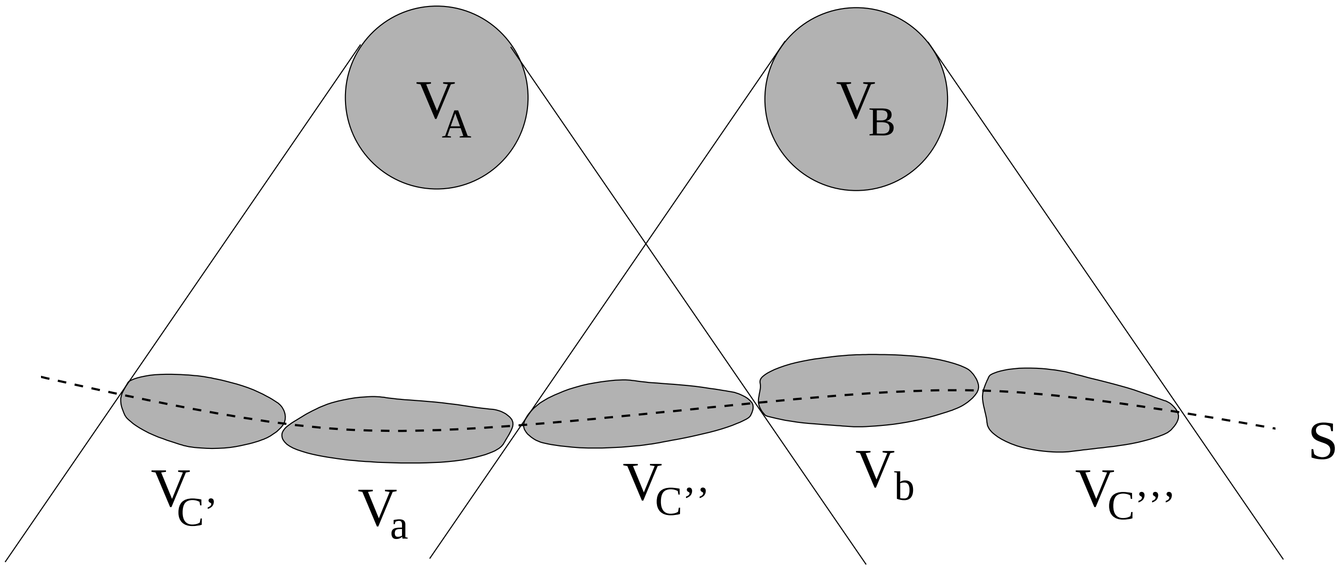
Localization of the common cause

Weak common cause: $\{C^W_{jkl} := C'_j C''_k C'''_l\}$



Localization of the common cause

Weak common cause: $\{C^W_{jkl} := C'_j C''_k C'''_l\}$



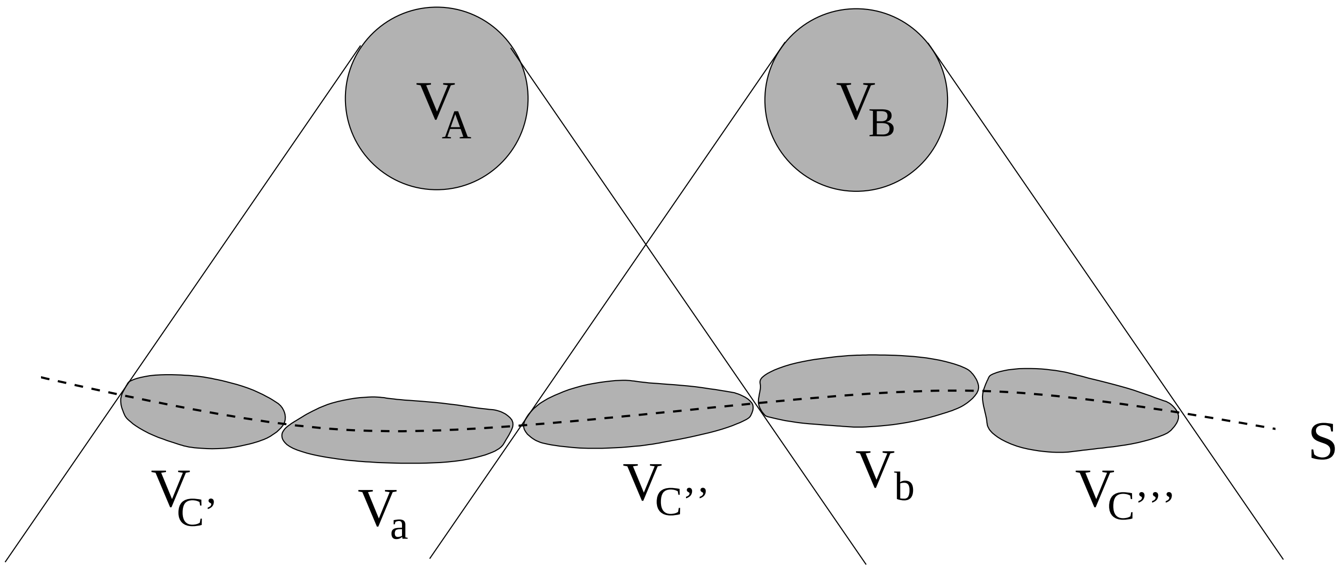
Localization of the common cause

Claim 1. The probabilistic characterization of the common cause cannot be justified by Bell's local causality, if $\{C_k^S\}$ is a non-atomic partition of $\mathcal{N}(V_C)$.

Localization of the common cause

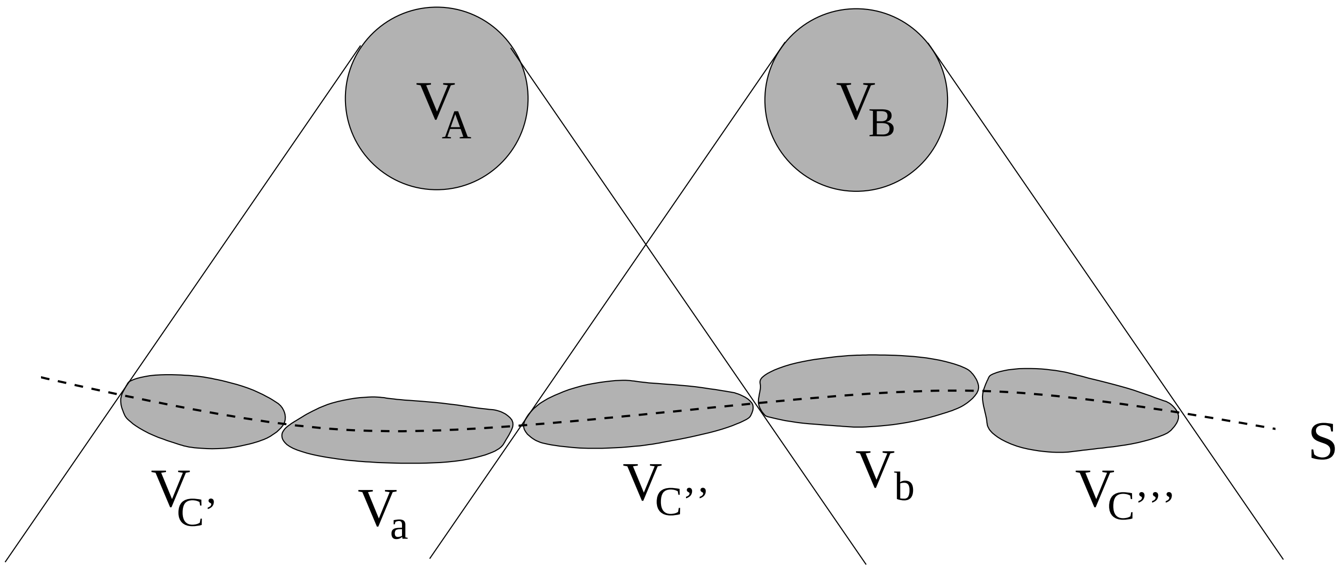
Claim 2. The probabilistic characterization of the common cause can be justified by Bell's local causality, if $\{C^W_{jkl} := C'_j C''_k C'''_l\}$ is a weak common cause where and $\{C''_j\}$ is an atomic partition of $\mathcal{N}(V_{C''})$.

Localization of the common cause



Question: What is the relation between the strong common cause and the weak common cause in the classical case?

Localization of the common cause



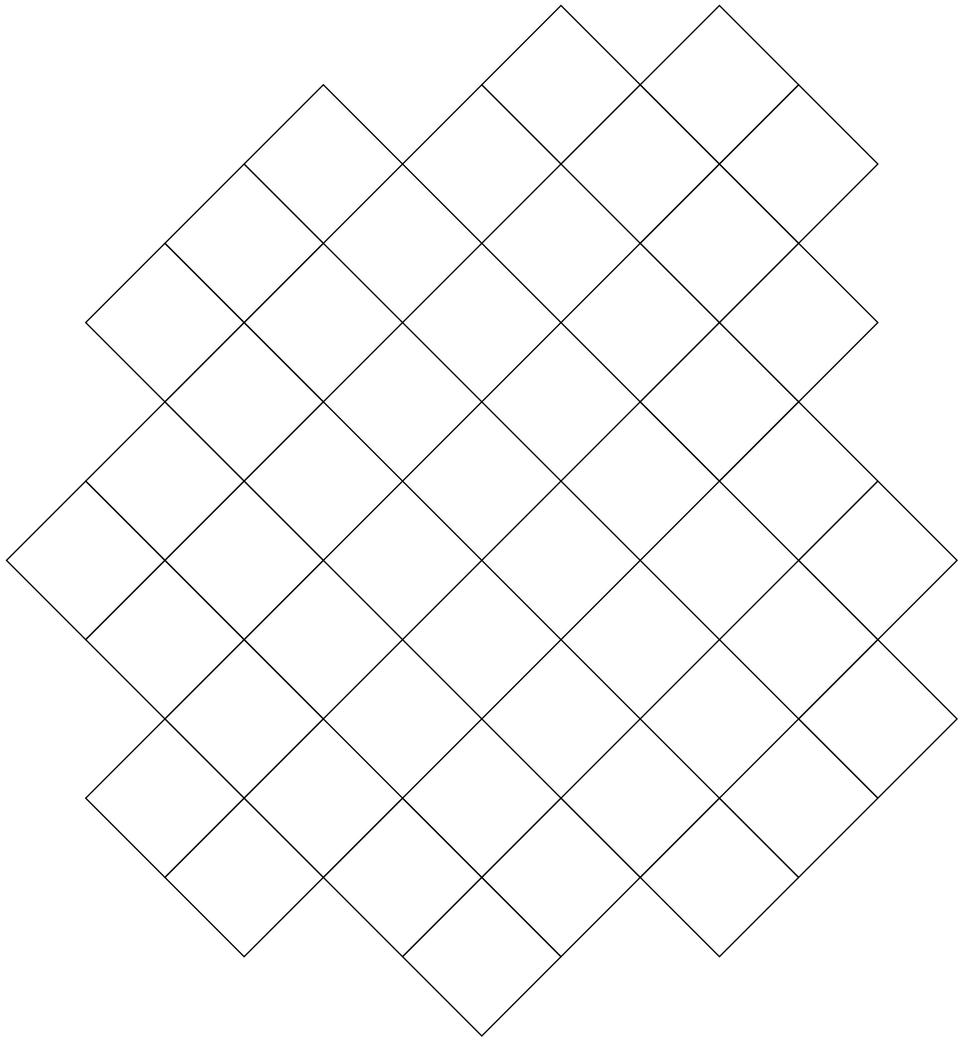
Proposition 1. Let $\{C^W_{jkl} := C'_j C''_k C'''_l\}$ be a *weak* common cause of the correlation (A, B) in the classical probability space (Σ, p). Then if local causality and independence

$$p(C'_j C''_k C'''_l) = p(C'_j)p(C''_k)p(C'''_l)$$

holds, then $\{C''_k\}$ is a *strong* common cause of the correlation.

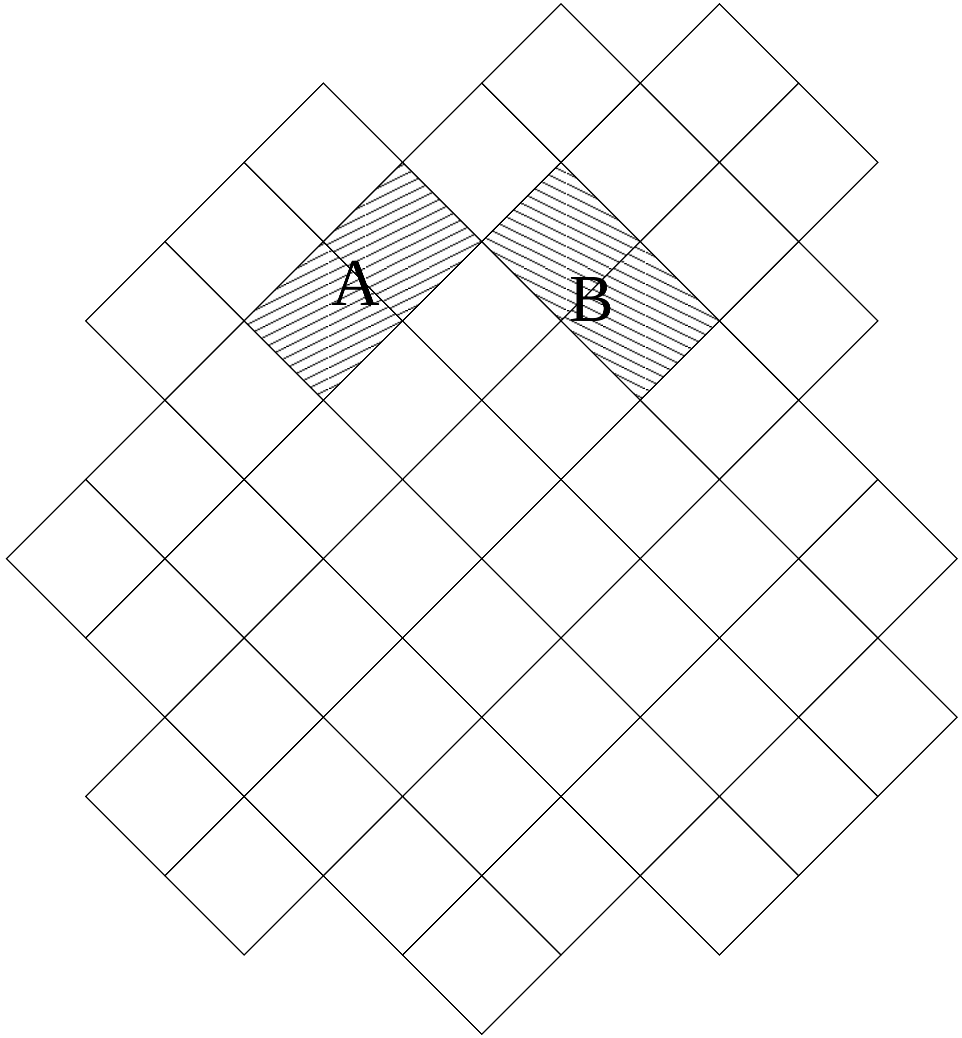
Localization of the common cause in AQFT

Two dimensional discrete Minkowski spacetime:



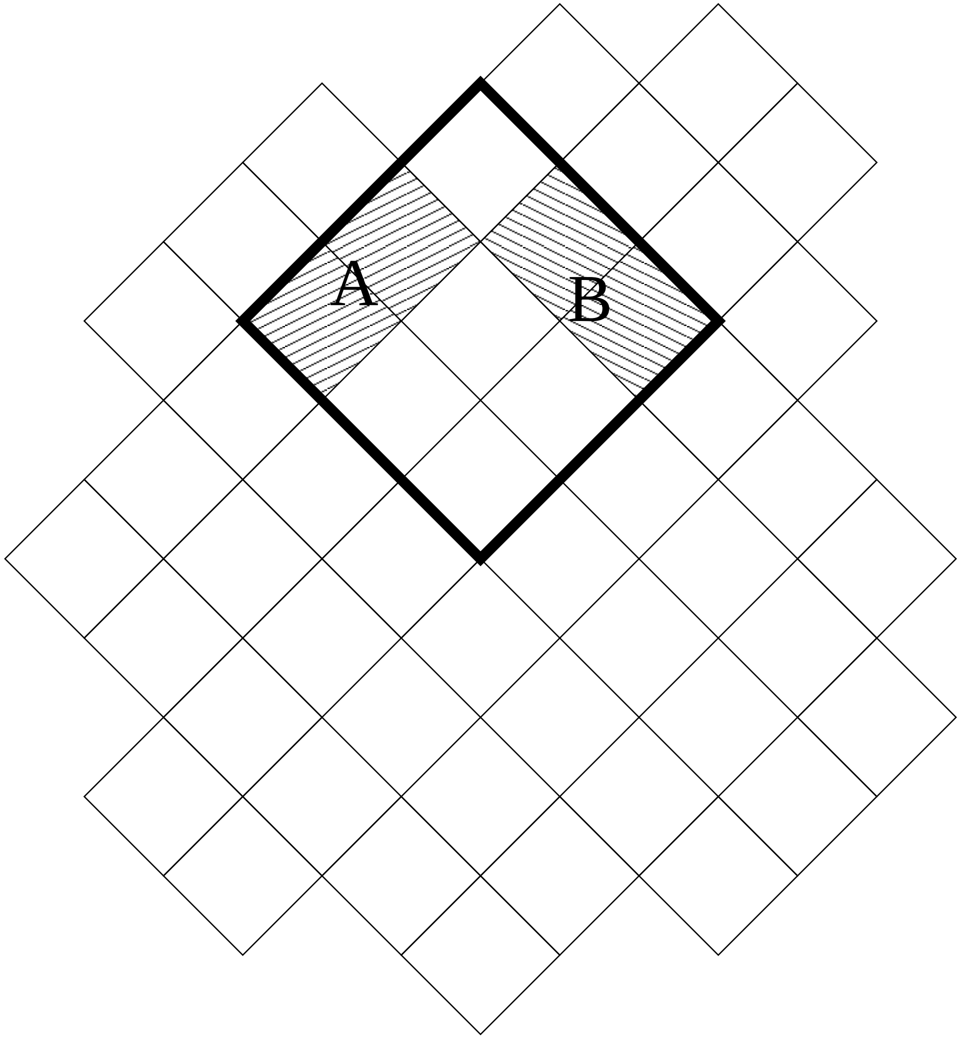
Localization of the common cause in AQFT

Two events:



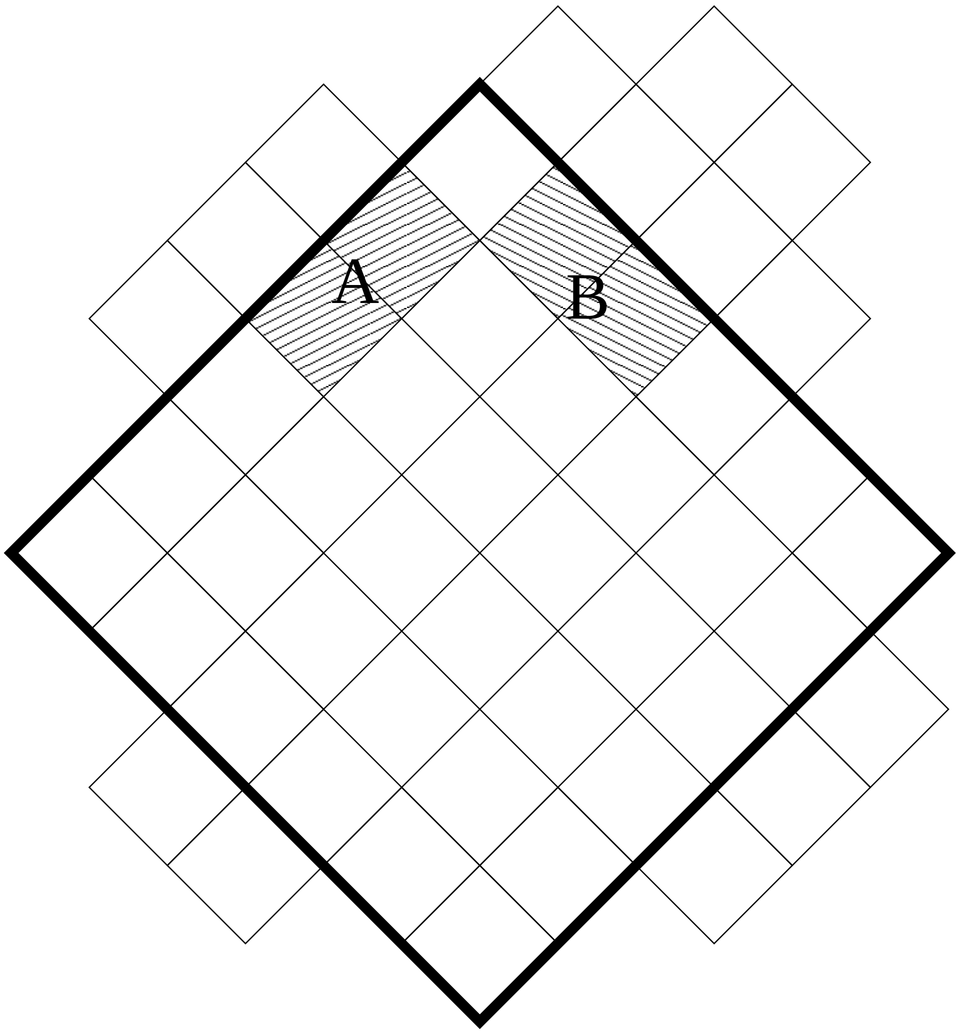
Localization of the common cause in AQFT

Correlation:



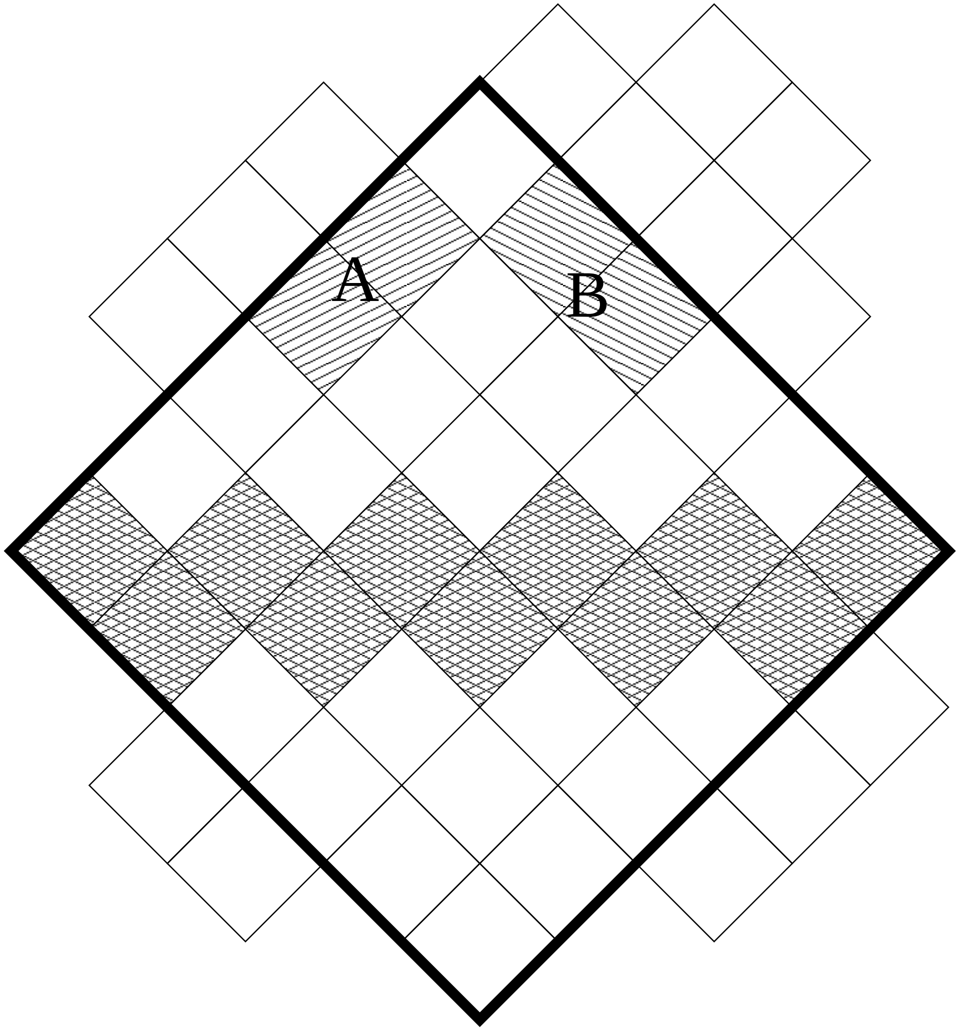
Localization of the common cause in AQFT

By isotony:



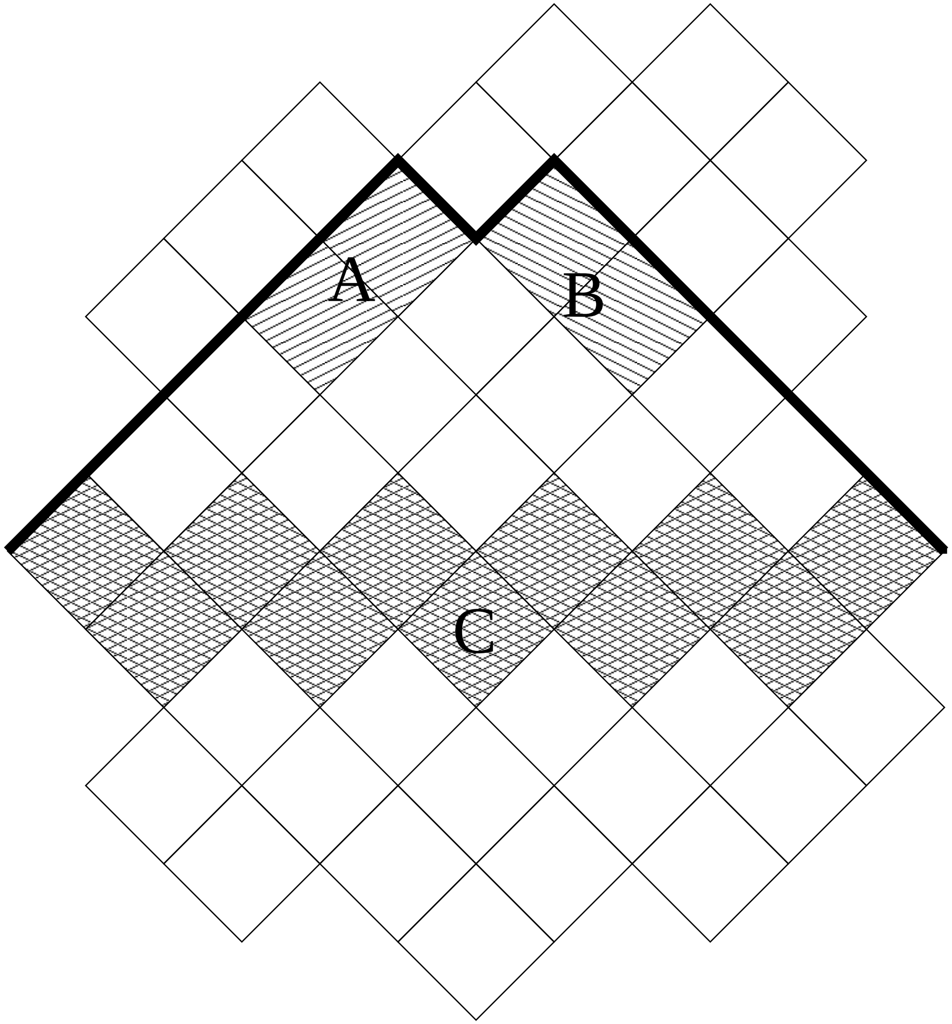
Localization of the common cause in AQFT

By local primitive causality:



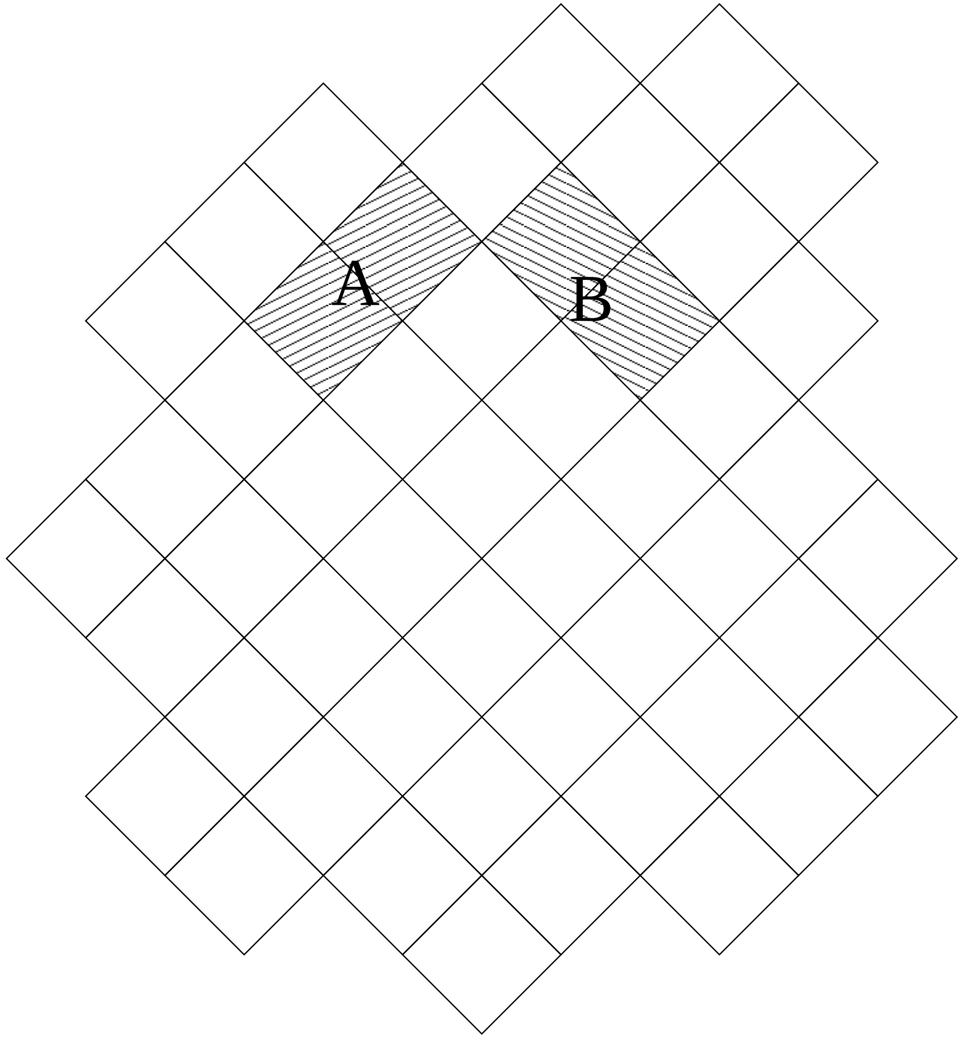
Localization of the common cause in AQFT

Weak common cause:



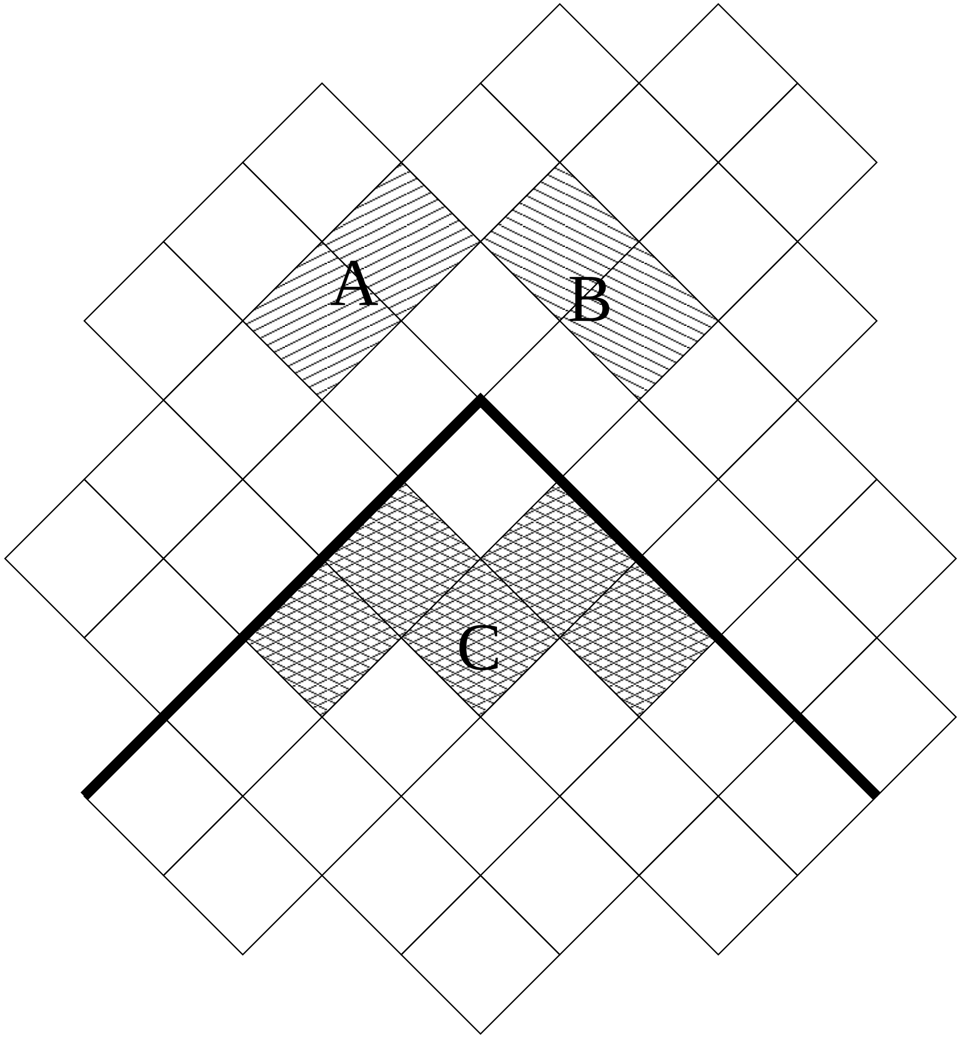
Localization of the common cause in AQFT

Why not a strong common cause?



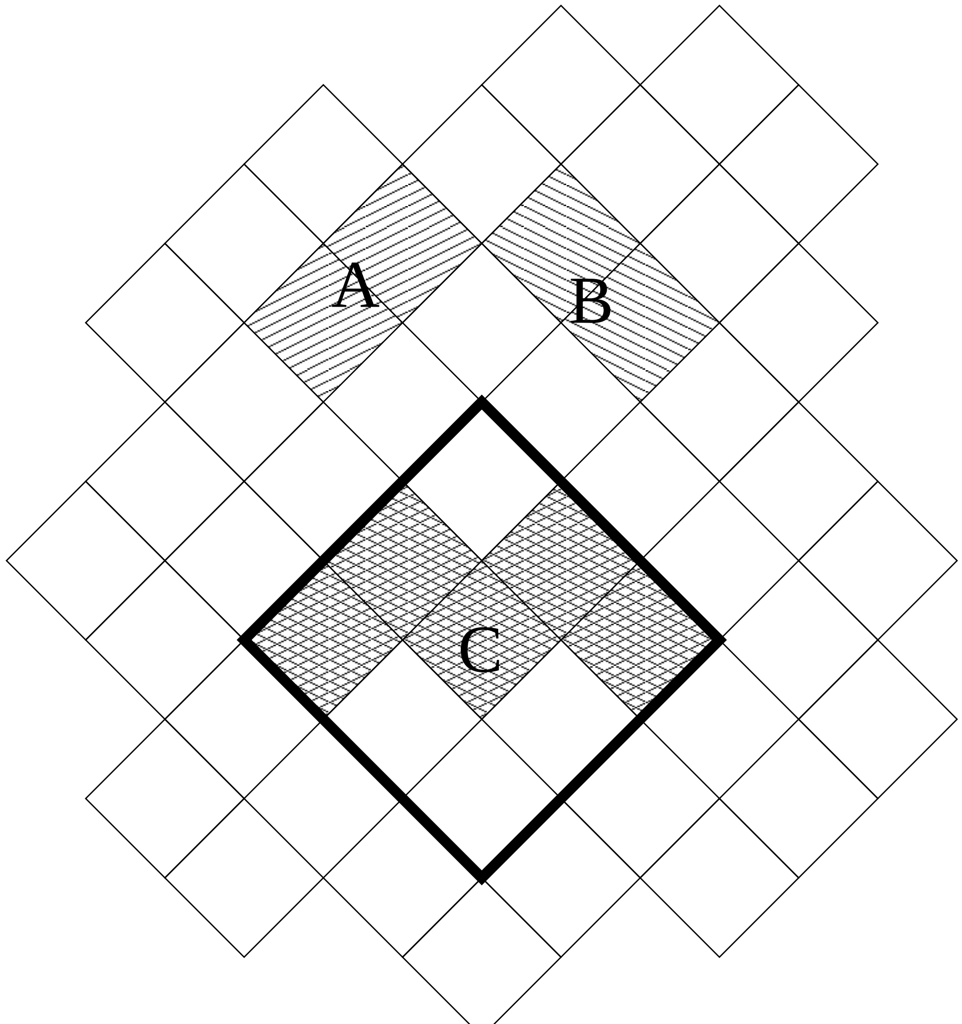
Localization of the common cause in AQFT

Strong common cause:



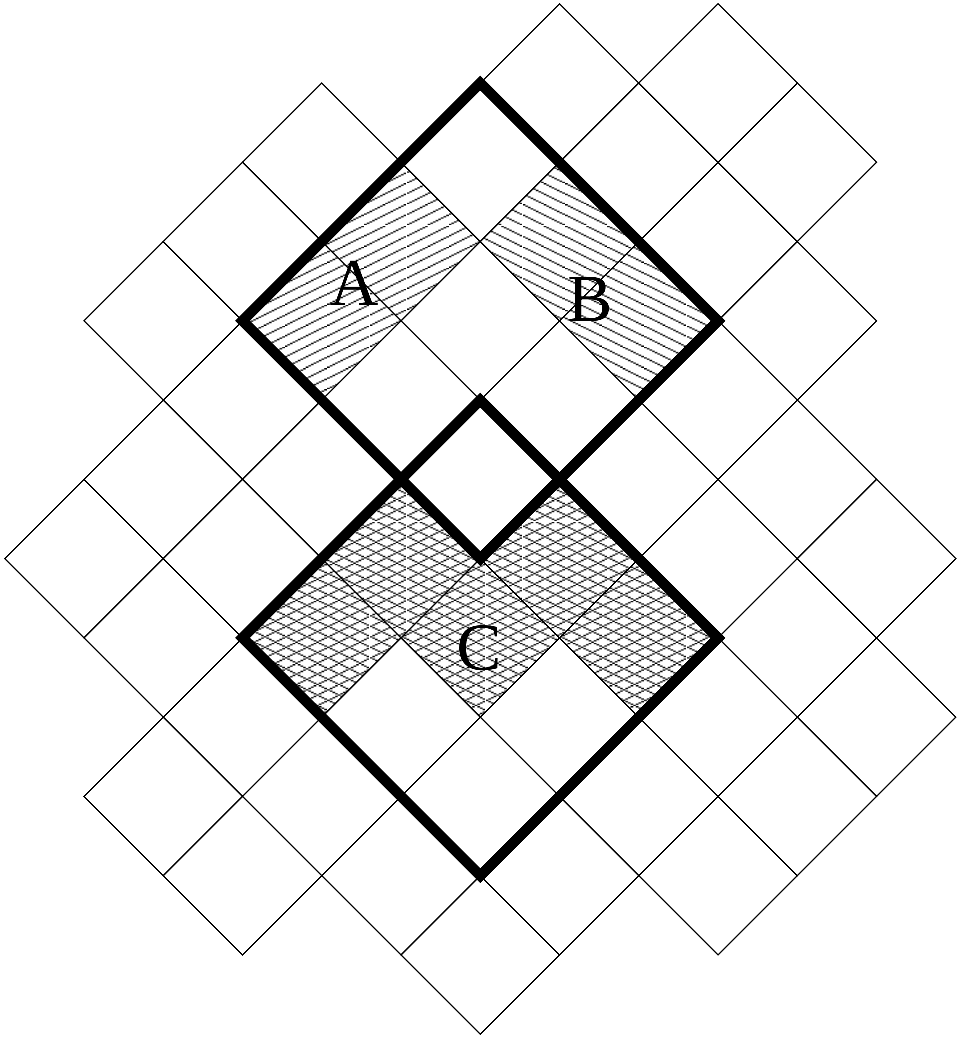
Localization of the common cause in AQFT

By local primitive causality:



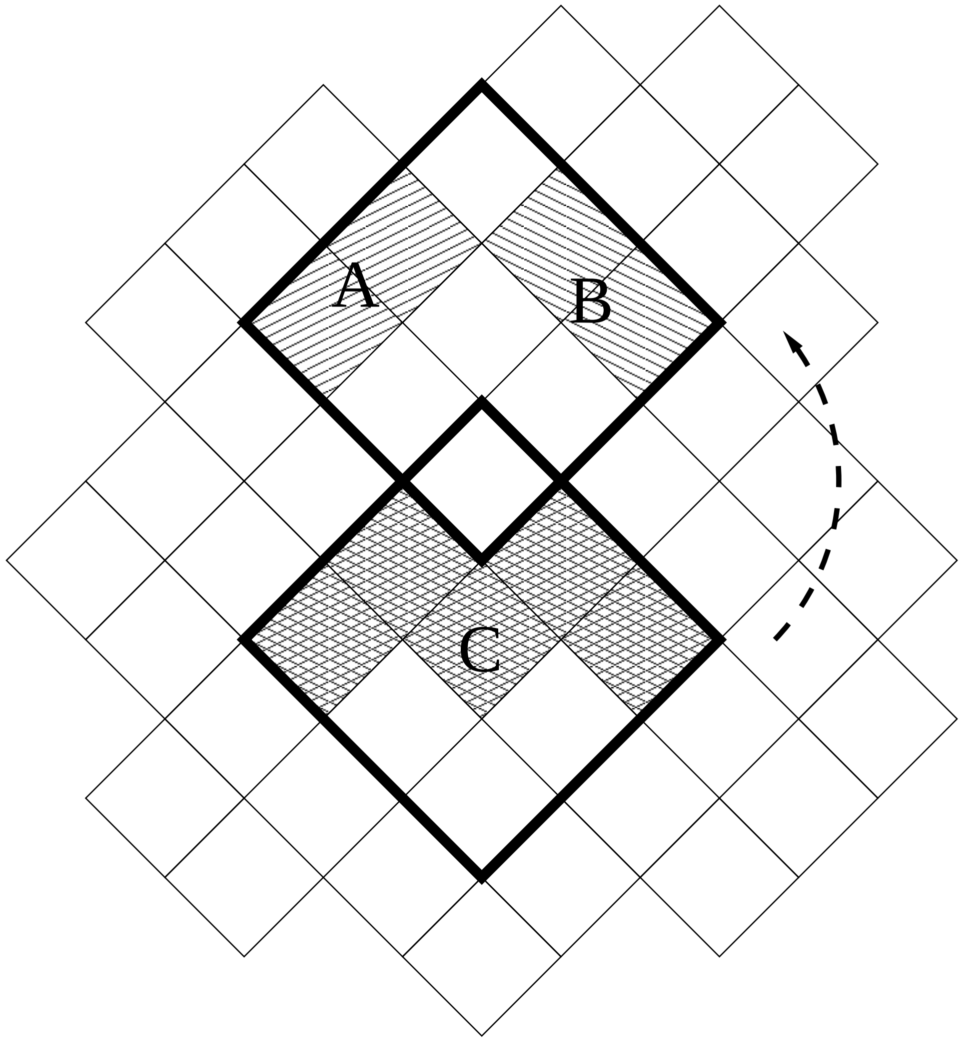
Localization of the common cause in AQFT

By isotony?



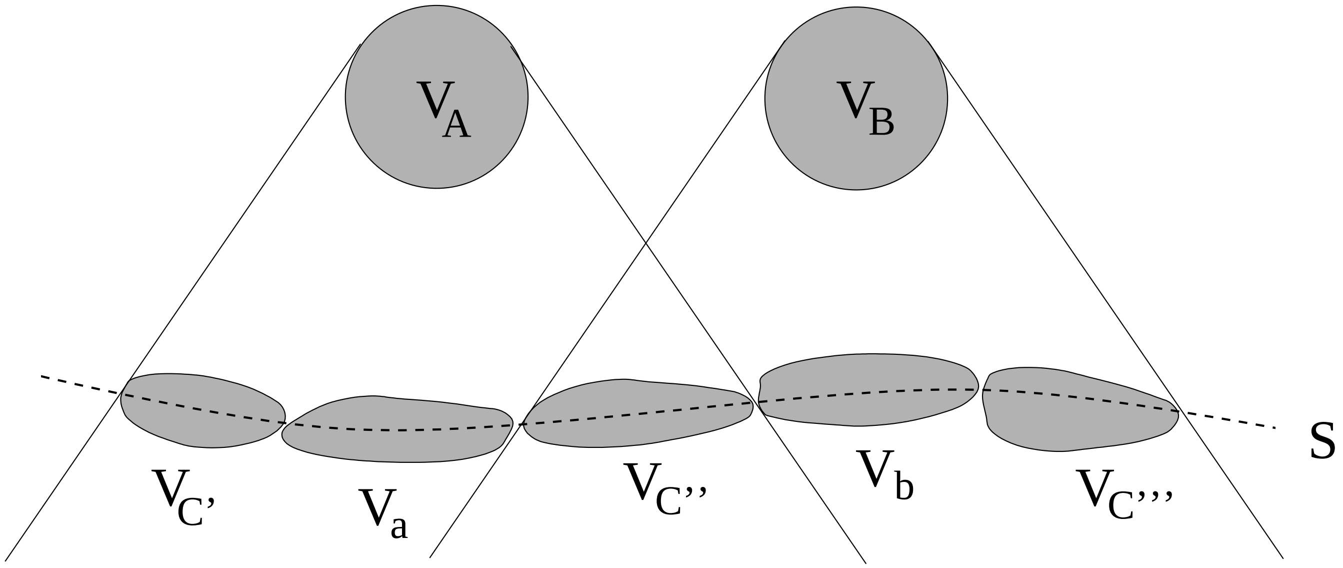
Localization of the common cause in AQFT

Dynamics is needed!



Conjecture

Question: Does Proposition 1 have anything to do with the fact that weak common causes are naturally arising in algebraic quantum field theory?



Conclusion

The probabilistic characterization of a

- strong, atomic common causes can,
- strong, non-atomic common causes cannot,
- weak, atomic common causes can,
- no-conspiracy cannot

be justified by Bell's local causality.

References

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Reichenbachian common cause

- **Classical probability space:** (Σ, p)
- **Positive correlation:** A, $B \in \Sigma$

$$p(AB) > p(A)\,p(B)$$

- **Reichenbachian common cause:** $C \in \Sigma$

$$\begin{aligned}
p(AB|C) &= p(A|C)p(B|C) \\
p(AB|\overline{C}) &= p(A|\overline{C})p(B|\overline{C}) \\
p(A|C) &> p(A|\overline{C}) \\
p(B|C) &> p(B|\overline{C})
\end{aligned}$$

Common cause system

- **Correlation:** A, $B \in \Sigma$

$$p(AB) \neq p(A)p(B)$$

- **Common cause system** (CCS): partition $\{C_k\}_{k \in K}$ in Σ

$$p(AB|C_k) \; = \; p(A|C_k)\, p(B|C_k)$$

- **Common cause:** CCS of size 2.

Non-classical common cause system

- **Non-classical probability space:** $(\mathcal{P}(\mathcal{N}), \phi)$
- **Correlation:** A, $B \in \mathcal{P}(\mathcal{N})$

$$\phi(AB) \neq \phi(A)\phi(B)$$

Non-classical common cause system

- **(Non-classical) CCS:** partition $\{C_k\}_{k\in K}$ in $\mathcal{P}(\mathcal{N})$

$$(\phi \circ E_c)(AB|C_k) \;=\; (\phi \circ E_c)(A|C_k)\,(\phi \circ E_c)(B|C_k)$$

- **Conditional expectation:**

$$E_c : \mathcal{N} \to \mathcal{C},\;\; A \mapsto \sum_{k\in K} C_k A C_k$$

- **Commuting / Noncommuting CCS:** $\{C_k\}_{k\in K}$ is commuting / not commuting with A and B

Common Cause Principles

Common Cause Principles: for any pair $A \in \mathcal{A}(V_A)$ and $B \in \mathcal{A}(V_B)$ of projections supported in spacelike separated regions and for every locally faithful state ϕ, there exists a *nontrivial* commuting/noncommuting common cause system $\{C_k\}_{k\in K} \subset \mathcal{A}(V_C)$ of the correlation such that V_C is in $P^W(V_A, V_B)$, $P^C(V_A, V_B)$ or $P^S(V_A, V_B)$.

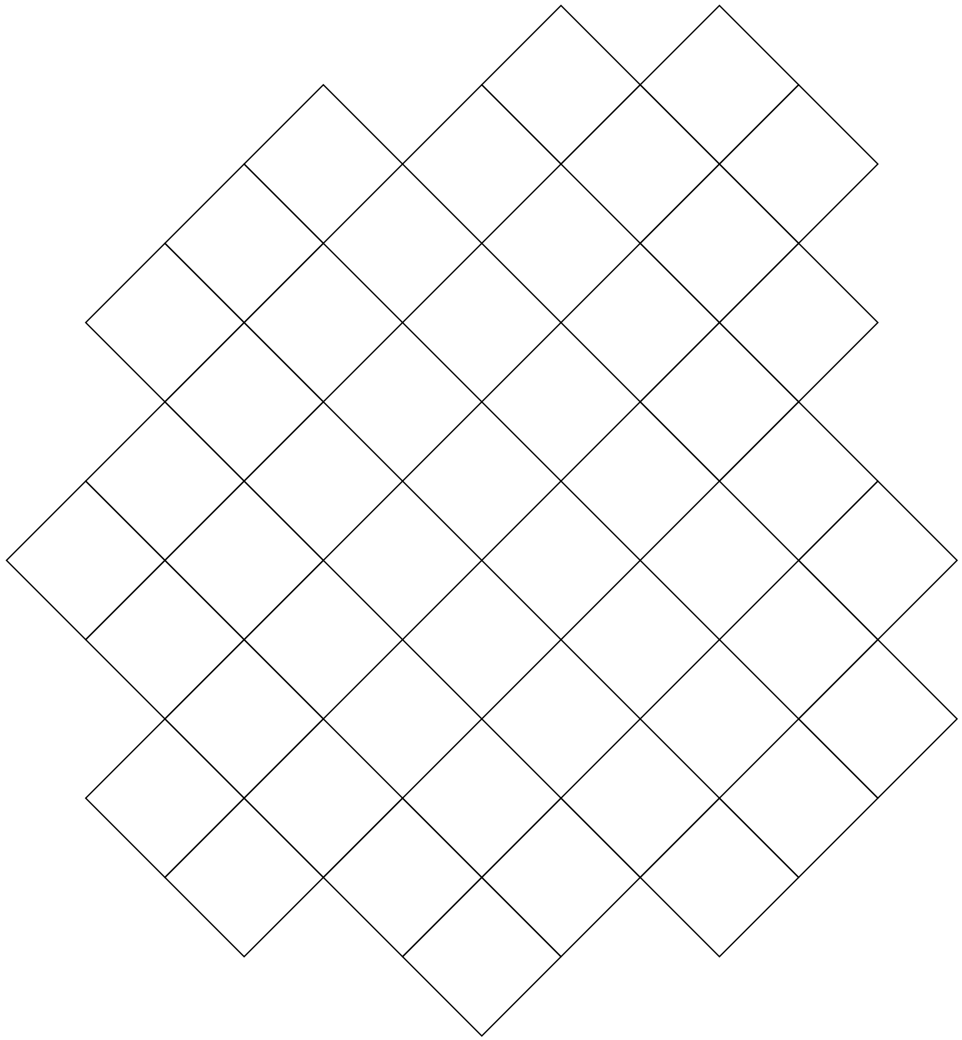
Common Cause Principles

- **Weak Commutative Common Cause Principle:** holds in Poincaré covariant AQFT (Rédei and Summers, 2002)
- **Weak Commutative Common Cause Principle:** does *not* hold in lattice AQFT (Hofer-Szabó and Vecsernyés, 2012)
- **Weak Noncommutative Common Cause Principle:** does hold in UHF-type AQFT (Hofer-Szabó and Vecsernyés, 2013)
- **(Strong) Common Cause Principles:** does *not* hold in AQFT (conjecture)

IV. Classical nets

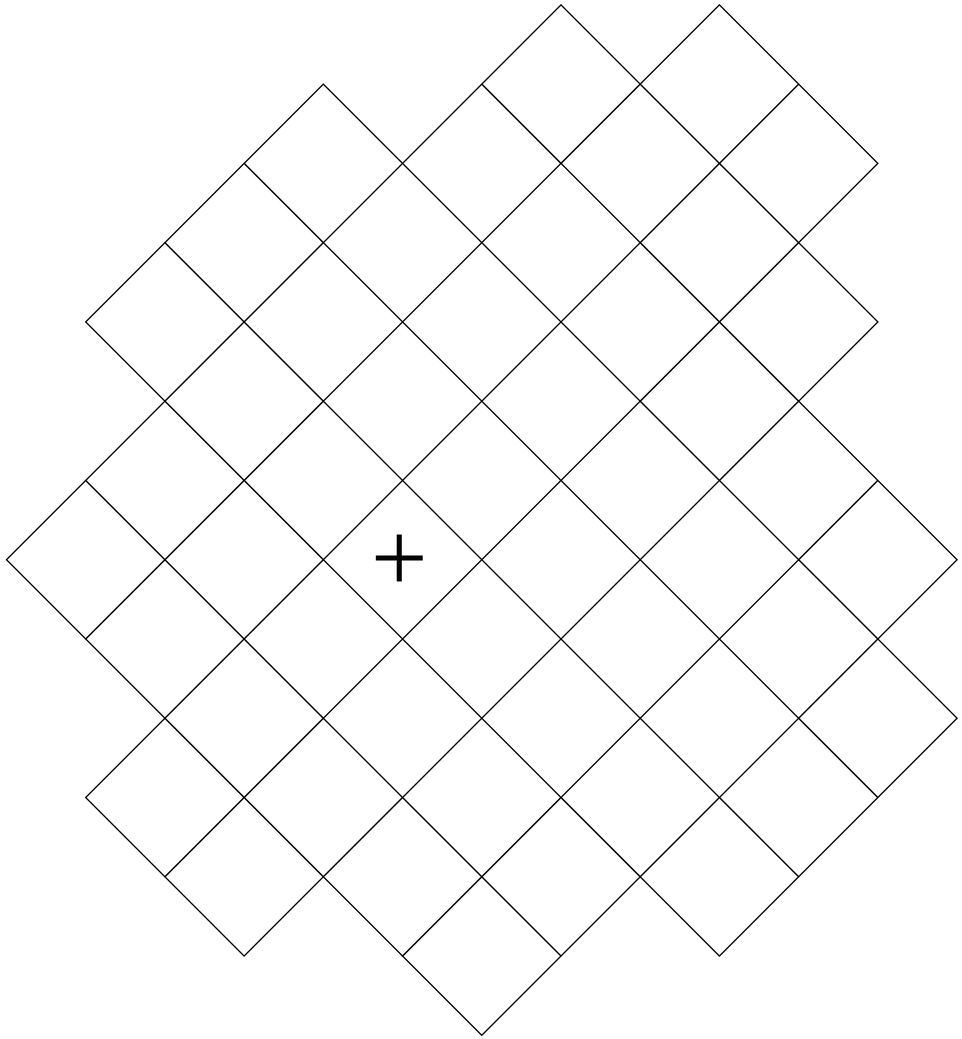
Classical nets

Two dimensional discrete Minkowski spacetime:



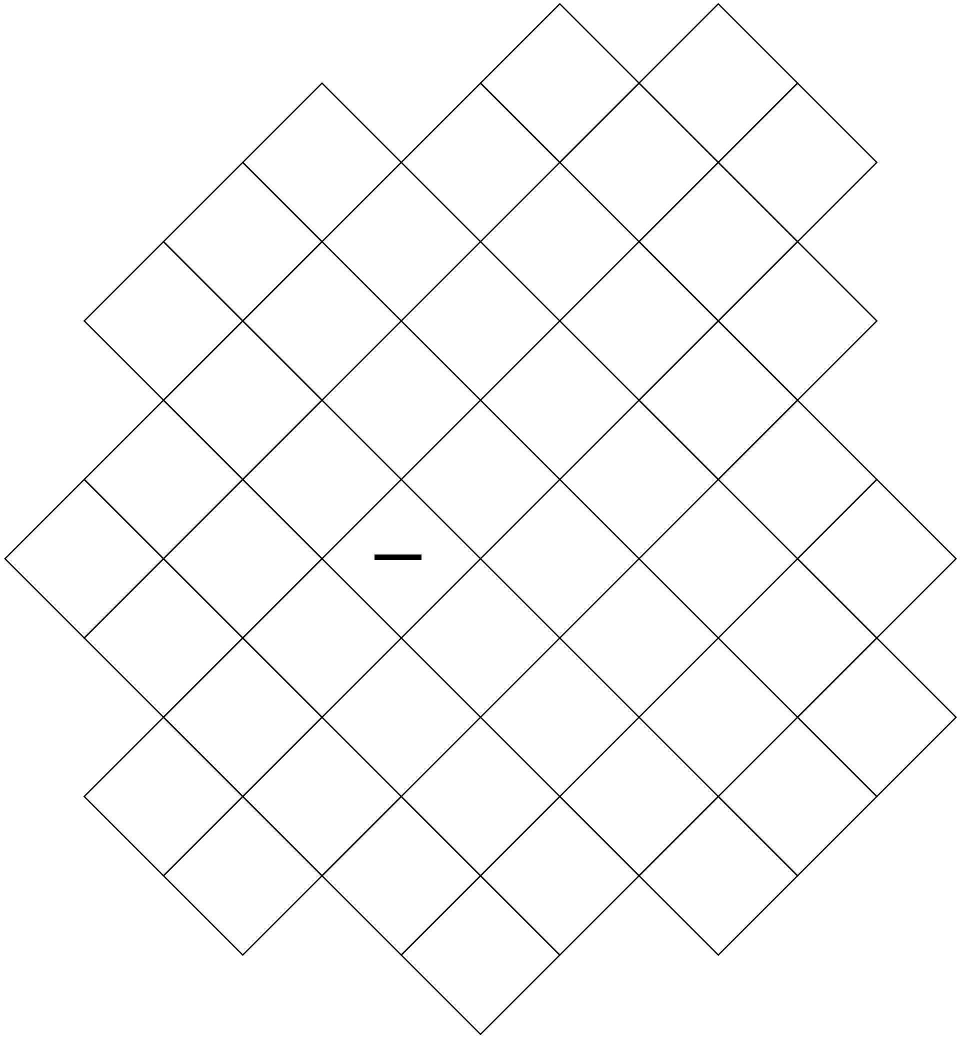
Classical nets

Local algebras:



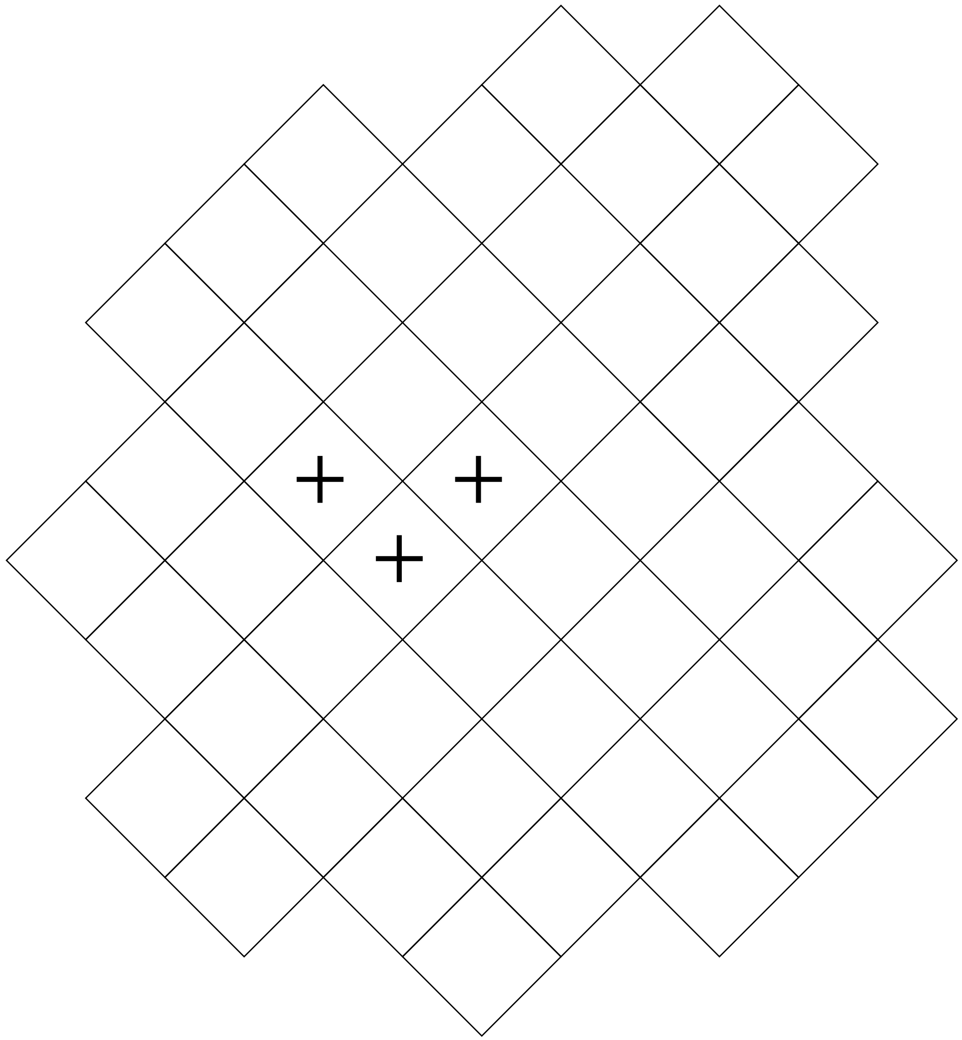
Classical nets

Local algebras:



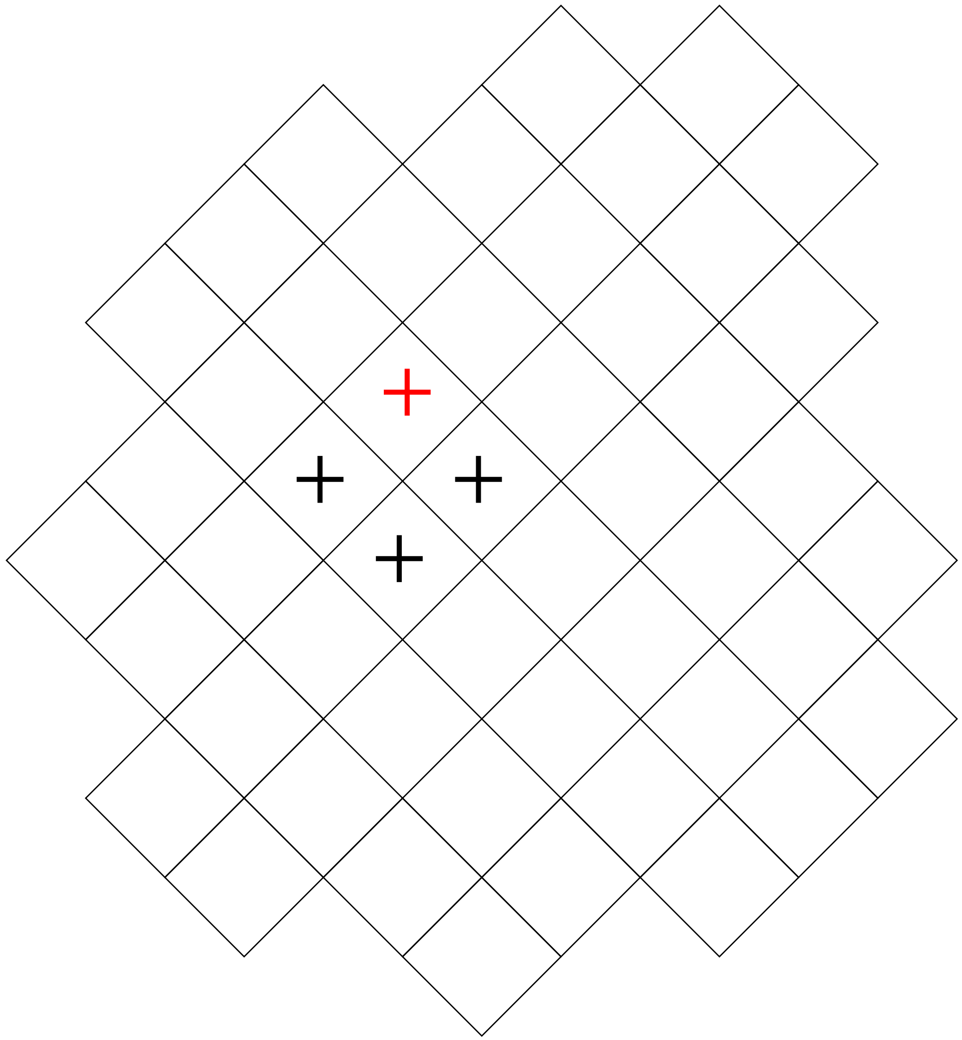
Classical nets

Deterministic dynamics:



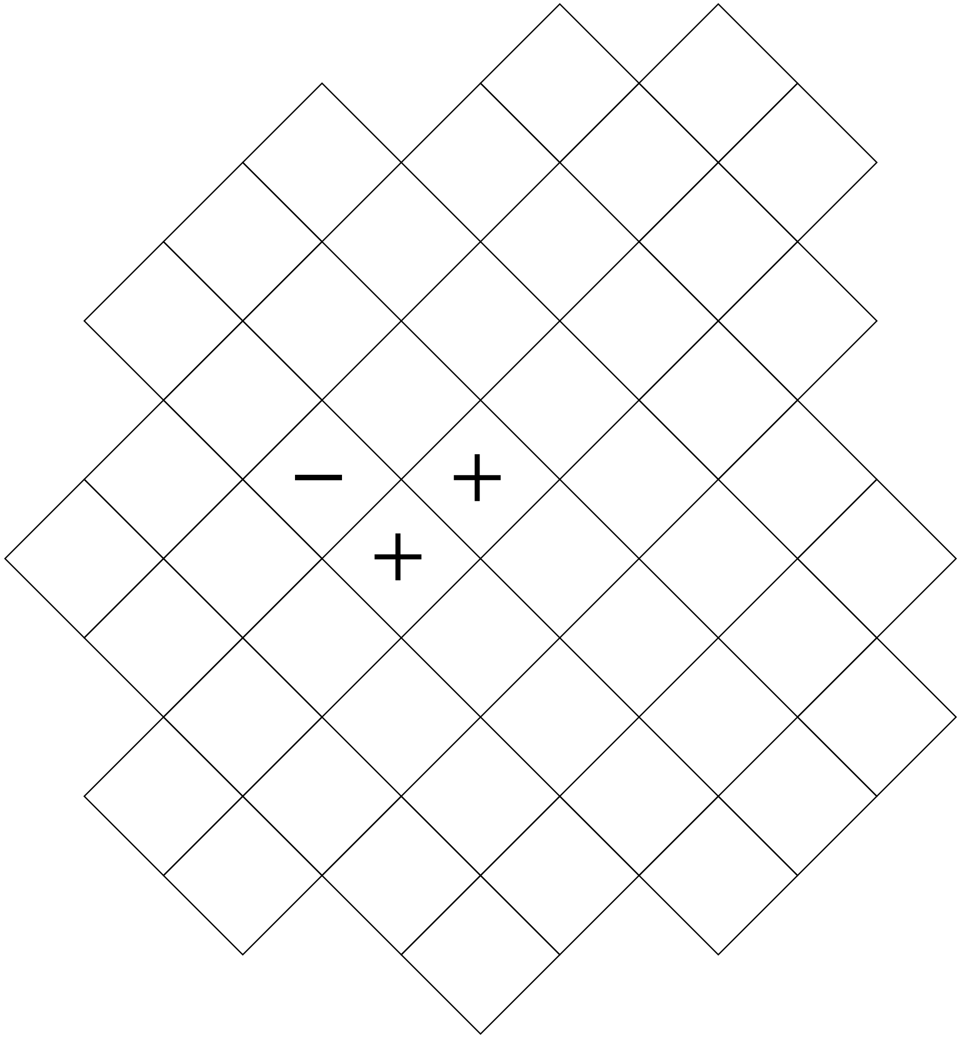
Classical nets

Deterministic dynamics:



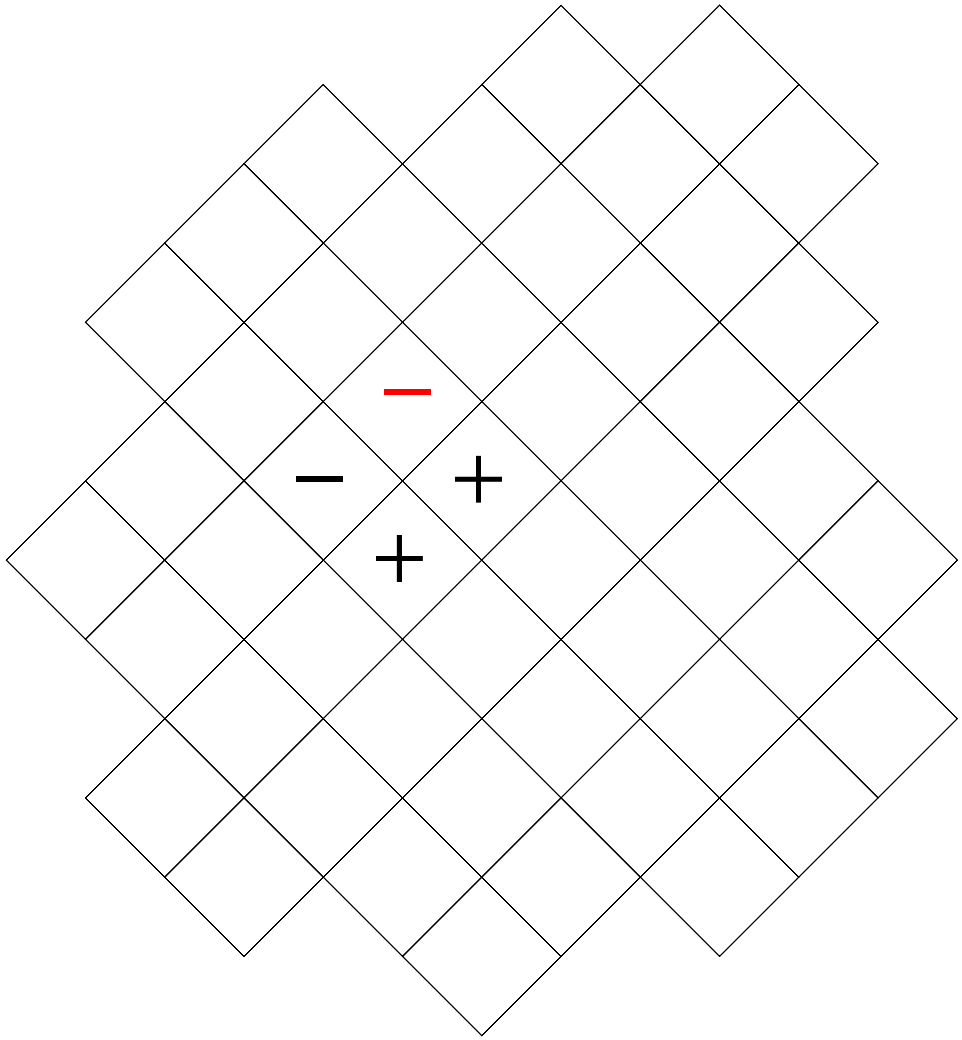
Classical nets

Deterministic dynamics:



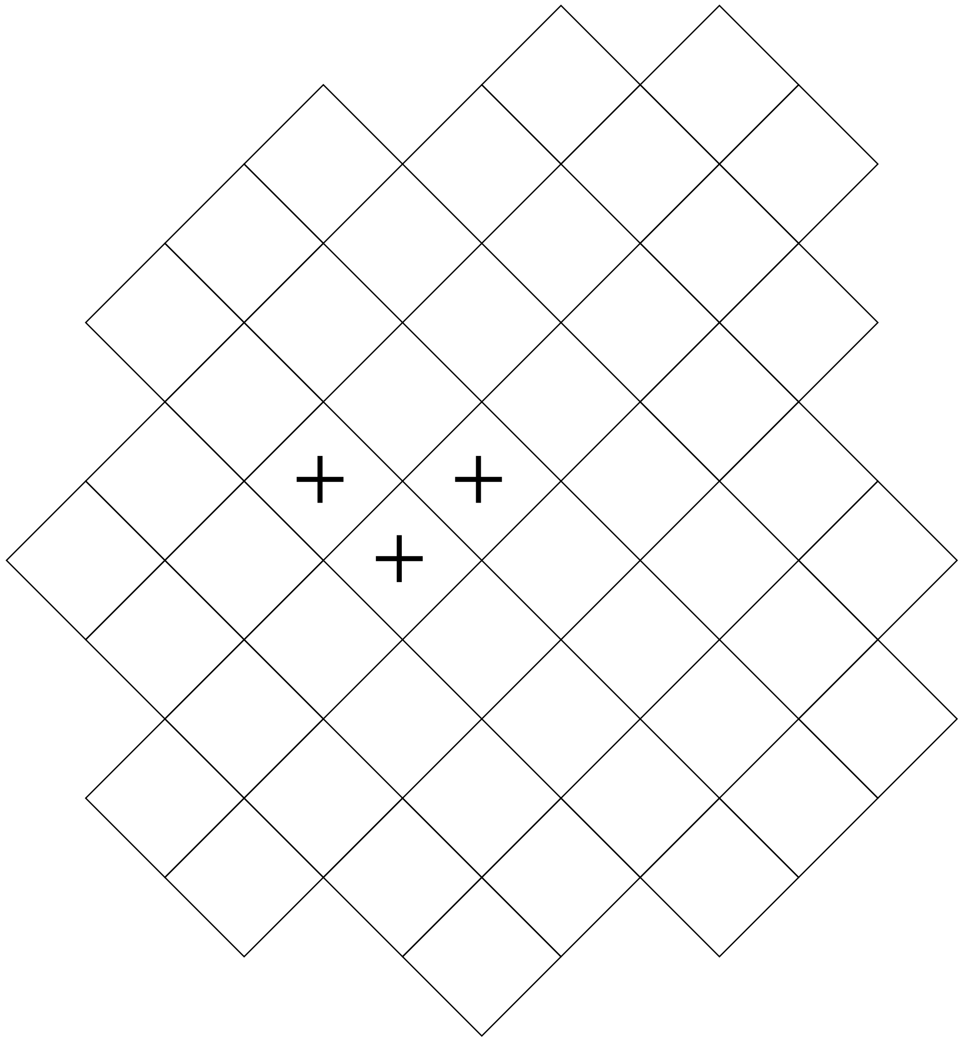
Classical nets

Deterministic dynamics:



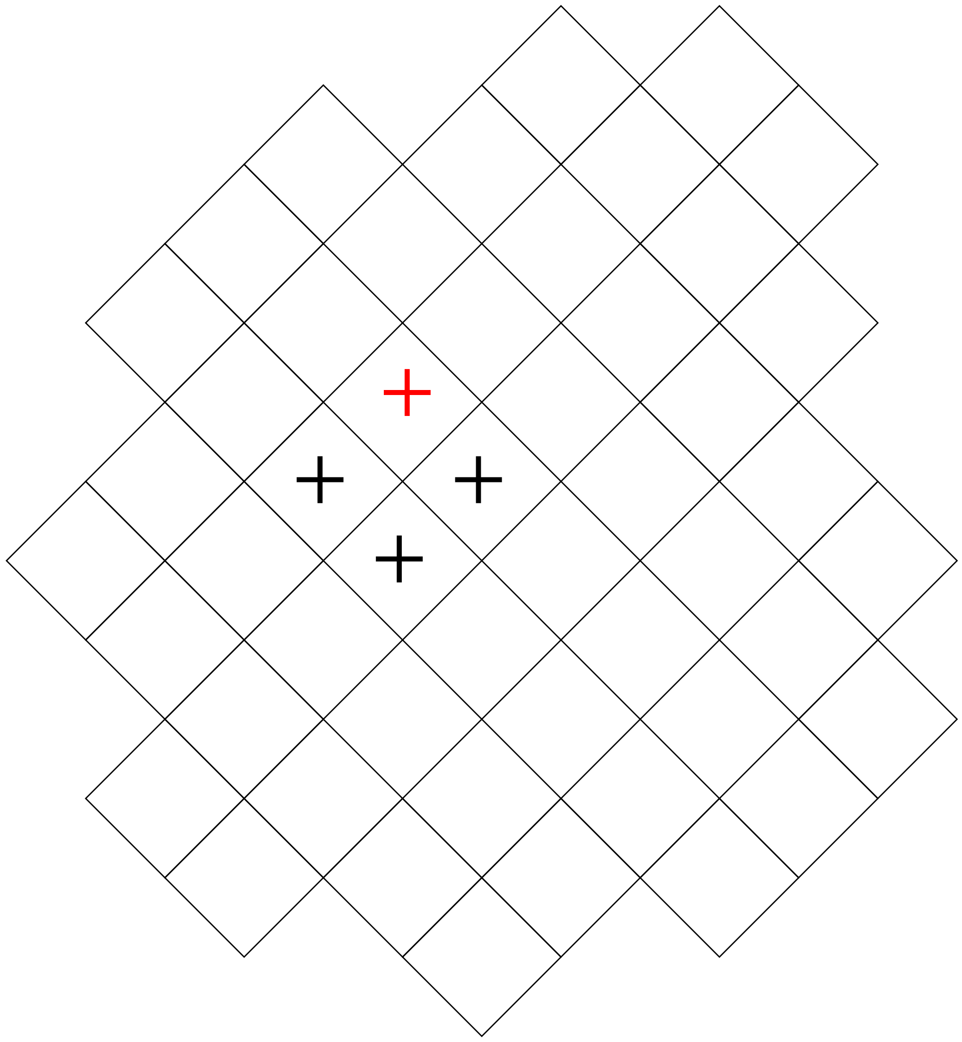
Classical nets

Stochastic dynamics:



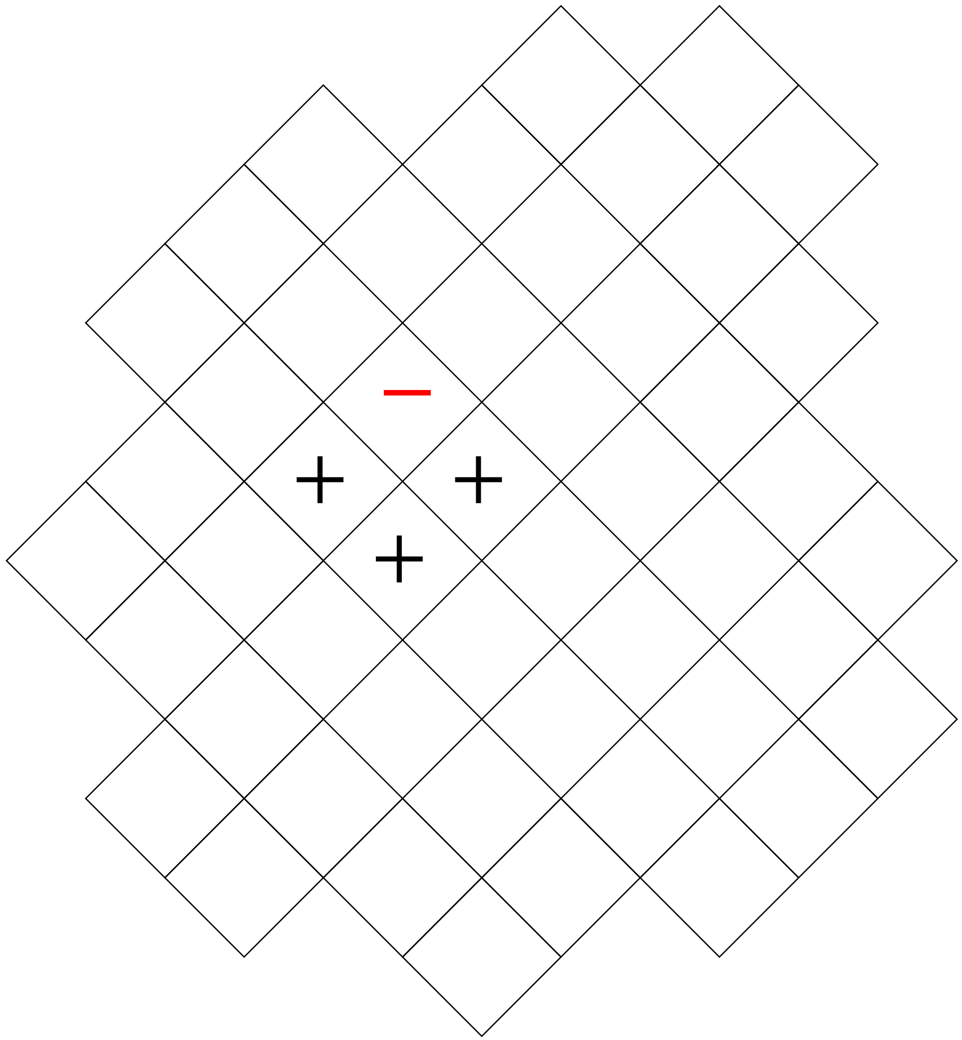
Classical nets

Stochastic dynamics: with probability p



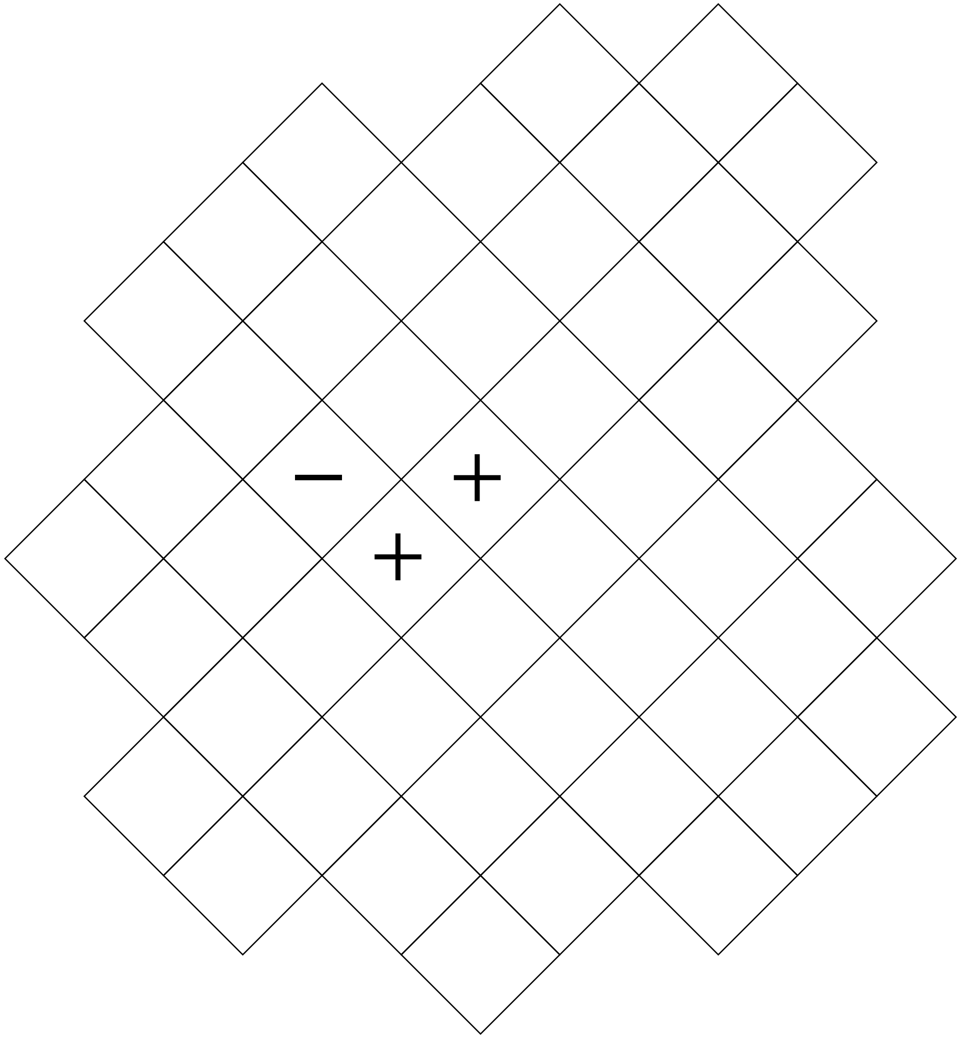
Classical nets

Stochastic dynamics: with probability $1 - p$



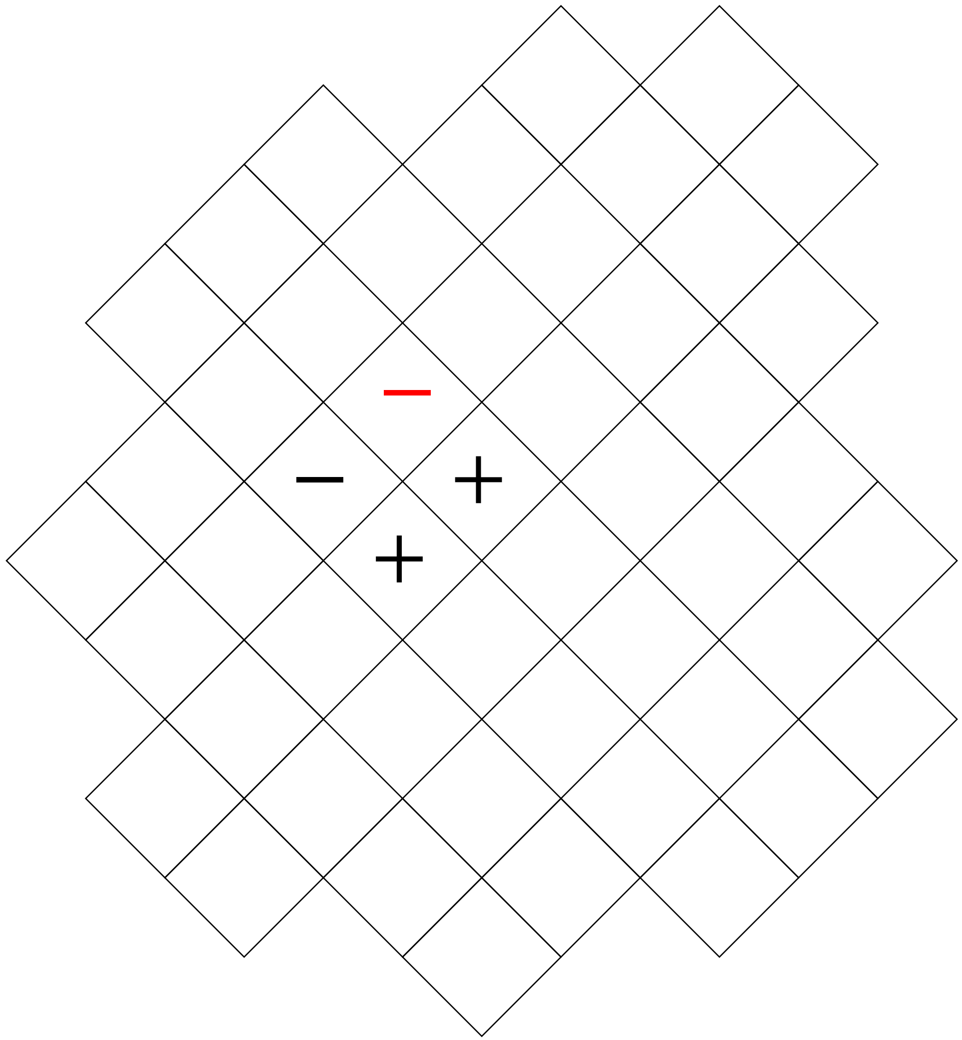
Classical nets

Stochastic dynamics:



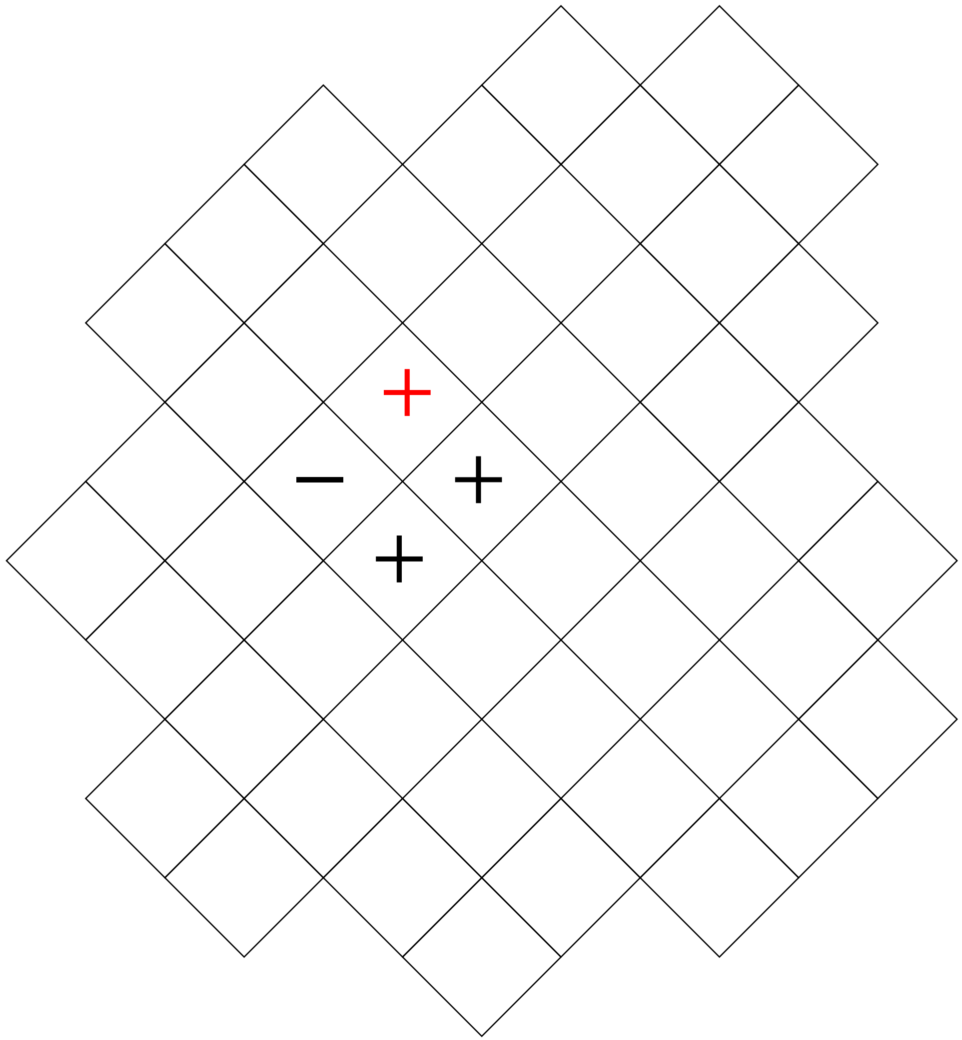
Classical nets

Stochastic dynamics: with probability p



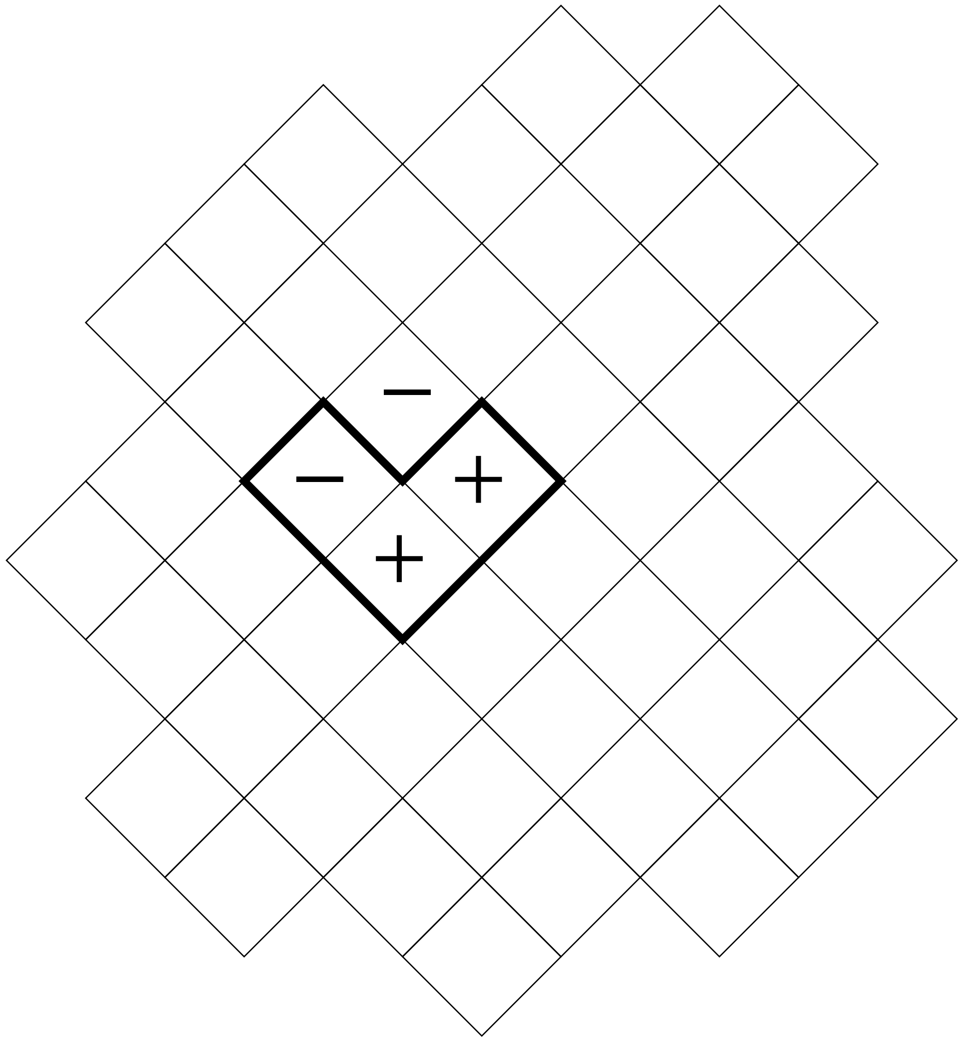
Classical nets

Stochastic dynamics: with probability $1 - p$



Classical nets

Local primitive causality does *not* hold:



Classical nets

But local causality *does* hold: