Reichenbach's Common Cause Principle

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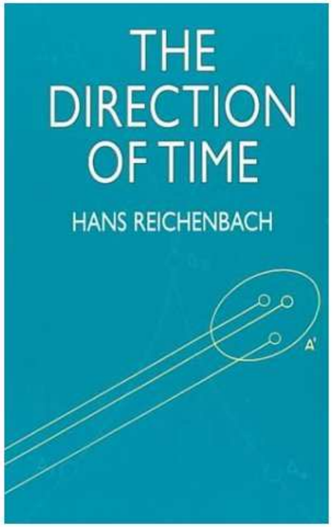
Project

I. Reichenbach's Common Cause Principle
II. The common cause and its generalizations
III. Common causal explanation and the EPR-Bell scenario

Reichenbach: The Direction of Time



Probabilistic causation

Correlation:

A B

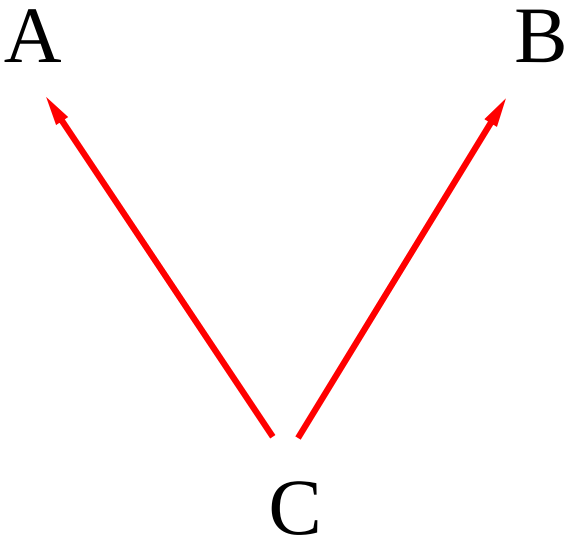
Probabilistic causation

Direct causal relation:

$$A \longrightarrow B$$

Probabilistic causation

Common cause:



The Common Cause Principle

Reichenbach's Common Cause Principle (RCCP): If there is a correlation between two events and a direct causal (or logical) connection between the correlated events can be excluded, then there exists a common cause of the correlation.

Question: What is a common cause?

Reichenbachian Common Cause

Classical probability space: (Σ, p)
Positive correlation: A, $B \in \Sigma$

$$p(AB) \quad > \quad p(A)\, p(B)$$

Reichenbachian common cause: $C \in \Sigma$

$$\begin{aligned} p(AB|C) &= p(A|C)p(B|C) \\ p(AB|C^{\perp}) &= p(A|C^{\perp})p(B|C^{\perp}) \\ p(A|C) &> p(A|C^{\perp}) \\ p(B|C) &> p(B|C^{\perp}) \end{aligned}$$

Examples

“Suppose both lamps in a room go out suddenly. We regard it as improbable that by chance both bulbs burned out at the same time and look for a burned out fuse or some other interruption of the common power supply. The improbable coincidence is thus explained as the product of a common cause.” (Reichenbach, 1956, p. 157)

“Or suppose several actors in a stage play fall ill showing symptoms of food poisoning. We assume that the poisoned food stems from the same source – for instance, that it was contained in a common meal – and then look for an explanation of the coincidence in terms of a common cause.” (Reichenbach, 1956, p. 157)

Positions on the status of RCCP

Bas C. van Fraassen (1982): RCCP does *not* hold in indeterministic situations

Nancy Cartwright (1987): RCCP *does* hold in indeterministic situations, but the common cause is *not* a screener-off

Elliott Sober (1987): RCCP does *not* hold even in deterministic situations

Jeremy Butterfield (1989): RCCP does *not* hold due to the EPR-Bell scenario

Common cause extensions

There exist probability spaces such that not all correlations have a common cause.

Proposition. Every classical probability space (Σ, p) is common cause extendable with respect to any finite set of correlated events.

Common cause closedness

Can we extend probability spaces such that *all* correlations have a common cause?

Propositions.

(i) (Probabilistically) non-atomic probability measure spaces are common cause closed.

(ii) Atomic probability measure spaces are common cause closed iff they have exactly one atom.

(iii) All atomic probability measure spaces can be embedded into a common cause closed non-atomic probability measure space.

Generalizations of the common cause

Reichenbach's definition of the common cause is inappropriate because it is

- restricted to a *single* common cause,
- restricted to *positive* correlations,
- silent about *spacetime* localization.

$$\begin{aligned} p(AB|C) &= p(A|C)p(B|C) \\ p(AB|C^{\perp}) &= p(A|C^{\perp})p(B|C^{\perp}) \\ p(A|C) &> p(A|C^{\perp}) \\ p(B|C) &> p(B|C^{\perp}) \end{aligned}$$

Solution: generalization

Common cause system

- **Correlation:** A, $B \in \Sigma$

$$p(AB) \neq p(A)p(B)$$

- **Common cause system** (CCS): partition $\{C_k\}_{k\in K}$ in Σ

$$p(AB|C_k) \quad = \quad p(A|C_k)\, p(B|C_k)$$

- **Common cause:** CCS of size 2.

Joint common cause system

- **Correlations:** $A_1,\, A_2,\, B_1,\, B_2 \in \Sigma$

$$p(A_m B_n) \neq p(A_m)\, p(B_n)$$

- **Joint CCS:** partition $\{C_k\}_{k\in K}$ in Σ

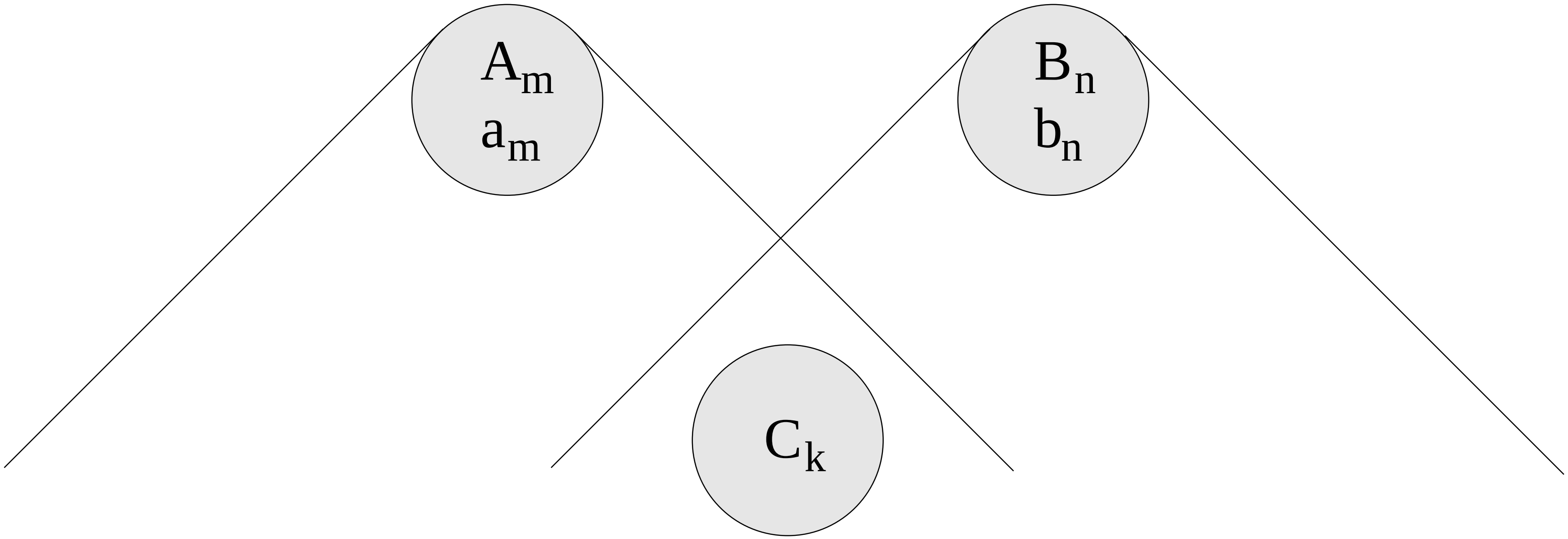
$$p(A_m B_n|C_k) \;=\; p(A_m|C_k)\, p(B_n|C_k)$$

Conditional probabilities

- **Measurement outcomes:** $A_m,\ B_n \in \Sigma$
- **Measurement choices:** $a_m,\ b_n \in \Sigma$
- **Conditional correlations:**

$$p(A_m B_n | a_m b_n) \neq p(A_m | a_m)\, p(B_n | b_n)$$

Localization of the common cause



Common causal explanation

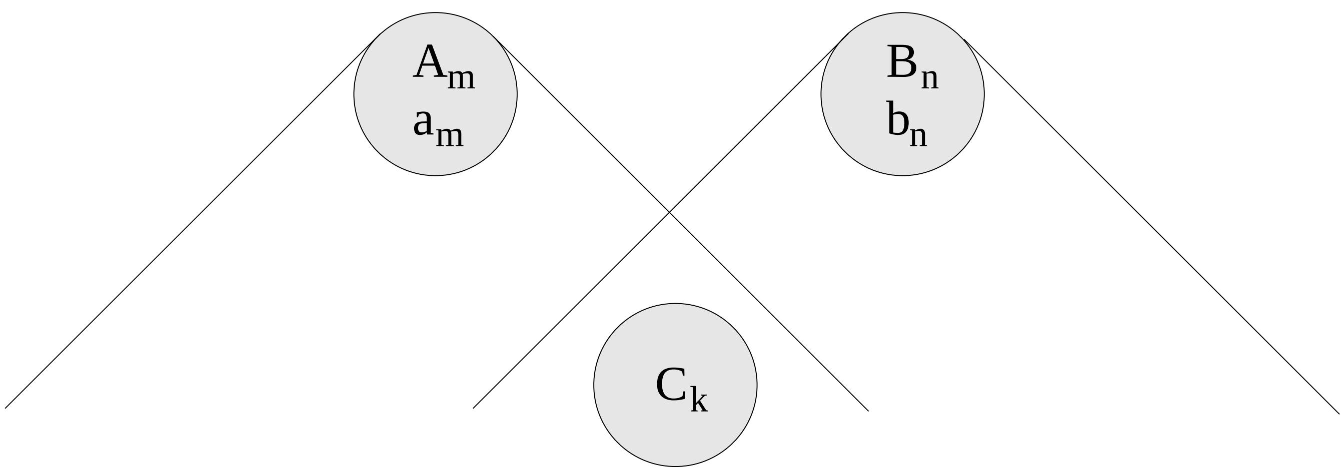
Local, non-conspriratorial joint common causal explanation: a partition $\{C_k\}$ in Σ

$$p(A_m B_n | a_m b_n C_k) = p(A_m | a_m b_n C_k)\, p(B_n | a_m b_n C_k) \qquad \text{(screening-off)}$$

$$p(A_m | a_m b_n C_k) = p(A_m | a_m b_{n'} C_k) \qquad \text{(locality)}$$

$$p(B_n | a_m b_n C_k) = p(B_n | a_{m'} b_n C_k) \qquad \text{(locality)}$$

$$p(a_m b_n C_k) = p(a_m b_n)\, p(C_k) \qquad \text{(no-conspiracy)}$$

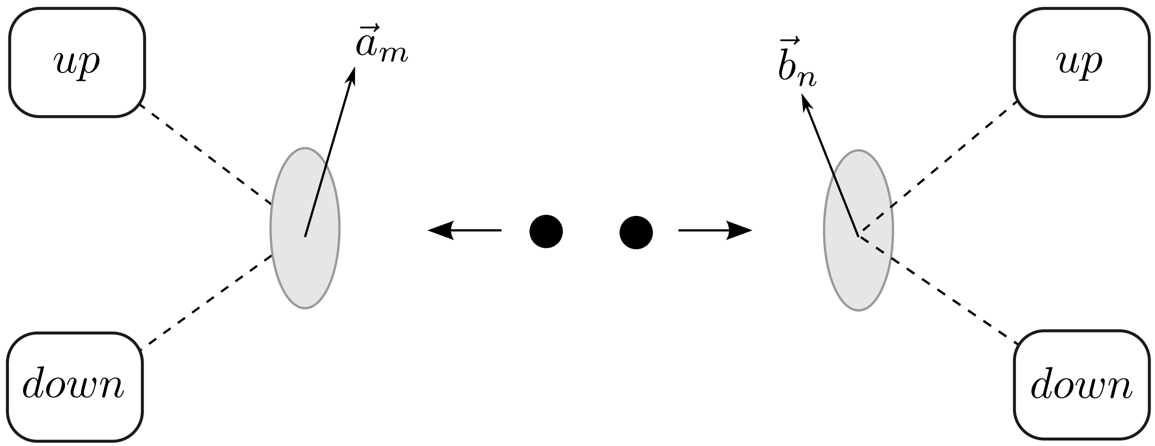


Bell inequality

$$\left.\begin{array}{r}\text{Joint CCS}\\ \text{Locality}\\ \text{No-conspiracy}\end{array}\right. \quad \Longrightarrow \quad \text{Bell inequality}$$

$$-1 \leqslant p(A_1B_1|a_1b_1) + p(A_1B_2|a_1b_2) + p(A_2B_1|a_2b_1)$$
$$-p(A_2B_2|a_2b_2) - p(A_1|a_1) - p(B_1|b_1) \leqslant 0$$

EPR correlations



Quantum probabilities:

$$
\begin{aligned}
q(A_m) &= \frac{1}{2} \\
q(B_n) &= \frac{1}{2} \\
q(A_m B_n) &= \frac{1}{4}\big(1 - cos(\theta_{a_m b_n})\big)
\end{aligned}
$$

where $\theta_{a_m b_n}$ is the angle between directions $\vec{a}_m$ and $\vec{b}_n$.

EPR correlations

Quantum probabilities are classical conditional probabilities:

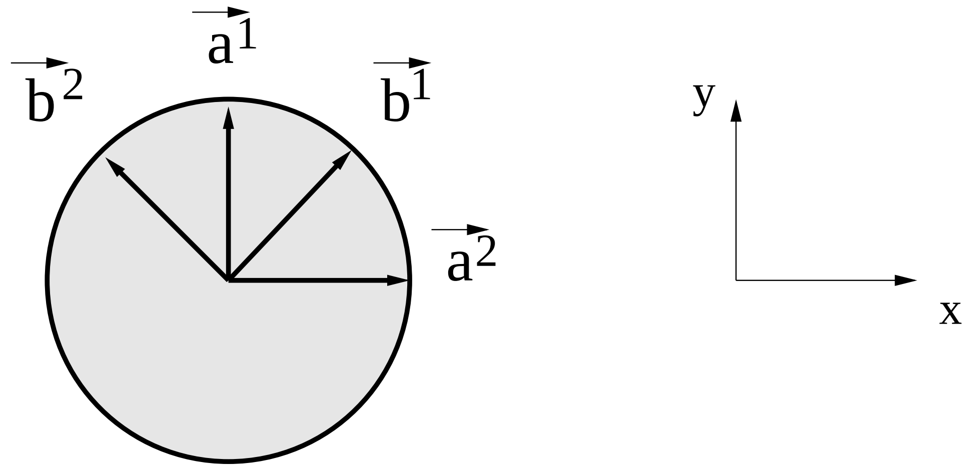
$$
\begin{aligned}
q(A_m) &\equiv p(A_m|a_m b_n) \\
q(B_n) &\equiv p(B_n|a_m b_n) \\
q(A_m B_n) &\equiv p(A_m B_n|a_m b_n)
\end{aligned}
$$

Question: Does the Bell inequality hold?

$$
\begin{aligned}
-1 \leqslant p(A_1 B_1|a_1 b_1) + p(A_1 B_2|a_1 b_2) + p(A_2 B_1|a_2 b_1) \\
-p(A_2 B_2|a_2 b_2) - p(A_1|a_1) - p(B_1|b_1) \leqslant 0
\end{aligned}
$$

EPR correlations

For the setting:



the Bell inequality is violated.

Standard conclusion

- The Bell inequality is a necessary condition for a set of correlations to have a common causal explanation.
- The Bell inequality is violated for appropriate setting.
- Therefore, RCCP is not valid in QM.

However ...

... there are extra assumptions:

- Generalized definition of the common cause
- Joint common causes / common cause systems
- Extra assumptions: locality and no-conspiracy
- Interpretation of quantum probabilities as classical conditional probabilities

”... assessing the status of the Principle of the Common Cause is a very subtle matter requiring a careful investigation of both the principle itself and the evidence for/against it provided by our best scientific theories.” (Hofer-Szabó, Rédei, Szabó, 2012)

More on that in ...