Interpretations of Quantum Theory

Gábor Hofer-Szabó
Research Center for the Humanities, Budapest
Email: szabo.gabor@abtk.hu

The aim of the course is to overview the **main interpretations of quantum theory**, such as the Copenhagen, the Modal, the Consistent Histories, the Relational, the Quantum Bayesian, the Ensemble, the Bohmian, and the Many-Worlds Interpretation

Prerequisites for the course: The course is **self-contained**; it does not presuppose the knowledge of quantum mechanics or the mathematical foundations thereof but it does require **hard work** from you during the semester

Grading: You will be graded based on the **weekly assignments** you find at the end of each lecture in this lecture notes. The length of the assignments will also be specified there. You must send each assignment **by Tuesday midnight** to the following email address: *hoferszabogabor@gmail.com*. **Late submissions** will not be accepted. To be graded, you need to turn in **all but two assignments**. There is no final exam

Web site of the course: http://hps.elte.hu/∼gszabo/InterpretationsofQM.html

Contents

1 Some Quantum Phenomena

We look at **eight experiments** presented in (Maudlin, 2019, Ch. 1)

1. Cathode ray tube

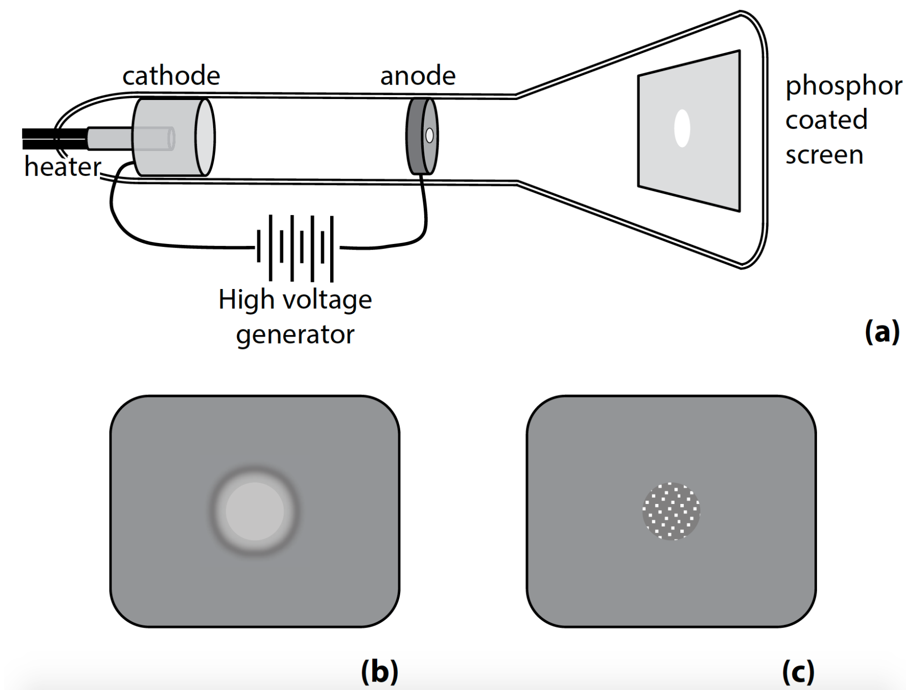


Figure 1: Cathode ray tube

Electrons: Heating the cathode boils off electrons, which, being negatively charged, are repelled by the negatively charged cathode and attracted to the positively charged anode, passing through the aperture and continuing on to the screen

2. Single slit

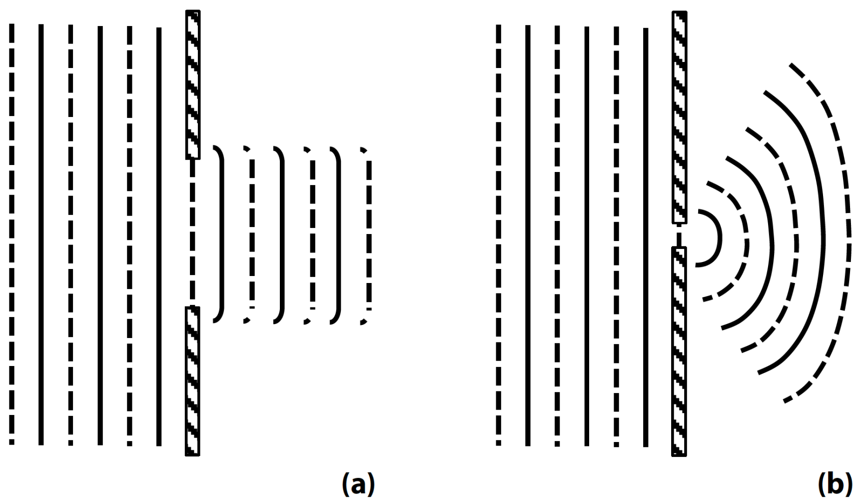


Figure 2: Single slit

Diffraction: making the slit thinner results in a wider spot on the screen

Individual particles: turning down the heating, one observes a series of individual flashes

Wave-particle duality: electrons produce phenomena associated both with waves and particles

3. Double slit

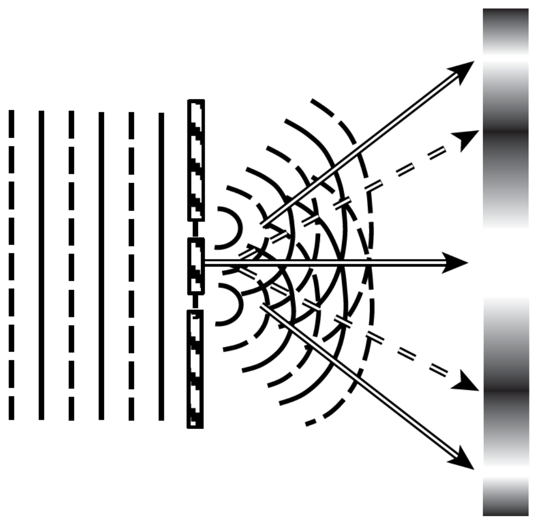


Figure 3: Double slit

Feynman: "*absolutely* impossible, to explain in any classical way"

Interference: Waves interfere because when they meet each other, they interact by **superposition**: if crests meet with crests or troughs with troughs, they reinforce; if crests meet troughs, they cancel out

Individual particles: turning down the heating, only individual dots appear on the screen

For each individual flash, the physical situation at the screen is **sensitive to the condition of both slits**

Different interpretations tackle this fact differently:

> **Bohmian interpretation:** the electron goes **through only one slit**. But then how the other slit being open affects it?
>
> **Objective collapse interpretation:** the electron goes **through both slits**. But then why are there discrete flashes?

4. Double slit with monitoring

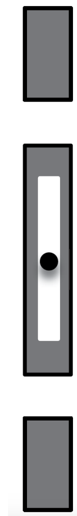


Figure 4: Double slit with monitoring

Why not **simply to check** on which slit the electron goes through?

Make a small, thin chamber in the screen between the two slits, and **place a proton** in a position exactly between the slits. Line the ends of the chamber with a substance that will **emit a flash** if a proton is absorbed

Half of the electrons will be "seen" to have gone through the upper slit and about half through the lower. But **the interference bands will disappear**

The behavior of the electrons changes from being **wavelike** to being **particle-like**

With reduced monitoring the **interference bands will slowly re-emerge**

Is maybe the **observer** central in QM?

> **Complementarity:** going back to the cathode ray experiment, **the better we determine the position** of the electron (the smaller the slit is), **the worse we can predict its momentum** (it will diffract the more)

5. Spin

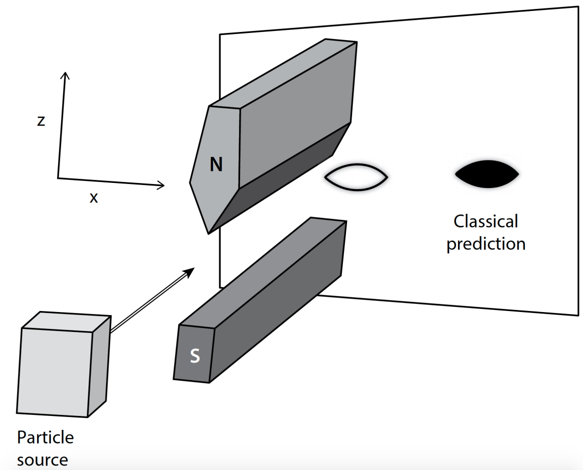


Figure 5: Stern-Gerlach apparatus

Spin: **intrinsic angular momentum** of the electron

Sending an electron through the **Stern-Gerlach apparatus** oriented in direction z, the beam divides into **two parts**, one deflected up, the other deflected down (**quantization of the spin**)

When passing the upper beam through a **second apparatus** oriented in **direction** z, all electrons will be deflected again upwards

When passing the upper beam through a **second apparatus** oriented in **direction** x, the beam splits 50-50

When further passing any of these latter two beams through a **third apparatus** again oriented in **direction** z, the beam splits again 50-50. The original preparation **has been lost**

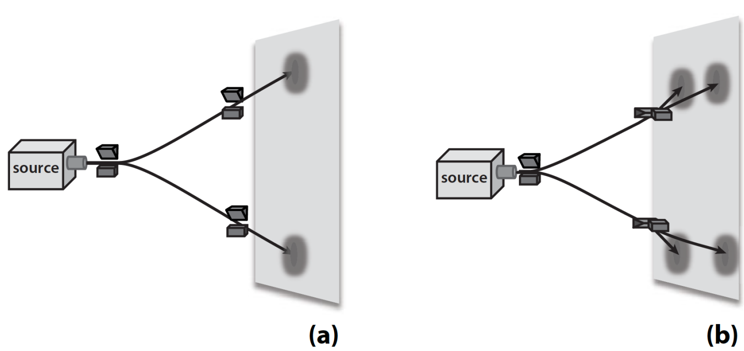


Figure 6: Two consecutive Stern-Gerlach apparata

6. **Mach-Zender interferometer**

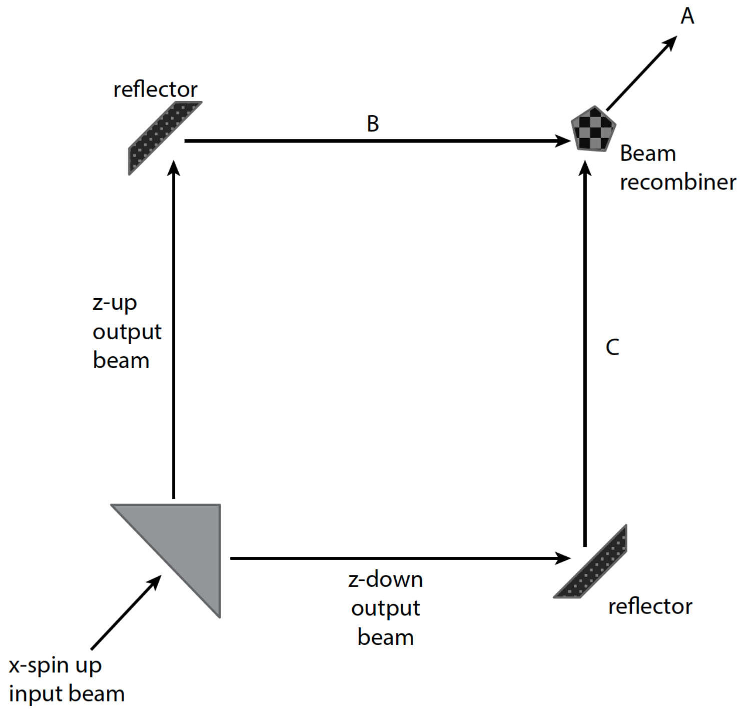


Figure 7: Mach-Zender interferometer

What if we let an x-spin up beam go through a z-oriented apparatus and **recombine** the two beams and let it go through again an x-oriented apparatus?

All electrons will be deflected up

The lost information about the original preparation is **restored by the recombination**

Now, apply a magnetic field **on the one route** which rotates each electron 360°. The originally x-**spin up** beam will be x-**spin down**.

Every electron is **sensitive** to the physical conditions along **both paths** in the interferometer

7. EPR experiment

Figure 8: EPR experiment

A pair of electrons is prepared in a **singlet state**

Each is **measured** by an apparatus oriented in direction z.

One finds **perfect anticorrelation** between the outcomes

EPR argument:

> **Assume locality:** electrons do no communicate with speed greater than light
>
> Then, the electrons must have **definite dispositions** to react to the magnets, otherwise is would be remarkable they output opposite outcomes.
>
> > **Example: two friends** subjected to questions in two different rooms
>
> This holds **for every direction**
>
> **Conclusion:** since these definite dispositions are **missing** from the quantum mechanical description, QM is **incomplete**

8. GHZ test

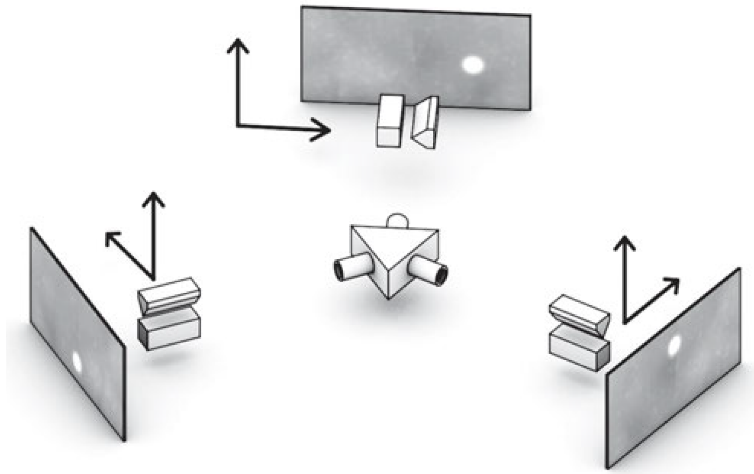


Figure 9: GHZ test

Three electrons are prepared in the **GHZ state**

Each is measured by an apparatus oriented either in direction x or y.

We pick four of the eight possible orientations:

$$X_1X_2X_3, \quad X_1Y_2Y_3, \quad Y_1X_2Y_3, \quad Y_1Y_2X_3$$

The **experimental result** is this:

> For $X_1X_2X_3$: we obtain an **odd number of up** outcome
>
> For $X_1Y_2Y_3$, $Y_1X_2Y_3$, $Y_1Y_2X_3$: an **even number of up** outcome
>
> **Perfect correlation**: if we know two of the outcomes, we can predict the third with certainty

Assuming locality, each of the electrons must have a **predetermined spin value**

But this is **logically impossible:**

> Adding up the "up" spin values along the four orientations yields: odd + even + even + even = an **odd** number of "up" outcomes
>
> However, we counted each value twice, so the total number of "ups" must be **even**. **Contradiction!**

GHZ test is a stronger than the EPR: there is **no way to avoid nonlocality**

Bell's inequalities: a similarly strong result for quantum nonlocality based on probabilities

Assignment 1. Watch the video: *Up and Atom: the Double Slit Experiment: Light As A Wave* and explain in 100-200 words what is an interference and under what conditions it appears. (Send it to: hoferszabogabor@gmail.com by Tuesday midnight.)

Readings

Maudlin, T., *Philosophy of Physics: Quantum theory*, (Princeton University Press, 2019), Ch. 1.

2 The Formalism of Quantum Mechanics

Basic concepts:	Classical mechanics	Quantum mechanics
System	Phase space	Hilbert space
States (pure/mixed)	Points in/probability measures on the phase space	Vectors in/density matrices on the Hilbert space
Observables	Functions on the phase space	Operators on the Hilbert space
Events	(Projections onto) subsets of the phase space	(Projections onto) linear subspaces of the Hilbert space
Dynamics	Time-evolution of the state	Time-evolution of the state

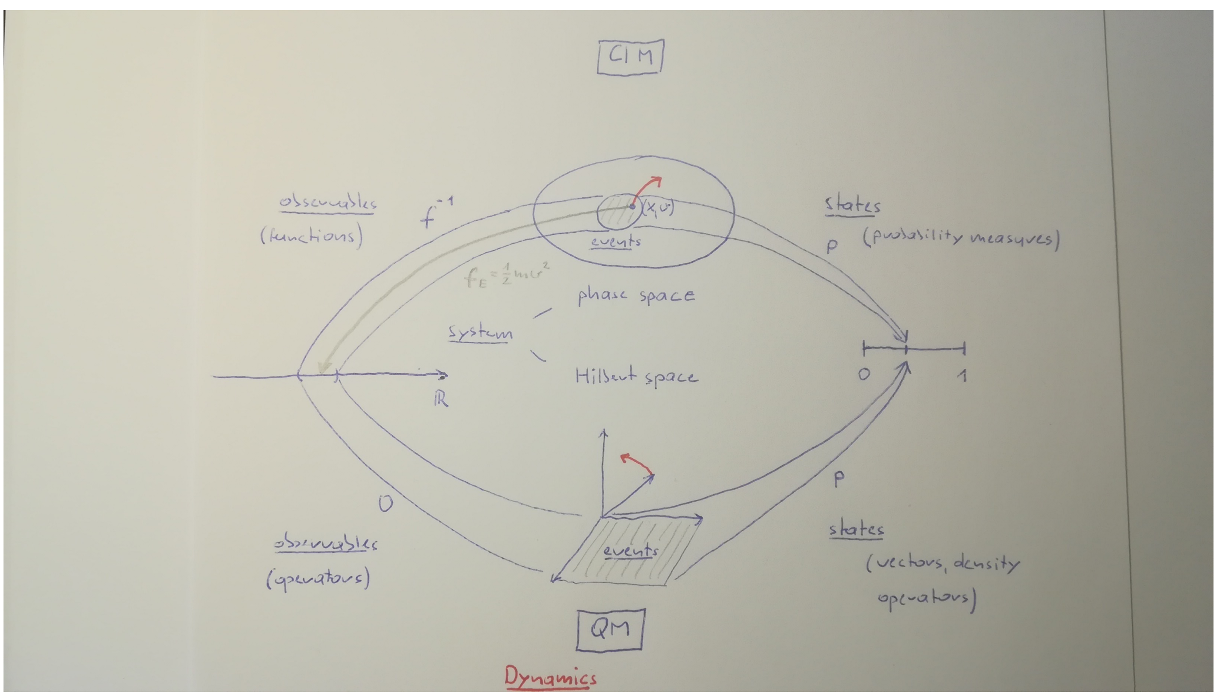


Figure 10: The structure of physical theories

I. Classical physics

Basic concepts: system, states, observables, events, dynamics

Unchanging properties: mass, spring constant $\to$ scalars: m, k

Changing properties: position, momentum $\to$ phase space: $(q,p) \in \mathbb{R}^2$

System: the possible states of the system are represented on a **phase space** or **state space**, $x \in \Omega$

Observables: real-valued functions on the phase space, e.g. total energy: $H(q,p) := \frac{p^2}{2m} + \frac{kq^2}{2}$

Events

Logical: Proposition: "The value of the total energy of the system lies in the interval 50-60": $H(q,p) \in [50,60]$

Ontological: Event: $H^{-1}([50,60]) := \{(q,p) \in \mathbb{R}^2 \,|\, H(q,p) \in [50,60]\}$, a subset of the phase space

Experimental: Yes-no question: "Does the value of the total energy of the system lie in the interval 50-60?": $(H, [50,60])$

Events form a **Boolean lattice**: $\mathcal{B}$

State

Specification of the properties: $q = 3$, $p = 5$ ($m = 2$, $k = 4$) ("property state")

Pure state: a function assigning to every event a definite truth value/to every observable a definite value

Mixed state: a probability measure assigning to every event a probability: $\mathcal{B} \to [0,1]$ such that

$$\begin{aligned} p(E \cup F) &= p(E) + p(F) \quad \text{if } E \cap F = \emptyset \\ p(\Omega) &= 1 \end{aligned}$$

Generally, σ-**additivity** is required

Dynamics: time-evolution of the state typically in form of partial differential equations (Hamilton equations)

An example: coin toss

Phase space: $\Omega = \{H, T\}$

Events: $\mathcal{B} = \{\emptyset, H, T, \Omega\}$

State: $p(\emptyset) = 0$, $p(H) = p(T) = \frac{1}{2}$, $p(\Omega) = 1$ (fair coin)

$p(\emptyset) = 0$, $p(H) = \frac{1}{4}$, $p(T) = \frac{3}{4}$, $p(\Omega) = 1$ (biased coin)

Probabilities in **vector space** ($\rightarrow$ superposition)

II. Quantum theory

Basic concepts: system, states, observables, events, dynamics

System: the possible states of the system are represented on a **Hilbert space**, $\mathcal{H}$

Ket vectors: $|\psi\rangle$

Bra vectors: $\langle\psi|$

Observables: self-adjoint operators on the Hilbert space

Events: subspaces of $\mathcal{H}$

Events form a non-Boolean lattice: **Hilbert lattice**, $\mathcal{P}(\mathcal{H})$

$\cap, \cup, ^{-}, \emptyset, \Omega \;\longrightarrow\; \wedge, \vee, ^{\perp}, \mathbf{0}, \mathbf{1}$

Non-distributivity, orthomodularity

Projections: self-adjoint and idempotent operators:

$P = |\psi\rangle\langle\psi|$

$Q = |\psi_1\rangle\langle\psi_1| + |\psi_2\rangle\langle\psi_2|$ where $|\psi_1\rangle \perp |\psi_2\rangle$

State

~~Specification of the properties~~

~~A function assigning to every event a definite truth value~~

A probability measure assigning to every event a probability: $\mathcal{P}(\mathcal{H}) \rightarrow [0,1]$ such that

$$\begin{aligned} p(P+Q) &= p(P) + p(Q) \quad \text{if } P \cdot Q = \mathbf{0} \\ p(\mathbf{1}) &= 1 \end{aligned}$$

Generally, σ**-additivity** is required

Pure state: given by a **unit vector (ray)** $|\psi\rangle \in \mathcal{H}$:

$$p_\psi(P) = \langle\psi|P\,\psi\rangle$$

Mixed state: given by a **density operator** (positive operator of trace 1) ρ on $\mathcal{H}$:

$$p_\rho(P) \;=\; Tr(\rho\, P)$$

Dynamics: Schrödinger equation:

$$i\hbar\frac{\partial}{\partial t}\psi \;=\; H\,\psi$$

or

$$\rho(t) \;=\; U(t)\,\rho\,U^\dagger(t)$$

where $U(t) = e^{-iHt/\hbar}$ if the Hamiltonian H is time-independent.

Assignment 2. Read Ch. 3 (p. 83-86) in Hughes, R. I. G., *The Structure and Interpretation of Quantum Mechanics* (Cambridge: Harvard University Press, 1989) and explain in 100-200 words why and how probabilities can be represented by vectors. You find the book here: **.** You get the login and password in class. (Send it to: hoferszabogabor@gmail.com by Tuesday midnight.)

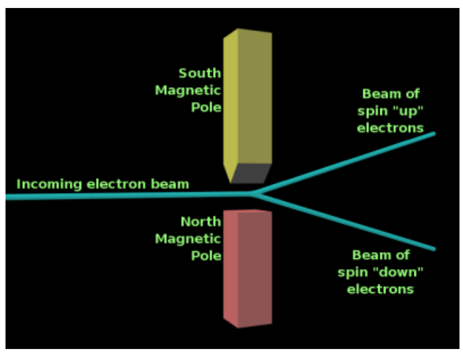
Readings

Hughes, R. I. G., *The Structure and Interpretation of Quantum Mechanics* (Cambridge: Harvard University Press, 1989).

3 The Quantum Theory of Spin

Electron spin (**qubit**) is the simplest two-dimensional quantum system represented by **Pauli matrices** in $\mathcal{H}_2$. One can nicely define **separability and entanglement** on two-qubit systems.

I. Spin is an internal angular momentum of elementary particles. The concept was first proposed and later work out by Pauli. It has been experimentally verified in the Stern-Gerlach experiment in 1922:



a beam of electrons is sent through an inhomogeneous magnetic field. The electrons will be **deflected either up or down**. The value of the spin of an electron can be $\pm\frac{1}{2}\hbar$ ($\hbar := \frac{h}{2\pi}$), but usually the natural unit $\frac{1}{2}\hbar = 1$ is used

Qubit: any two-level quantum system

Hilbert space: $\mathcal{H}_2$

Orthonormal basis (ONB):

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Another ONB:

$$|+\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big) \qquad |-\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle - |1\rangle\big)$$

Events: (1-dimensional) projections onto $\mathcal{H}_2$

$$P_{\mathbf{n}^\pm} \;=\; \frac{1}{2}(\mathbf{1} \pm \sigma_{\mathbf{n}})$$

For example:

$$\begin{aligned}
P_0 &= |0\rangle\langle 0| \\
P_+ &= |+\rangle\langle +| = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|0\rangle\langle 1| + \frac{1}{2}|1\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1| \\
P_\psi &= |\psi\rangle\langle\psi|
\end{aligned}$$

Observables: composed from events:

$$\begin{aligned}
\sigma_z &= +1\cdot P_+ + -1\cdot P_- = |0\rangle\langle 0| - |1\rangle\langle 1| \\
\sigma_x &= |+\rangle\langle +| - |-\rangle\langle -|
\end{aligned}$$

In the ONB $\{|0\rangle, |1\rangle\}$, every observable is the real linear combination of the **Pauli matrices** and the identity

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \mathbf{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Eigenvectors of the Pauli matrices:

$$\begin{aligned}
\sigma_x |x^\pm\rangle &= \pm |x^\pm\rangle \\
& \quad |+\rangle := |x^+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \\
& \quad |-\rangle := |x^-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \\
\sigma_y |y^\pm\rangle &= \pm |y^\pm\rangle \\
& \quad |y^+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix} \\
& \quad |y^-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix} \\
\sigma_z |z^\pm\rangle &= \pm |z^\pm\rangle \\
& \quad |1\rangle := |z^+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\
& \quad |0\rangle := |z^-\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}
\end{aligned}$$

General spin operators:

$$\sigma_{\mathbf{n}} = \boldsymbol{\sigma}\cdot\mathbf{n}$$

where $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ and $\mathbf{n} \in \mathbb{R}^3$, $|\mathbf{n}| = 1$

Some relations $(i, j, k \in \{x, y, z\})$:

$$\begin{aligned} \sigma_i^2 &= \mathbf{1} \\ [\sigma_i, \sigma_j] := \sigma_i\sigma_j - \sigma_j\sigma_i &= 2i\,\varepsilon_{ijk}\sigma_k \\ \{\sigma_i, \sigma_j\} := \sigma_i\sigma_j + \sigma_j\sigma_i &= 2i\,\delta_{ij}\mathbf{1} \\ \mathrm{Tr}\,\sigma_i &= 0 \\ \mathrm{Det}\,\sigma_i &= -1 \end{aligned}$$

State:

Pure states: unit vectors $|\psi\rangle \in \mathcal{H}_2$. For example:

$$|\psi\rangle = \frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle = \frac{1+\sqrt{3}}{\sqrt{8}}|+\rangle + \frac{1-\sqrt{3}}{\sqrt{8}}|-\rangle$$

Mixed state: density operator on $\mathcal{H}_2$. For example:

$$\rho = \frac{1}{4}|0\rangle\langle 0| + \frac{3}{4}|1\rangle\langle 1|$$

Mixed states are **convex combination** of other states; pure states are not

Probabilities:

$$\begin{aligned} p_\psi(P_0) &= \langle\psi|P_0\psi\rangle = \langle\psi|\big(|0\rangle\langle 0|\big)|\psi\rangle = \langle\psi|0\rangle\langle 0|\psi\rangle = |\langle\psi|0\rangle|^2 = \frac{1}{4} \\ p_\rho(P_0) &= Tr(\rho\,P_0) = Tr\Big[\big(\frac{1}{4}|0\rangle\langle 0| + \frac{3}{4}|1\rangle\langle 1|\big)|0\rangle\langle 0|\Big] = Tr\big(\frac{1}{4}|0\rangle\langle 0|\big) = \frac{1}{4} \\ &= \langle 0|\big(\frac{1}{4}|0\rangle\langle 0|\big)|0\rangle + \langle 1|\big(\frac{1}{4}|0\rangle\langle 0|\big)|1\rangle = \frac{1}{4}|\langle 0|0\rangle|^2 = \frac{1}{4} \end{aligned}$$

What is then the **difference**?

$$\begin{aligned} p_\psi(P_+) &= \langle\psi|P_+\psi\rangle = \langle\psi|\big(|+\rangle\langle +|\big)|\psi\rangle = \langle\psi|+\rangle\langle +|\psi\rangle = |\langle\psi|+\rangle|^2 = \frac{(1+\sqrt{3})^2}{8} \\ p_\rho(P_+) &= Tr(\rho\,P_+) = Tr\Big[\big(\frac{1}{4}|0\rangle\langle 0| + \frac{3}{4}|1\rangle\langle 1|\big)|+\rangle\langle +|\Big] \\ &= Tr\Big[\big(\frac{1}{4}|0\rangle\langle 0| + \frac{3}{4}|1\rangle\langle 1|\big)\big(\frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|0\rangle\langle 1| + \frac{1}{2}|1\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1|\big)\Big] \\ &= Tr\Big[\frac{1}{8}|0\rangle\langle 0| + \frac{1}{8}|0\rangle\langle 1| + \frac{3}{8}|1\rangle\langle 0| + \frac{3}{8}|1\rangle\langle 1|\Big] = \frac{1}{2} \end{aligned}$$

II. Composite systems

System: $S_1 + S_2$ represented by the tensor product space $\mathcal{H}_1 \otimes \mathcal{H}_2$

Orthonormal basis:

$$\{|0\rangle \otimes |0\rangle\,, |0\rangle \otimes |1\rangle\,, |1\rangle \otimes |0\rangle\,, |1\rangle \otimes |1\rangle\}$$

abbreviated as

$$\{|00\rangle\,, |00\rangle\,, |00\rangle\,, |00\rangle\}$$

Another ONB:

$$\{|{+}{+}\rangle\,, |{+}{-}\rangle\,, |{-}{+}\rangle\,, |{-}{-}\rangle\}$$

Events: projections onto $\mathcal{H}_1 \otimes \mathcal{H}_2$

For example:

$$\begin{aligned} P_0 \otimes P_0 &= |00\rangle\langle 00| \\ \mathbf{1} \otimes P_{z^+} &= |00\rangle\langle 00| + |10\rangle\langle 10| \\ P_+ \otimes P_0 &= |{+}0\rangle\langle {+}0| \end{aligned}$$

and more complicated projections...

States

Pure states: unit vectors $|\Psi\rangle \in \mathcal{H}_1 \otimes \mathcal{H}_2$.

1. **Separable (= product) states:** express probabilistic independence

$$|\Psi\rangle \quad = \quad |\psi_1\rangle \otimes |\psi_2\rangle$$

For example:

$$|00\rangle\,, |01\rangle\,, |0{+}\rangle$$

2. **Entangled states:** express correlation

$$|\Psi\rangle \quad \neq \quad |\psi_1\rangle \otimes |\psi_2\rangle$$

For example: **four Bell states** forming an ONB:

$$\begin{aligned} |\Phi^{\pm}\rangle &= \frac{1}{\sqrt{2}}\big(\,|00\rangle \pm |11\rangle\,\big) \\ |\Psi^{\pm}\rangle &= \frac{1}{\sqrt{2}}\big(\,|01\rangle \pm |10\rangle\,\big) \end{aligned}$$

$|\Psi^-\rangle$ is the **singlet state** which can also be written as

$$|\Psi^-\rangle \;=\; \frac{1}{\sqrt{2}}\big(\,|{+}{-}\rangle - |{-}{+}\rangle\,\big)$$

GHZ state (in $\mathcal{H}_2 \otimes \mathcal{H}_2 \otimes \mathcal{H}_2$):

$$|\Psi_{GHZ}\rangle \;=\; \frac{1}{\sqrt{2}}\big(\,|000\rangle + |111\rangle\,\big)$$

Mixed states: density operators on $\mathcal{H}_1 \otimes \mathcal{H}_2$.

1. **Separable states:**

$$\rho \;=\; \sum_i p_i\, \rho_i^1 \otimes \rho_i^2 \qquad \text{where } \sum_i p_i = 1$$

For example:

$$\rho_{sep} \;=\; \frac{1}{2}\,|01\rangle\langle 10| + \frac{1}{2}\,|10\rangle\langle 01|$$

Special case: **product state:**

$$\rho \;=\; \rho^1 \otimes \rho^2$$

For example: **maximally mixed state**

$$\frac{\mathbf{1}_4}{4} \;=\; \frac{\mathbf{1}_2}{2} \otimes \frac{\mathbf{1}_2}{2} = \frac{1}{4}\big(\,|00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 10| + |11\rangle\langle 11|\,\big)$$

2. **Entangled states:** not separable

Schrödinger on entanglement (*Verschränkung*):

> "When two systems, of which we know the states by their respective representatives, enter into temporary physical interaction due to known forces between them, and when after a time of mutual influence the systems separate again, then they can no longer be described in the same way as before, viz. by endowing each of them with a representative of its own. I would not call that *one* but rather *the* characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought." (Schrödinger, 1935)

Werner state: mixture of the singlet state and the maximally mixed state

$$\rho_{Werner} \;=\; p\,|\Psi^-\rangle\langle\Psi^-| + (1-p)\frac{\mathbf{1}_4}{4}$$

is **separable** if $p \in [0, \frac{1}{3}]$ and **entangled** if $p \in (\frac{1}{3}, 1]$ (but does **not violate** the CHSH inequality for $p \in (\frac{1}{\sqrt{2}}, \frac{1}{3}]$!)

Probabilities:

$$\begin{aligned}
p_{\Psi^-}(P_{00}) &= \langle \Psi^- | P_{00} \Psi^- \rangle = \langle \Psi^- | \big(|00\rangle\langle 00| \big) |\Psi^-\rangle = \langle \Psi^- |00\rangle \langle 00|\Psi^-\rangle = |\langle \Psi^- |00\rangle|^2 = 0 \\
p_{\rho_{sep}}(P_{00}) &= Tr(\rho_{sep}\, P_{00}) = Tr\Big[\big(\frac{1}{2} |01\rangle\langle 10| + \frac{1}{2} |10\rangle\langle 01| \big) |00\rangle\langle 00| \Big] = 0
\end{aligned}$$

GHZ test: we rewrite the GHZ state in the $|x^\pm x^\pm x^\pm\rangle$ and $|y^\pm y^\pm x^\pm\rangle$ **basis:**

$$\begin{aligned}
|\Psi_{GHZ}\rangle &= \frac{1}{\sqrt{2}} \big(|000\rangle + |111\rangle \big) \\
&= \frac{1}{2} \big(|x^+x^+x^+\rangle + |x^-x^-x^+\rangle + |x^-x^+x^-\rangle + |x^+x^-x^-\rangle \big) \\
&= \frac{1}{2} \big(|y^-y^-x^-\rangle + |y^+y^+x^-\rangle + |y^+y^-x^+\rangle + |y^-y^+x^+\rangle \big)
\end{aligned}$$

Measuring $X_1X_2X_3$, the probability of obtaining an **odd** number of spin-up outcomes is 1: $\langle x^+x^+x^+ | x^+x^+x^+\rangle = \frac{1}{4}$ probability for three ups, $\frac{3}{4}$ probability for one up

Measuring $Y_1Y_2X_3$, the probability of obtaining an **even** number of spin-up outcomes is 1: $\langle y^-y^-x^- | y^-y^-x^-\rangle = \frac{1}{4}$ probability for zero up, $\frac{3}{4}$ probability for two ups

Assignment 3. Calculate the following probabilities:

$$\begin{aligned}
p_{\Psi^-}(P_{++}) &= ? \\
p_{\rho_{sep}}(P_{++}) &= ? \\
p_{\rho_{Werner}}(P_{00}) &= ?
\end{aligned}$$

(Send it to: hoferszabogabor@gmail.com by Tuesday midnight.)

Readings

Hughes, R. I. G., *The Structure and Interpretation of Quantum Mechanics* (Cambridge: Harvard University Press, 1989).

4 The Interpretations and the Measurement Problem

An interpretation of a physical theory tells us **what the world is like if the theory is true**. Interpretations of QM can be classified on whether they take the wave function to be **ontic or epistemic**. **ψ-ontic interpretations** run into difficulties with the measurement. They can be further classified on their response to the **measurement problem**

What is an Interpretation?

Van Fraassen (1991, 242): Interpretation: "Under what conditions is this theory true? What does it say **the world is like**?"

Structure-specification:

Basic concepts:	**Classical mechanics**	**Quantum mechanics**
System	Phase space	Hilbert space
States (pure/mixed)	Points in/probability measures on the phase space	Vectors in/density matrices on the Hilbert space
Observables	Functions on the phase space	Operators on the Hilbert space
Events	(Projections onto) subsets of the phase space	(Projections onto) linear subspaces of the Hilbert space
Dynamics	Time-evolution of the state	Time-evolution of the state

Semantic: assigning truth values to the propositions of a theory

Classical physics:

"Observable f in state (q,p) has value x" is true just in case $f(q,p) = x$

"The value of the observable f in state (q,p) lies in the interval Δ" is true just in case $f(q,p) \in \Delta$ or equivalently

$(q,p) \in f^{-1}(\Delta)$

"The probability that the value of the observable f lies in the interval Δ is r" is true just in case $p\left(f^{-1}(\Delta)\right) = r$

Quantum mechanics:

"Observable A in state ψ has value x" is true just in case $A\psi = x\psi$

"The value of the observable A in state ψ lies in the interval Δ" is true just in case $P^A_\Delta \psi = \psi$ (where P^A_Δ is the spectral projection of A with spectrum Δ)

"The probability that the value of the observable A lies in the interval Δ is r" is true just in case $p\left(P^A_\Delta\right) = r$

Ruetsche (2011, 2): "to interpret a physical theory is to identify the **set of worlds possible according to that theory**... what the theory's possible worlds are depends only on its laws"

Redhead (2089, 44): Interpretation: "some account of the nature of the external worlds and/or our epistemological relation to it that **serves to explain** how it is that the statistical regularities predicted by the formalism with the minimal statistical interpretation come out the way they do."

Correspondence rules: The choice between interpretations of quantum mechanics is a choice between different correspondence rules which establish a correspondence between mathematical quantities and physical entities

Specifically: What does the probability $Tr(\rho\, P^A_i)$ or $\langle\psi|P^A_i\psi\rangle$ and the terms in it refer to?

Note: After specifying the meaning of the above terms, the meaning of every other term (e.g. **Heisenberg's uncertainty relation**) composed of these will also be fixed.

Interpretations of quantum theory

I. ψ-epistemic interpretations: the wave function **is not part** of the ontology

- **Operationalist (minimalist) interpretation:** Textbook representation of states, observables, dynamics • QM is about predicting the statistics of measurement outcomes of systems prepared in certain quantum states
- **Copenhagen interpretation:** Textbook representation of states, observables, dynamics • The wave function collapses at measurements • Observables have definite

values only in eigenstates • Nature is intrinsically indeterministic • QM applies to individual systems • In describing micro-objects, one needs to apply complementary descriptions and use classical concepts

Neo-Copenhagen interpretations:

- **Modal Interpretations:** There is no collapse • There are two states: the quantum state and the property state • This latter specifies the set of definite properties of the system
- **Consistent histories interpretation:** "Copenhagen done right" • Micro-objects have a history: a time-sequence of properties • QM describes these micro-objects in terms of consistent sets of histories (frameworks) • Quantum paradoxes arise from combining properties that belong to incompatible frameworks
- **Relational interpretation:** QM is not about properties of objects but about relations between objects • Measurement is an ordinary physical interaction • "Absolute" or "observer-independent" state of a quantum system has no meaning
- **Quantum Bayesianism:** The wave function encodes an observer's state of knowledge about a quantum system • Collapse is the change of the observer's knowledge • QBism is an information-theoretic approach to QM
- **Ensemble interpretation:** The wave refers to an ensemble of identically prepared systems • Collapse is the change of the ratio of properties in the ensemble

II. ψ-ontic interpretations: the wave function **is part** of the ontology: these interpretations can be grouped on their response to the **measurement problem**

The measurement problem: if QM is also valid for the **measurement apparatus**, **ψ-ontic interpretations** run into difficulties in accounting for the measurement process due to the **linearity of the Schrödinger dynamics:**

$$\begin{aligned}|\psi_1\rangle &\xrightarrow{\text{Schrödinger}} |\psi_1'\rangle\\ |\psi_2\rangle &\xrightarrow{\text{Schrödinger}} |\psi_2'\rangle\\ \frac{1}{\sqrt{2}}\big(|\psi_1\rangle + |\psi_2\rangle\big) &\xrightarrow{\text{Schrödinger}} \frac{1}{\sqrt{2}}\big(|\psi_1'\rangle + |\psi_2'\rangle\big)\end{aligned}$$

This causes a problem **at the measurement:**

System + Apparatus: $\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_A$

$$\begin{aligned}
|0\rangle \otimes |a_R\rangle &\xrightarrow{\text{Schrödinger}} |0\rangle \otimes |a_0\rangle \\
|1\rangle \otimes |a_R\rangle &\xrightarrow{\text{Schrödinger}} |0\rangle \otimes |a_1\rangle \\
\frac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big) \otimes |a_R\rangle &\xrightarrow{\text{Schrödinger}} \Psi = \frac{1}{\sqrt{2}}\big(|0\rangle \otimes |a_0\rangle + |1\rangle) \otimes |a_1\rangle\big)
\end{aligned}$$

where $|a_R\rangle$ is the 'ready state' of the apparatus

However, we never see the apparatus in **superposition** Ψ (we cannot make an interference between $|a_0\rangle$ and $|a_1\rangle$)[1]

The post-measurement state is the **mixed state** $\frac{1}{2}P_0 + \frac{1}{2}P_1$

But the **Schrödinger evolution** takes pure states into pure states

What shall we do?

Schrödinger's cat:

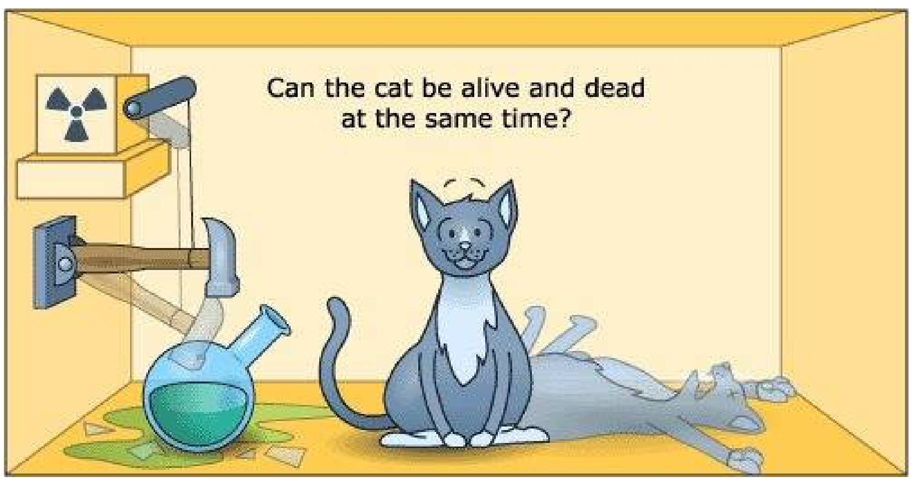


Figure 11: Schrödinger's cat

In **Bell's version**: the cat gets milk or not

The cat example is a *reductio ad absurdum*:

> The quantum mechanical description **cannot be complet**e since we always observe the cat in a **definite state**. Superposition expresses just our **ignorance** ("a shaky or out-of-focus photograph") and not a smeared out reality ("a snapshot of clouds"):

[1]Or can we? One can create a superposed state by coupling the centre of mass and the internal degrees of freedom of a $^9Be^+$ ion and perform measurements that are sensitive to this superposition

> It is typical of these cases that an indeterminacy originally restricted to the atomic domain becomes transformed into macroscopic indeterminacy, which can then be resolved by direct observation. That prevents us from so naively accepting as valid a 'blurred model' for representing reality. In itself it would not embody anything unclear or contradictory. There is a difference between a shaky or out-of-focus photograph and a snapshot of clouds and fog banks (Schrödinger, 1935)

Since there is **no sharp line between micro- and macrosystem**, also the microsystem has to be in a definite state.

If the wave function is just about our ignorance, there is no problem with the collapse

This was the **original Born interpretation** in 1926. Later it has been absorbed in the Copenhagen interpretation

However, **the ignorance interpretation is not right**: in the two-slit experiment we would not get inference if the wave function would just code out ignorance of which slit the particles go through. Similarly, many no-go theorems show the same

Bell, 1986: "Either the wave function, as given by the Schrödinger equation, is not everything, or it is not right"

1. **"is not everything": Hidden-variable models**
   - **Bohmian mechanics:** The wave function evolves unitarily and never collapses • There are also hidden variables: the position of the particles • Particles move along definite trajectories guided by their pilot waves
2. **"is not right":**
   - **Collapse interpretations:** The wave function evolves unitarily and also collapses • This collapse is a spontaneous, stochastic and dynamical process
3. **Both everything and right:**
   - **Many-worlds interpretation:** The wave function evolves unitarily and never collapses • There are no hidden variables • At the moment of measurement, the Universe splits into separate, equally real worlds

Assignment 4. What is the measurement problem in quantum mechanics? Explain it in 100-200 words and send it to: hoferszabogabor@gmail.com by Tuesday midnight.

Readings

Pykacz, J., "A Brief Survey of Main Interpretations of Quantum Mechanics", *Quantum Physics, Fuzzy Sets and Logic* (Springer Brief, 2005).

Ruetsche, L., *Interpreting Quantum Theories* (Oxford University Press, 2015).

5 The Operationalist and the Realist Interpretations

The operationalist and the realist interpretations provide **different meanings** to the terms featuring in the formalism of QM.
On the **operationalist interpretation**, the quantum states refer to preparations, the self-adjoint operators refer to measurements, spectral projections refer to measurement outcomes and the expression $Tr(\rho\, P_i^A)$ refers to probability of certain outcome statistics of measurements performed on systems previously prepared in certain ways.
On the **realist (ψ-ontic) interpretation**, the quantum states refer to the state of the micro-object, the self-adjoint operators refer to observables (physical quantities) of these micro-objects, spectral projections refer to the values of these observables and the expression $\langle\psi|P_i^A\psi\rangle$ refers to the probability of the system's corresponding observable having a certain value outcome

I. The operationalist (instrumentalist, minimalist, verificationalist, empiricist, anti-realist) interpretation associates (non-uniquely) the terms in the formalism of QM as follows

$$\begin{aligned}
\rho &\longrightarrow S_\rho : \text{ preparation} \\
A &\longrightarrow M_A : \text{measurement} \\
P_i^A &\longrightarrow X_i^A : \textit{ith} \text{ outcome of the measurement } M_A
\end{aligned}$$

Born rule: the expression $Tr(\rho\, P_i^A)$ is the probability of getting the *ith* outcome if the measurement M_A represented by an operator

$$A = \sum_i a_i P_i^A$$

is performed on a system previously prepared in a state S_ρ represented by the density operator ρ:

$$Tr(\rho\, P_i^A) = p(X_i^A | M_A \wedge S_\rho)$$

The operational interpretation is **minimalist** in the sense that it is minimally needed for QM to be empirically adequate

The operational interpretation is essentially is the **Born statistical interpretation**

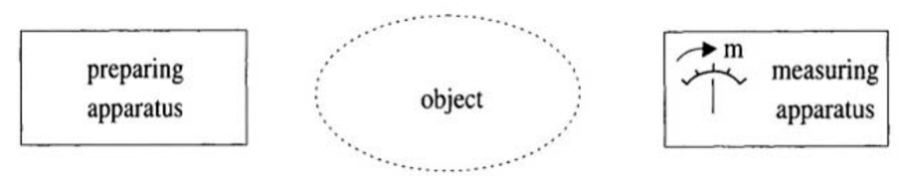


Figure 12: Operationalist interpretations

The probability in the operational interpretation is **frequentist**: $p(X_i^A|M_A \wedge S_\rho)$ is tested by **repeating the experiment many times** say and counting the relative frequencies $\frac{N_i}{N}$

The operational interpretation is a **verificationist/empiricist interpretation**: the emphasis is on the directly observable part of reality

In the operational interpretation the mathematical formalism describes only correlations between **directly observable events**: settings of the knobs of preparing apparata and the readings of pointer positions of measuring instruments

Note that there is **no observable** $\mathcal{A}$ corresponding to a quantity of the micro-object is featuring in the operational interpretation, only the operator A and the measurement M_A realizing it

In the operationalist interpretation the **ontology** contains only preparations and measurements (macroscopic operations)

The operational interpretation **does not deny** that the phenomena described by quantum mechanics are caused by microscopic objects. It claims, however, that these **microscopic objects** and how they cause the macroscopic phenomena **are not described by quantum mechanics** (but maybe by some subquantum (hidden variable) theories)

The terminology in which a Hermitian operator is called an **'observable' fits perfectly** into the operational interpretation. In Heisenberg's original invention of **matrix mechanics**, the theory was formulated so as to contain observable quantities only.

Time evolution is not a description of the state evolution of the microscopic object, but simply **another preparation procedure**—obtained by performing the procedure represented by ρ and waiting some time, $U(t)\rho\, U^\dagger(t)$; **or another retarded measurement**—waiting some time and then performing the measurement, $U^\dagger(t)A\, U(t)$

II. Realist (ψ-ontic) interpretations associate the quantities in the quantum formalism with the state, physical quantity and value the quantity of the micro-object:

$$\begin{array}{rcl} \rho & \longrightarrow & s_\rho : \text{state of the micro-object} \\ A & \longrightarrow & \mathcal{A} : \text{observable (physical quantity) of the micro-object} \\ P_i^A & \longrightarrow & a_i \text{ or } v(\mathcal{A}) = a_i\text{: the event that the value of the observable } \mathcal{A} \text{ is } a_i \end{array}$$

Born rule: the expression $Tr(\rho\, P_i^A)$ represents the probability of the observable $\mathcal{A}$ of the micro-object in state ρ having the value a_i:

$$Tr(\rho\, P_i^A) = p(a_i|s_\rho)$$

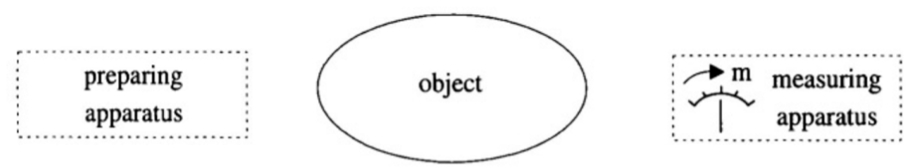


Figure 13: Realist interpretations

Most of physicist are realist: they consider it self-evident that, since quantum mechanics has especially been devised to describe micro-phenomena, it must describe micro-objects: quantum mechanical wave function should replace the classical phase space point as a description of a microscopic particle and observables are viewed upon as properties of the microscopic particles

The standard terminology reinforces this realist attitude: the electron "**is** in a certain quantum state", or "**has** a certain value of momentum"

> **Einstein:** "Physics is an attempt conceptually to grasp reality as it is thought independent of its being observed"
>
> **Bell:** "To restrict quantum mechanics to be exclusively about piddling laboratory operations is to betray the great enterprise."

Motivation: We are not satisfied by merely accepting physics as a description of phenomena. We also ask from our theories **explanations**. The microscopic object has such an explanatory function: it constitutes a causal relation between preparation and measurement

Two different realist interpretations:

1. **Objectivistic-realist interpretation:** the quantum mechanical properties of the object are thought to be **objective properties, possessed by the object inde-**

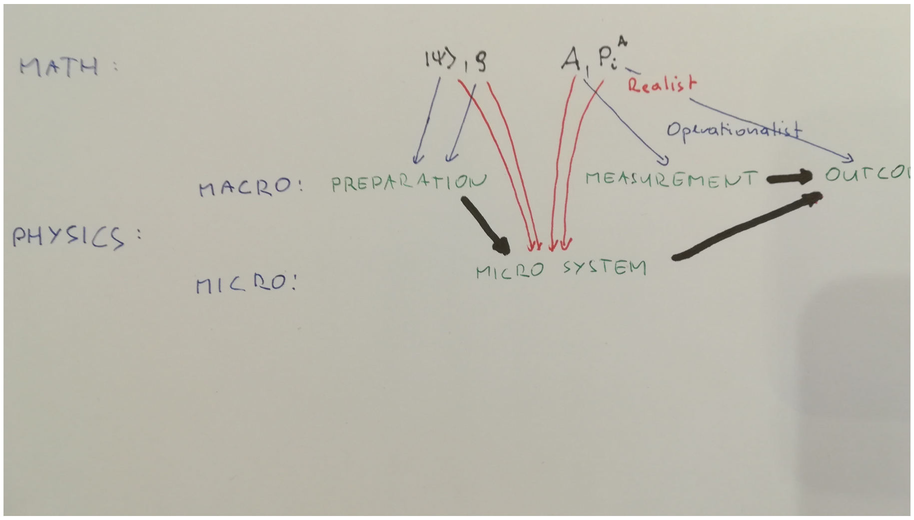


Figure 14: Operationalist and Realist interpretations

pendently of observation, prior to measurement, the object could be considered as an isolated or closed system ("possessed value principle")

2. **Contextualistic-realist interpretation:** the object is thought to have its quantum mechanical properties only within the context of the measurement

   Quantum mechanics is thought not to describe a closed system, but an **open system**, co-determined by its environment

   Many elements of the **Copenhagen interpretation** are contextualistic-realist

Problems (Norsen 135):

The **ontology** of the wave function if not clear: it lives in the **configuration space** which "does not smell like something real" (Einstein, 1926)

The collapse implies **nonlocality**

Measurement problem: there are **no definite measurement outcomes**

The most **no-go theorems** are directed against the objectivistic-realist interpretation

In the realist interpretation, terms have a **double referent**: ρ refers to both the preparation and the state of the system, and A refers to both the measurement and the quantity. To make this notation **consistent**, one introduces the:

Principle of faithful measurement: $p(X_i^A|a_j \wedge M_A) = \delta_{ij}$: the outcome of the measurement is numerically equal to the value the observable had immediately preceding the measurement

Principle of faithful preparation: $p(\psi|S_{\psi'}) = \delta_{\psi\psi'}$

Assignment 5. What is the difference between the operationalist and the realist interpretations of quantum mechanics? Explain it in 100-200 words and send it to: hoferszabogabor@gmail.com by Tuesday midnight.

Readings

de Muynck, W. M., *Foundations of Quantum Physics, an Empiricist Approach* (Kluwer, 2002).

6 The Copenhagen Interpretation

The Copenhagen (Orthodox) Interpretation is the **“official” interpretation** of QM since the Solvay conference in 1927. It has been elaborated, defended and indoctrinated by Bohr, Heisenberg, Born, Pauli, Dirac and von Neumann.

Figure 15: Bohr, Heisenberg and Pauli

Text book principles:

A quantum system is represented by a complex Hilbert space

A composite systems is represented by the tensor product of the corresponding Hilbert spaces

States of a system are represented by unit vectors (rays) in or density operators on the Hilbert space

Observables (physical quantities) of the system are associated with self-adjoint operators acting on the Hilbert space

Dynamics: the state of the system evolves in time according to the Schrödinger equation

Born rule: the expression $Tr(\rho\, P_i^A)$ is the probability that *after* the measurement M_A the value of $\mathcal{A}$ of the system prepared in in state ρ will be a_i is:

$$Tr(\rho\, P_i^A) = p(a_i | M_A \wedge S_\rho)$$

Projection postulate or collapse: if a system in state ρ is measured, its state will stochastically jump into the state

$$\rho \xrightarrow{M_A} \rho' = \frac{P_i^A\, \rho\, P_i^A}{Tr(P_i^A\, \rho\, P_i^A)}$$

with probability $Tr(\rho\, P_i^A)$.

The eigenstate-eigenvalue link: the system has (in the present) a definite value of the observable $\mathcal{A}$ iff its state vector $|\psi\rangle$ is an eigenstate of A. In this case, the definite value is the eigenvalue a_i:

$$v_\psi(\mathcal{A}) = a_i \iff A\,|\psi\rangle = a_i\,|\psi\rangle$$

Remarks:

The Copenhagen interpretation is a **Ψ-epistemic interpretation**. The wave function is not part of the ontology. It does not refer to a property of the micro-object (as in the Ψ-epistemic interpretations), neither to the preparation (as in the operational interpretation). It refers to whole experimental arrangement (including obviously the micro-object).

The Copenhagen interpretation **does not make a clear difference between preparations and measurements** (Ballentine) since—due to Bohr's holistic and contextualistic understanding of the experimental process—it does not associate the wave function with the former and the self-adjoint operators with the latter. Both the wave function and the operators refer to the whole experimental arrangement including the preparation and the measurement. Therefore, it is maybe better to talk about $(M + S)_{\rho,A}$ as the holistic experimental arrangement represented by ρ and A

The Copenhagen interpretation is a kind of **combination of the operational and the Ψ-ontic interpretation**. It resembles the operational interpretation in that the wave function and the operators symbolically represents (Halvorson, ???) the whole experimental arrangement including the preparation and the measurement. It resembles the Ψ-ontic interpretation in that it assigns values (at least in eigenstates) to observables.

The Born rule follows from the **projection postulate** and the **eigenstate-eigenvalue link**: measuring $\mathcal{A}$ on a system in state ψ, the system will jump into one of the eigenstates ψ_i of A with probability $|\langle\psi_i|\psi\rangle|^2$and will have the possessed value a_i. One can also assume that upon measurement the probability of the system's having a possessed value is the same as the probability of the corresponding measurement outcome:

$$p(a_i|M_A \wedge S_\psi) = p(X_i^A|M_A \wedge S_\psi)$$

Strictly speaking this fact **does not follow from the principle of faithful measurement**, since this principle says that measuring the system having a fixed value the measurement will reveal this value, whereas here the value and the outcome is established at the same act of measurement.

Other principles (heuristics):

Indeterminism: Nature is intrinsically indeterministic and acausal

Individuality: The wave function refers to the state of an individual system not to that of an ensemble. The Born probabilities refer to individual systems. Probabilities are propensities

Classicality: QM is primarily about describing a microsystem in terms of experimental arrangements and results. To do this unambiguously, one needs to use classical concepts, ordinary language and classical physics. Statements about the system outside the measurement context have no meaning. "No elementary phenomenon is a phenomenon until it is a registered (observed) phenomenon" (Wheeler)

Complementarity: When describing a microsystem, the appropriate description depends on the experimental context. Observables corresponding to mutually exclusive laboratory arrangements cannot be defined (contextuality). However, to fully characterize a system, one needs mutually exclusive experimental contexts (wave–particle duality). This feature of the micro-world is represented by the non-commutativity of observables in the formalism

Observation is an interaction between the system and the measuring apparatus. Due to the finiteness of the quantum interaction (h), the interaction introduces an uncontrollable influence on the system causing an irreversible collapse of the wave function onto an eigenstate of the observable. It is the observation itself which selects the measured observable (the preferred basis) of the system. During the observation, potentialities turn into actualities

Completeness: QM is complete: there are no experimentally verifiable predictions about atomic phenomena that cannot be described by QM ($\neq$ Einstein's completeness condition)

Correspondence: In the appropriate limit, QM comes to resemble classical physics and reproduces the classical predictions

Heisenberg's uncertainty relation:

$$\Delta A_\psi \cdot \Delta B_\psi \geqslant \frac{1}{2} |\langle C \rangle_\psi|$$

If ψ is an **eigenstate** of A or B, then the right hand side is 0

The uncertainty relation can be derived from the **noncommutativity** of A and B:

$$[A, B] = i\, C$$

For discrete spectrum, C **cannot be the unit operator**, $\mathbf{1}$, but for continuous spectrum it can. The **most famous** uncertainty relation is

$$\Delta Q_\psi \cdot \Delta P_\psi \geqslant \frac{1}{2}\hbar$$

Interpretation of Heisenberg's uncertainty relation: scattering, incombatility, complementarity, inaccuracy?

Note that **the uncertainty relation gets interpreted** as soon as the terms in it are interpreted. Since in every interpretation the term $\langle\psi|A\psi\rangle$ is already interpreted, so also the uncertainty relation

It is **not a constraint on measurements but on preparations:** one cannot prepare a state such that the product of the standard deviation (scattering) of two measurements represented by noncommuting operators and performed separately on an ensemble of systems prepared in state ψ is below a certain threshold

It is **not about complementarity:** not a constraint on **simultaneous measurability** of the observables $\mathcal{A}$ and $\mathcal{B}$ on an **individual** system

It is **not about mutual disturbance of the measurements:** $\mathcal{A}$ and $\mathcal{B}$ can be measured on two different sub-ensembles.

It is **not a constraint on the accuracy of simultaneous measurements:** $\delta\mathcal{A} \cdot \delta\mathcal{B} \geqslant \frac{1}{2}\hbar$. This inequality can be **violated**. **Ballentine** (1970, 365) has shown that the position and momentum of a particle diffracting through a slit can be determined more precisely than $\hbar$ (Muynck, p. 230)

Consequently, it is **not a constraint on the attribution of well-defined values to individual systems**

Einstein-Bohr debate

It happened during the **5th Solvay Conference** in Brussels in 1927

General structure:

Einstein attacked the **completeness** of the Copenhagen interpretation by showing up (often using the assumption of locality) elements of reality which are missing from the quantum mechanical description

Bohr however systematically **misunderstood** Einstein's examples. Since he did not believe on unobservable terms he understood Einstein as trying to determine the quantum mechanical quantities **more precisely than it is allowed by Heisenberg's uncertainty relation**

Diffraction (one-slit) experiment:

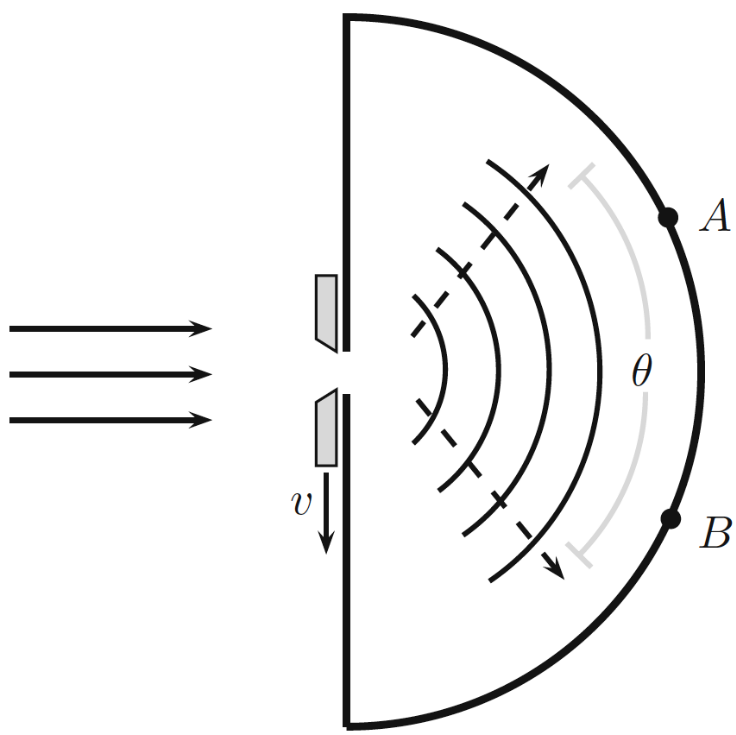


Figure 16: Diffraction

Einstein: The diffracted spherical wave function assign a given probability both to point A and B. Since after the particles hitting the screen, there is a perfect anticorrelation between for the particle arriving at point A and at point B, therefore there is either a kind of **spooky action at a distance**, or the particle had a **definite location** all along (such that the description in terms of a diffracting wave was incomplete)

Bohr: One cannot define the position and momentum of the particle more precisely than Heisenberg's uncertainty relation $\delta z \cdot \delta p_z \gtrsim h$. The vertical inaccuracy of position of the particle at the moment of going through the slit is $\delta z = a$ where a is the width of the slit. The particle's original momentum is p. When it goes though the slit, it gets deflected by an angle of θ. The maximum of this θ can be determined by considering the particle as a plane wave diffracting through a slit and determining the edge of the first reinforcement region: $sin\theta = \frac{\lambda}{a}$. Then, using the de Broglie relation $p = \frac{\lambda}{d}$, the maximum of the z-component of the deflected momentum is:

$$\delta p_z \quad = \quad p\, sin\theta = \frac{h}{\lambda}\frac{\lambda}{a} = \frac{h}{a}$$

Thus, $\delta z \cdot \delta p_z \approx h$.

> This is a strange argument. When the particle hits the screen, we can calculate the ???

EPR argument

The EPR argument is Einstein's ultimate attempt in the discussion with Bohr to prove the **incompleteness QM** by showing up a situation where **incompatible properties** of a system can be measured/defined more precisely than allowed by Heisenberg's **uncertainty relation**.

Einstein was an **objective realist**, while Bohr was a **contextualistic-realist** firmly sticking to the idea that properties can be attributed to a system only in a given experimental arrangement

In the earlier *Gedankenexperiments*, Bohr could always counter Einstein's examples by pointing out the **disturbing effect** of the measuring apparatus on the system. In the EPR argument, the system **does not interact** with any measuring instrument due to **spatial separation**

The argument:

> **Completeness Criterion:** "every element of the physical reality must have a counterpart in the physical theory"
>
> **Reality Criterion:** "If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of reality corresponding to that quantity"

The Reality Criterion is a special case of the **Common Cause Principle** (Gömöri, Hofer-Szabó, 2021)

Assumptions:

Perfect correlation: established by a singlet state

$$|\Psi_{12}\rangle = \frac{1}{\sqrt{2}}\big(\,|01\rangle - |10\rangle\,\big) = \frac{1}{\sqrt{2}}\big(\,|{+}{-}\rangle - |{-}{+}\rangle\,\big)$$

Locality: performing a measurement on system 1 has no causal influence on the elements of reality of system 2 (this assumption is buried in the original EPR paper)

Argument:

Measuring the z-spin on system 1 and obtaining "up", we can predict with certainty the z-spin outcome of system 2

Due to **Locality**, measurement on system 1 did not disturb the system 2

So we can apply the **Reality Criterion**: there must be an element of reality determining the z-spin outcome of system 2

Note: In the Copenhagen interpretation, **the only place for an element of reality** is the value of an observable in an eigenstate. Thus, each below version of the argument will use the **eigenstate-eigenvalue link**

Version A (using time): This element of reality must have been there even before the measurement. But then the state of the system was described by $|\Psi_{12}\rangle$ which is not an eigenstate of σ_z, therefore does not describe this element of reality

Version B (using incompatible measurements): We could have measured the x-spin or any other spin direction instead of the z-spin. This would imply that there must be an element of reality for every (incompatible) observables. But QM does not describe these elements of reality

> Einstein: whether we use incompatible observables or not "*ist mir wurst*"

Version C (using the projection postulate): Before measuring z-spin on system 1, system 2 is not in an eigenstate of σ_z, hence the value of the z-spin for system 2 is not determinate. After measuring the z-spin on system 1 and obtaining "up", system 2—due to the projection postulate—will jump into the state $|\psi_2\rangle = |1\rangle$ and—due to the eigenstate-eigenvalue link—will have the value "z-spin up". Thus, due the measurement on system 1, the possessed value of system 2 has changed, violating locality

Conclusion: QM is incomplete

Remark:

Determinism ("God doesn't play dice") **is not assumed** in the argument. It is a consequence of the **Reality Criterion**

Bohr's contextualistic answer: "Of course there is ... no question of a mechanical disturbance of the system under investigation during the last critical stage of the measuring procedure. But even at this stage there is essentially the question of *an influence on the very conditions which define the possible types of predictions regarding the future behavior of the system*"

The eigenstate-eigenvalue link (Arthur Fine): A quantum system has a determinate value of a property, which is represented by a self-adjoint operator if and only if the system's state vector is an eigenstate of that operator

Dirac: "The expression that an observable 'has a particular value' for a particular state is permissible in quantum mechanics in the special case when a measurement of the observable is certain to lead to the particular value, so that the state is an eigenstate of the observable."

Indeterminate value: does not exist or is not fixed?

Incompatible observables never have simultaneous values

Most of the time and for mosts of the observables there is no definite value

Combined with the **projection postulate**, the eigenstate-eigenvalue link states that **after the measurement** of an observable a system will possess a definite value corresponding to the eigenstate into which the system jumped into

An argument for the Eigenstate-eigenvalue link (Fletcher and Taylor, 2021):

The **"if"-part** can be supported by the **EPR Criterion of Reality:**

> "If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of reality corresponding to that quantity"

In an eigenstate we can predict with certainty the measurement outcome, therefore there must be an element of reality which is the definite value of the observable

The **"only if"-part** can be supported by the **EPR Criterion of Completeness:**

> If a physical theory is complete, then "every element of the physical reality must have a counterpart in the physical theory"

Beyond the Eigenstate-eigenvalue link, there is no other postulates by which the definite value would be represented

Which observables have definite value in a given pure state?

Classical states are property states: they specify the maximal list of properties of the system at a particular time

Property state on the orthodox Dirac-von Neumann interpretation corresponding to $|\psi\rangle$: $\{P \in \mathcal{P}(\mathcal{H}) : P_\psi \leqslant P\}$ (subspaces containing the ray of ψ)

Determinate events: $\mathcal{L}_\psi = \{P \in \mathcal{P}(\mathcal{H}) : P_\psi \leqslant P \text{ or } P_\psi \leqslant P^\perp\}$

Equivalently: $\mathcal{L}_\psi = \{P \in \mathcal{P}(\mathcal{H}) : Tr(PP_\psi) = 1 \text{ or } 0\}$

$\mathcal{L}_\psi$ is an **orthomodular sublattice** of $\mathcal{P}(\mathcal{H})$

It has an **"umbrella-shape"**

At the measurement the Eigenstate-eigenvalue link becomes problematic

System + Apparatus: $\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_A$

$$\begin{aligned} |0\rangle \otimes |a_R\rangle &\longrightarrow |0\rangle \otimes |a_0\rangle \\ |1\rangle \otimes |a_R\rangle &\longrightarrow |0\rangle \otimes |a_1\rangle \\ \frac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big) \otimes |a_R\rangle &\longrightarrow \Psi = \frac{1}{\sqrt{2}}\big(|0\rangle \otimes |a_0\rangle + |1\rangle) \otimes |a_1\rangle\big) \end{aligned}$$

But the determinate sublattice $\mathcal{L}_\Psi$ does not contain the projections $P_0 \otimes \mathbf{1}$ and $P_1 \otimes \mathbf{1}$ since

$$Tr\big((P_0 \otimes \mathbf{1})\, P_\Psi\big) = \langle \Psi | (P_0 \otimes \mathbf{1}\big)\Psi\rangle = \frac{1}{2}$$

Therefore, the **observable does not have a determinate value** after the measurement

Solution: two processes

1. When the system is left alone: **unitary (Schrödinger) evolution:**

$$\rho \xrightarrow{\text{unitary}} \rho(t) = U(t)\,\rho\, U^{\dagger}(t)$$

where $U(t) = e^{-iHt/\hbar}$ if the Hamiltonian H is time-independent.

2. When the system is measured: **projection postulate (collapse, Lüder's rule):**

$$\rho \xrightarrow{\text{collapse}} \rho' = \frac{P_i^A\,\rho\,P_i^A}{Tr(P_i^A\,\rho\,P_i^A)}$$

where P_i^A are the spectral projections of the measured observable $\mathcal{A}$

That is, upon measurement the system projects into a mixed state:

$$\frac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big) \xrightarrow{\text{collapse}} \frac{1}{2}P_0 + \frac{1}{2}P_1$$

The system has determinate values; the Eigenstate-eigenvalue link can be upheld.

Consistency

System | Apparatus$_1$ + Apparatus$_2$: measuring the System by Apparatus$_1$ results in a mixed state:

$$\frac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big) \xrightarrow{\text{collapse}} \frac{1}{2}P_0 + \frac{1}{2}P_1$$

System + Apparatus$_1$ | Apparatus$_2$: letting the composite system (System + Apparatus$_1$) evolve unitarily and then measuring it by Apparatus$_2$:

$$\frac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big)\otimes|a_R\rangle \xrightarrow{\text{unitary}} \frac{1}{\sqrt{2}}\big(|0\rangle\otimes|a_0\rangle + |1\rangle\big)\otimes|a_1\rangle\big) \xrightarrow{\text{collapse}} \frac{1}{2}\big(P_{0a_0} + P_{1a_1}\big)$$

where $|a_0\rangle$ and $|a_1\rangle$ are pointer states of Apparatus$_1$. Taking the reduced state (partial state) of the system, we get $\frac{1}{2}P_0 + \frac{1}{2}P_1$, just as above. ✓

von Neumann: we are free to move the cut

"Let us now compare these circumstances with those which actually exist in nature or in its observation. First, it is inherently entirely correct that the measurement or the related process of the subjective perception is a new entity relative to the physical environment and is not reducible to the latter. Indeed, subjective perception leads us into the intellectual inner life of the individual, which is extra-observational by its very nature (since it must be taken for granted by any conceivable observation or experiment) .... Nevertheless, it is a fundamental requirement of the scientific viewpoint - the so-called principle of the **psycho-physical parallelism** – that it must be possible so to describe the extra-physical process of the subjective perception as if it were in reality in the physical world – i.e., to assign to its parts equivalent physical processes in the objective environment, in ordinary space .... In a simple example, these concepts might be applied about as follows: We wish to measure a temperature. If we want, we can pursue this process numerically until we have the temperature of the environment of the mercury container of the thermometer, and then say: this temperature is measured by the thermometer. But we can carry the calculation further, and from the properties of the mercury, which can be explained in kinetic and molecular terms, we can calculate its heating, expansion, and the resultant length of the mercury column, and then say: this length is seen by the observer. Going still further, and taking the light source into consideration, we could find out the reflection of the light quanta on the opaque mercury column, and the path of the remaining light quanta into the eye of the observer, their refraction in the eye lens, and the formation of an image on the retina, and then we would say: this image is registered by the retina of the observer. And were our physiological knowledge more precise than it is today, we could go still further, tracing the chemical reactions which produce the impression of this image on the retina, in the optic nerve tract and in the brain, and then in the end say: these chemical changes of his brain cells are perceived by the observer. But in any case, no matter how far we calculate – to the mercury vessel, to the scale of the thermometer, to the retina, or into the brain, at some time we must say: and this is perceived by the observer. That is, we must always divide the world into two parts, the one being the observed system, the other the observer. In the former, we can follow up all physical processes (in principle at least) arbitrarily precisely. In the latter, this is meaningless. The boundary between the two is arbitrary to a very large extent. ... That this boundary can be pushed arbitrarily deeply into the interior of the body of the actual observer is the content of the principle of the psycho-physical parallelism – but this does not change the fact that in each method of description the boundary must be put somewhere, if the method is not to proceed vacuously, i.e., if a comparison with experiment is to be possible. Indeed, experience only makes statements of this type: an observer has made a certain (subjective) observation; and never any like this: a physical quantity has a certain value.

Now quantum mechanics describes the events which occur in the observed portions of the world, so long as they do not interact with the observing portion, with the aid of the process 2 [unitary dynamics], but as soon as such an interaction occurs, i.e., a measurement, it requires the application of process 1 [projection]. The dual form is therefore justified. However, the danger lies in the fact that the principle of the psycho-physical parallelism is violated, so long as it is not shown that the boundary between the observed system and the observer can be displaced arbitrarily in the sense given above."

What is the collapse?

Bohr: does not take collapse as a physical process (because he took the wave function—living in the configuration space—to be only a symbolic representation of the atomic processes)

Heisenberg: collapse is a physical process and cannot be analyzed any further because of its indeterministic nature

von Neumann, Wigner: human consciousness causes the collapse (Wheeler: 'observer-participancy')

Collapse and nonlocality

In the **realist interpretation** where ψ refers to a property/propensity of an individual system/ensemble of systems, the **collapse** is an objective process and it **implies non-locality**: for a composite system in singlet state, after measuring the z-spin of system/ensemble 1 and the state obtaining $+1$, the state of system/ensemble 2 which is spatially separated from system/ensemble 1 jumps abruptly from $\frac{1}{2}P_0 + \frac{1}{2}P_1$ into $|1\rangle$. Assuming **eigenstate-eigenvalue link**, this non-local change of state results in **non-local change of possessed values** of system/ensemble 2

In the **operational interpretation** where ψ refers to a preparation procedure, the collapse is just a new preparation procedure (just like the Schrödinger evolution of the state) based on some extra machination (e.g. measuring a subsystem and obtaining a result). These machinations have **no non-local influence** on remote measurements (**no-signaling**). Note, that **the operational interpretation is also non-local** but not because of the collapse but because of certain correlation of spacelike separated measurement outcomes which do not have a common causal explanation (Bell inequalities). Since the operational interpretation is minimal (in the sense that any interpretation of QM has to include it), therefore **QM itself is non-local**

In the **Ψ-epistemic interpretation**, where ψ is just a book-keeping device, the **collapse does not imply non-locality** (nevertheless, these interpretations are also non-local)

In the **Copenhagen interpretation** where the wave function and the observables together and undistinguished refer to the whole experimental situation, the collapse refers to a new experimental situation. Thus, non-locality disappears in a kind of **contextualistic realism**

Wigner's friend: When does the collapse occur?

A quantum system is in a **superposition** $\frac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big)$

There are **two observers:** F (friend), **inside** the laboratory and W (Wigner), **outside** the laboratory

F measures the system; the system **collapses** to the mixed state $\frac{1}{2}P_0 + \frac{1}{2}P_1$ (due to the projection postulate)

Before entering the lab, W describes the state of the joint system as $\frac{1}{\sqrt{2}}\big(|0\rangle|F_0\rangle + |1\rangle|F_1\rangle\big)$ (based on the unitary evolution)

After entering the lab, W describes the state of the joint system as collapsing to $\frac{1}{2}P_0 P_{F_0} + \frac{1}{2}P_1 P_{F_1}$(based on the projection postulate) and consequently the state of the quantum system as $\frac{1}{2}P_0 + \frac{1}{2}P_1$

Question: When does the collapse of the quantum system happen? When F measured or when W measured?

Wigner's answer: Since W can ask F whether he was sure in the outcome *before* being asked (and would answer 'yes'), the collapse occurred when first measured. The collapse is due to the systems interaction with a **conscious being**

Rovelli's answer: both descriptions are right—**the quantum state is relative to the observer**

Everett's answer: these is **no collapse**. After the measurement of F, the system will be in the **superposition** $\frac{1}{\sqrt{2}}\big(|0\rangle|F_0\rangle + |1\rangle|F_1\rangle\big)$. W can **verify** this by making **interference** measurement on $S+F$ or making a composite measurement such that one basis vector is $\frac{1}{\sqrt{2}}\big(|0\rangle|F_0\rangle + |1\rangle|F_1\rangle\big)$. Thus, **both branches must exist**

Collapse has been introduced to **maintain the eigenstate-eigenvalue link at the measurement**

Before completely abandoning the Eigenstate-eigenvalue link, let us see its **possible generalizations**. Maybe the set of determinate properties were too meagre and one needs to define it using **both the quantum state and the preferred observable** $\longrightarrow$ Modal interpretation

Positivism?

The Copenhagen interpretation was highly diverse in the 20's and 30's. The core idea was **Bohr's complementarity idea** according to which—due to the entanglement between the system and the measurement apparatus—the ideal of a detached observer is inaccessible in QM. Consequently, we need to describe the system taking into account the whole experimental arrangement described by classical language. The positivist attitude of von Neumann, Jordan and Wigner were only side interpretations. After WWII, however, Heisenberg put a twist on the original Bohrian idea by describing the process of measurement as a **collapse** of the wave function, stressing the subjective element of **observation** and using a more **positivistic** language. The unique-looking Copenhagen interpretation is the creature of Heisenberg further promoted by the criticism of Bohm, Feyerabend, Hanson and Popper (Howard, 2004).

Bohr's contextualistic-realism:

> "The quantum postulate implies that any observation of atomic phenomena will involve an interaction with the agency of observation not to be neglected. Accordingly, an independent reality in the ordinary physical sense can neither be ascribed to the phenomena nor to the agencies of observation... This situation has far-reaching consequences. On one hand, the definition of the state of a physical system, as ordinarily understood, claims the elimination of all external disturbances. But in that case, according to the quantum postulate, any observation will be impossible, and, above all, the concepts of space and time lose their immediate sense. On the other hand, if in order to make observation possible we permit certain interactions with suitable agencies of measurement, not belonging to the system, an unambiguous definition of the state of the system is naturally no longer possible, and there can be no question of causality in the ordinary sense of the word. The very nature of the quantum theory thus forces us to regard the space-time co-ordination and the claim of causality, the union of which characterizes the classical theories, as complementary but exclusive features of the description, symbolizing the idealization of observation and definition respectively." (Bohr [1927] 1934)

Heisenberg's positivism:

> "one can never observe more than one point in the orbit of the electron; therefore, there is no orbit in the ordinary sense" (Heisenberg, 1958)

Bohr's philosophy also has **neo-Kantian** roots:

> "Kant conceived the apparatus of observation as an inner mental faculty, analogous to a pair of spectacles that mediated and in particular gave form to and interpreted raw sense impressions. Neo-Kantians projected the interpretative aspect of vision outwards, reconceiving it as a bodily, and specifically physiological process. Bohr took this further by including observation as [affecting] not merely what we see but also the terms in which we describe it" (Krips, 2008)

Jordan's positivist position:

> "Observations not only disturb what is to be measured, they produce it. ... We compel [the particle] to assume a definite position."

Characterization of the Copenhagen interpretation:

It does not make a clear difference between **preparations** and **measurements** (Ballentine, 1970) since—due to Bohr's holistic understanding of the experimental process—it does not associate the wave function with the former and the self-adjoint operators with the latter.

It also does not accept that all observables have a **possessed value** before the measurement

From these two, it follows that they understood measurement process not as

determinative but as a **preparative** process where not the **pre-measurement** properties should be determined but **post-measurement** properties should be filtered out

Heisenberg explains the **uncertainty relation** via the γ-**microscop** example in this sense: due to the measurement **disturbance** one cannot **prepare** an object's **final** position and momentum more precisely than the threshold specified in the uncertainty relation

Einstein: the Copenhagen interpretation is a "**tranquilizing philosophy**"

Assignment 6. Read the above von Neumann text about the free move of the cut between the observer and the system. Evaluate von Neumann's argument in 100-200 words; or read the paper Don Howard, *Who Invented the "Copenhagen Interpretation"? A study in mythology* and tell the true story of the Copenhagen interpretation in 100-200 words. (Send it to: hoferszabogabor@gmail.com by Tuesday midnight.)

Readings

Bub, J., *Interpreting the Quantum World* (Cambridge University Press, 1997).

Fletcher, S. C., Taylor D. E, "Quantum indeterminacy and the eigenstate-eigenvalue link," Synthese, 199, 11181–11212 (2021).

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Howard, D., "Who Invented the "Copenhagen Interpretation"? A study in mythology," Philosophy of Science, 71, 669-682 (2004).

Krips, H., "Measurement in Quantum Theory," *Stanford Encyclopedia of Philosophy* (2008).

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7 The Modal Interpretations

Modal interpretations are **neo-Copenhagen interpretations:** they generalize the **eigenstate-eigenvalue link** but reject **collapse**. They are **realist** interpretations, kind of hidden-variable models. They try to squeeze out the maximal classical **possibility structure** from $\mathcal{P}(\mathcal{H})$. They were born as a response to the **Kochen-Specker theorems.**

Central idea: there are **two states**

> **Dynamical (theoretical) state:** the quantum state ψ
>
> **Property (value, physical) state:** specifies all the **possible** (occurrent and non-occurrent) **events** (all the possessed properties)

Possible events: not the whole $\mathcal{P}(\mathcal{H})$ but only an orthomodular sublattice, the **determinate sublattice of possibilities**, $\mathcal{L}(\psi, O)$

> $\mathcal{L}(\psi, O)$ is determined by the quantum state ψ (just as in the eigenstate-eigenvalue link) **plus a preferred observable** O at time t

Property states: 2-valued homomorphisms (yes-no maps) on $\mathcal{L}(\psi, O)$

Range of possibilities for the system is defined by these homomorphisms

Actual properties are selected by a **2-valued homomorphism**

Why modal?

> The **possibility structure** of a quantum world is represented by **the dynamically evolving (non-Boolean) determinate sublattice** $\mathcal{L}(\psi, O)$—in contrast to the classical world where the possibility structure is fixed for all time as the **Boolean algebra of subsets of a phase space**
>
> The dynamical evolution of **the quantum state tracks the evolution of possibilities** (and probabilities defined over these possibilities) through the evolution of the determinate sublattice $\mathcal{L}(\psi, O)$, rather than actualities; while the dynamically evolving **classical state tracks the evolution of actual properties**.

Properties of $\mathcal{L}(\psi, O)$

It is called a **faux-Boolean algebra**

Construction: Take any set, S, of mutually orthogonal projections in $\mathcal{P}(\mathcal{H})$. Let $S^{\perp}$ be the set of all projections orthogonal to the span of the elements of S. Take the union of S and $S^{\perp}$ and close it under the operations join, meet, and orthocomplementation.

$\mathcal{L}(\psi, O)$ is the **largest structure** in $\mathcal{P}(\mathcal{H})$ on which **classical probability** can be defined as $p(P) = \langle\psi|P\,\psi\rangle$ where $P \in \mathcal{L}(\psi, O)$: the S part is classical, the $S^{\perp}$ part is non-classical but gets probability 0 (the mixed part causes no problem)

Different model interpretations define $\mathcal{L}(\psi, O)$ differently:

1) **Van Fraassen's version:**

The **von Neumann-Lüders measurement** takes the state of the joint system $\mathcal{H}_S \otimes \mathcal{H}_A$ system in state $\psi = \sum_i c_i\ |\psi_i\rangle$ into

$$\Psi \;=\; \sum_i c_i\ |\psi_i\rangle\,|a_i\rangle$$

Determinate properties: the faux-Boolean algebra generated by the set $\{P_{\psi_i} \otimes P_{a_i}\}$ and $(\vee_i(P_{\psi_i} \otimes P_{a_i}))^{\perp}$

Problem: it uses the von Neumann-Lüders measurement

2) **Kochen-Dieks-Healey's version:**

Schmidt decomposition: Any pure state Ψ in $\mathcal{H}_S \otimes \mathcal{H}_A$ can be expressed as

$$\Psi \;=\; \sum_i c_i\ |\psi_i\rangle\,|a_i\rangle$$

for some orthonormal basis $\{|\psi_i\rangle\}$ in $\mathcal{H}_S$ and $\{|a_i\rangle\}$ in $\mathcal{H}_A$. The decomposition is **unique** iff $c_i \neq c_j$ for all $i \neq j$.

Determinate properties: the Boolean algebra generated by the set $\{P_{\psi_i}\}$

Equivalently: $\mathcal{L}_{modal}(\rho_S) = \{P \in \mathcal{P}(\mathcal{H}_S) : Tr(PP_{\rho_i}) = 1 \text{ or } 0\}$ where ρ_S is the reduced density operator of Ψ and P_{ρ_i} are spectral projections of ρ_S

Equivalently: $\mathcal{L}_{modal}(\Psi, \rho_S \otimes \mathbf{1})\big|_S$—the sublattice $\mathcal{L}_{modal}(\Psi, \rho_S \otimes \mathbf{1})$ restricted to $\mathcal{H}_S$

3) **Bub's version:**

R_i: the orthogonal eigensubspaces of the observable O

R_ψ: the subspace corresponding to ψ

$R_{\psi_i} := (R_\psi \vee R_i^\perp) \wedge R_i$: the subspace obtained by projecting ψ onto the subspace R_i

P_{ψ_i}: projection onto R_{ψ_i}

Determinate events: $\mathcal{L}(\psi, O)$ generated by the orthogonal projections $\{P_{\psi_i}\}$ and all the projections in (smaller than) $(\vee_i P_{\psi_i})^\perp$

Generalized "umbrella" with many handles

$\mathcal{L}(\psi, \mathbf{1}) = \mathcal{L}_\psi$

Modal interpretations embrace **complementarity**

> **Bohr:** "Recapitulating, the impossibility of subdividing the individual quantum effects and of separating a behaviour of the objects from their interaction with the measuring instruments serving to define the conditions under which the phenomena appear implies an ambiguity in assigning conventional attributes to atomic objects which calls for a reconsideration of our attitude towards the problem of physical explanation. In this novel situation, even the old question of an ultimate determinacy of natural phenomena has lost its conceptual basis, and it is against this background that the viewpoint of complementarity presents itself as a rational generalization of the very ideal of causality."

The preferred determinate observable **changes with time** as the state ψ evolves

Problem: perspectivalism, that is, the failure of property composition and decomposition): P can be a determinate property on $\mathcal{H}_S$ without $P \otimes \mathbf{1}$ being a determinate property on $\mathcal{H}_S \otimes \mathcal{H}_A$, or conversely. For example, if

$$\rho_{S+A} \;=\; \frac{1}{8} P_0 \otimes P_{a_0} + \frac{3}{8} P_0 \otimes P_{a_1} + \frac{1}{6} P_1 \otimes P_{a_0} + \frac{2}{6} P_1 \otimes P_{a_1}$$

then the reduced density operator is

$$\rho_S \;=\; \frac{1}{2}\mathbf{1}$$

so the system has only the trivial properties **1** and **0**, but the composite system has $P_0 \otimes P_{a_0}$ and $P_0 \otimes P_{a_1}$ and hence their sum, $P_0 \otimes \mathbf{1}$.

In the orthodox interpretation, property composition and decomposition does hold since $Tr(\rho_S P) = 1$ or 0 iff $Tr(\rho_{S+A}(P \otimes \mathbf{1})) = 1$ or 0. (This is **not** the same as the measurement problem for the Eigenstate-eigenvalue link) ???

Assignment 7. (Send it to: hoferszabogabor@gmail.com by Tuesday midnight.)

Readings

Bub, J., *Interpreting the Quantum World* (Cambridge University Press, 1997).

Dickson, W. M., *Quantum Chance and Non-locality* (Cambridge University Press, 1998).

Lombardini, O. and Dieks, D., "Modal Interpretations of Quantum Mechanics," *Stanford Encyclopedia of Philosophy* (2021).

Juan Sebastián Ardenghi, Olimpia Lombardi, and Martín Narvaja, "Modal Interpretations and Consecutive Measurements," in EPSA 2011: Perspectives and Foundational Problems in Philosophy of Science, V. Karakostas, and D. Dieks (eds.), Berlin: Springer, pp. 207–217.

de Muynck, W. M., *Foundations of Quantum Physics, an Empiricist Approach,* Ch. 6.6 (Kluwer, 2002).

8 Decoherence

Quantum systems are open systems, immersed in the surrounding environment and interact continuously with it. Decoherence brings about a local suppression of interference between preferred states selected by the interaction with the environment. Decoherence is responsible for the transition from quantum to classical. The key insight is to look at $\mathcal{H}_S \otimes \mathcal{H}_A \otimes \mathcal{H}_E$ instead of $\mathcal{H}_S \otimes \mathcal{H}_A$

Decoherence program: started in the 1980s by Zeh, Zurek, Schlosshauer

The role of environment:

> "In classical physics, the environment is usually viewed as a kind of disturbance, or noise, that perturbs the system under consideration in such a way as to negatively influence the study of its "objective" properties. Therefore science has established the idealization of isolated systems, with experimental physics aiming at eliminating any outer sources of disturbance as much as possible in order to discover the "true" underlying nature of the system under study.
>
> The distinctly nonclassical phenomenon of quantum entanglement, however, has demonstrated that the correlations between two systems can be of fundamental importance and can lead to properties that are not present in the individual systems." (Schlosshauer, 2005)

Measurement problem:

System + Apparatus: $\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_A$

Pre-measurement:

$$\left(\sum_i c_i \left|s_i\right\rangle\right) \left|a_R\right\rangle \xrightarrow{\text{Schrödinger}} \sum_i c_i \left|s_i\right\rangle \left|a_i\right\rangle$$

where $|a_R\rangle$ is the 'ready state' of the apparatus

Two problems: (The solution to both problems will be the introduction of a third party, the **environment** E)

1. **Problem of definite outcome:** How can it be that we never see the apparatus in a superposition?

   Solution: environment-induced decoherence: The interaction between the system (+ apparatus) the environment **suppresses the interference** between different states of the system. This process takes place on **extremely short time scales** and requires the presence of only a **minimal environment**. **Global phase** coherence is not actually destroyed; it **remains fully present** in the total system-environment composition

2. **Problem of preferred outcome:** What picks out the basis?

   Namely, The expansion of the final composite state is not unique, and therefore the measured observable is not uniquely defined either. What pick out the right basis? To ensure distinguishable outcomes, the **pointer states** of the apparatus **must be orthogonal.** Then, we can apply the

   Schmidt decomposition: Any pure state $\Psi_{S+\mathcal{A}}$ in $\mathcal{H}_S \otimes \mathcal{H}_A$ can be expressed as

   $$\Psi_{S+A} \;=\; \sum_i c_i \,|s_i\rangle\,|a_i\rangle$$

   for some orthonormal basis $\{|s_i\rangle\}$ in $\mathcal{H}_S$ and $\{|a_i\rangle\}$ in $\mathcal{H}_A$. However, the decomposition is **unique** if and only if $c_i \neq c_j$ for all $i \neq j$.

   Solution: environment-induced superselection. The environment selects out the **pointer states** that are robust. These states are determined by the form of the interaction between the system and its environment and are suggested to correspond to the "classical" states of our experience

Decoherence presupposes the **division of the world into system(s) and environment**

Zeh: the **locality of the observer** defines an observation in the sense that any observation arises from the ignorance of a part of the universe; and that this also defines the "facts" that can occur in a quantum system

Partial trace:

Let a **composite system** $S + A$ be in a (pure) **singlet state:**

$$|\Psi_{S+A}\rangle = \frac{1}{\sqrt{2}}\big(\,|01\rangle - |10\rangle\,\big)$$

The state of **the composite system** can be represented by **the density**

operator:

$$\rho_{S+\mathcal{A}} = |\Psi_{S+A}\rangle\langle\Psi_{S+A}| \;\;=\;\; \frac{1}{2}\big(\,|01\rangle\langle 01| - |01\rangle\langle 10| - |10\rangle\langle 01| + |10\rangle\langle 10|\,\big)$$

Suppose that we **observe only subsystem** S. The **density operator of subsystem** $\mathcal{S}$ is obtained by the **partial trace** over A:

$$\rho_S \;\;=\;\; Tr_A(\rho_{S+A}) = \frac{1}{2}\big(\,|0\rangle\langle 0| + |1\rangle\langle 1|\big)$$

Subsystem $\mathcal{S}$ is in **mixed state**

Note that $|0\rangle$ and $|1\rangle$ are **orthogonal** in $\mathcal{H}_A$; otherwise we would have obtained also **inference terms:** $|0\rangle\langle 1|$ and $|1\rangle\langle 0|$ in ρ_S

General von Neumann measurement:

System + Apparatus + Environment: $\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_A \otimes \mathcal{H}_E$

Pre-measurement:

$$\left(\sum_i c_i\;|s_i\rangle\right)|a_R\rangle\,|e_0\rangle \xrightarrow{\text{Schrödinger 1}} \left(\sum_i c_i\;|s_i\rangle\,|a_i\rangle\right)|e_0\rangle \xrightarrow{\text{Schrödinger 2}} \sum_i c_i\;|s_i\rangle\,|a_i\rangle\,|e_i\rangle$$

where $|e_0\rangle$ is the initial state of the environment. Typically, the $|e_i\rangle$ will be product states of many microscopic subsystem states corresponding to the individual parts that form the environment

Environment-induced decoherence:

We **observe only** $S{+}A$ and the many degrees of freedom of the **environment** E **remain unobserved**

Measuring the system-apparatus component, $S+A$, the partial trace will be:

$$\rho_{S+A} \;\;=\;\; Tr_E(\rho_{S+A+E}) = \sum_{i,j} c_i c_j^*\;|s_i\rangle|a_i\rangle\langle s_j|\langle a_j|\,\langle e_j|e_i\rangle$$

which contains **inference terms** $|s_i\rangle|a_i\rangle\langle s_j|\langle a_j|$ if $\langle e_j|e_i\rangle \neq 0$ for some i,j.

However, $\langle e_j|e_i\rangle(t) \longrightarrow \delta_{ij}$ very rapidly due to decoherence

Typical decoherence time scales:

Environment	Dust grain	Large molecule
Cosmic background radiation	1	10^{24}
Photons at room temperature	10^{-18}	10^{6}
Best laboratory vacuum	10^{-14}	10^{-2}
Air at normal pressure	10^{-31}	10^{-19}

Environment-induced superselection (einselection):

The interaction between the apparatus and the environment singles out a set of mutually commuting observables due to the

Tridecompositional uniqueness theorem: If Ψ_{S+A+E} can be decomposed into the diagonal ("Schmidt") form $\sum_i c_i \left|s_i\right\rangle \left|a_i\right\rangle \left|e_i\right\rangle$, then the expansion is **unique** (provided the three sets are linearly independent)

The inclusion of a **third system** is necessary to remove the basis ambiguity

The environment plays a double role: it selects a preferred pointer basis, and it guarantees its uniqueness via the tridecompositional uniqueness theorem

Stability criterion:

The process $\xrightarrow{\text{Schrödinger 1}}$ above expresses **faithful measurement**.

The process $\xrightarrow{\text{Schrödinger 2}}$ assumes that the interaction with the environment **does not disturb the established correlation** between the state of the system and the corresponding pointer state.

This should be regarded as the **definition of the preferred basis.**

The pointer state projections operators **commute with the apparatus-environment interaction Hamiltonian.** This Hamiltonian determines the **preferred pointer basis**.

> "System-environment interaction Hamiltonians frequently describe a scattering process of surrounding particles (photons, air molecules, etc.) interacting with the system under study. Since the **force laws** describing such processes typically **depend on some power of distance** (such as $\propto r^{-2}$ in Newton's or Coulomb's force law), the interaction Hamiltonian will usually **commute** with the position basis such that, according the commutativity requirement, the **preferred basis** will be in position space. The fact that position is frequently the determinate property of our experience can then be explained by referring to the dependence of most interactions on distance...

> Since the form of the interaction Hamiltonians usually depends on familiar “classical” quantities, the preferred states will typically also correspond to the small set of **“classical” properties**.” (Schlosshauer, 2005)

The environment carries out a **nondemolition measurement** on the apparatus

Environment-assisted invariance (envariance):

Aim: to deduce the Born rule based on a particular symmetry property of entangled quantum states

Zurek: existential interpretation:

“This approach... defines the **reality, or objective existence**, of a state as the possibility of finding out what the state is and simultaneously leaving it unperturbed, similar to a classical state. Zurek assigns a “relative objective existence” to the **robust states** (identified with elementary “events”) selected by the environmental stability criterion. By measuring properties of the system-environment interaction Hamiltonian and employing the robustness criterion, the observer can, at least in principle, determine the set of observables that can be measured on the system without perturbing it and thus find out its “objective” state. Alternatively, the observer can take advantage of the **redundant records of the state** of the system as monitored by the environment. By intercepting parts of this environment, for example, **a fraction of the surrounding photons**, he can determine the state of the system essentially without perturbing it” (Schlosshauer, 2005)

Assignment 8. Watch the video: *How Quantum Mechanics produces REALITY & perhaps ARROW of TIME* and explain in 100-200 words how collapse is related to the direction of time. (Send it to: hoferszabogabor@gmail.com by Tuesday midnight.)

Readings

Schlosshauer, M., “Decoherence, the measurement problem, and interpretations of quantum mechanics” *Rev. Mod. Phys.* 76, 1267 (2005).

Zurek, W. H., “Decoherence, einselection and the quantum origin of the classical” *Rev. Mod. Phys. 75, 715* (2003).

Riedel, C. J., Zurek, W. H., and Zwolak, M., “Objective past of a quantum universe: Redundant records of consistent histories” *Phys. Rev. A 93, 032126* (2016).

9 Consistent Histories

The

Time sequence: $t_1 < t_2 < ... < t_n$

Classical history: sequence of properties at successive times: H H T H T T H T H H

Boolean combination of histories

Fine/coarse-graining

Complete set of mutually orthogonal projectors: $\{P_k\}_k$ on a Hilbert space $\mathcal{H}$

Quantum history: $H_\alpha = P(1)_{\alpha_1} \odot P(2)_{\alpha_2} \odot ... \odot P(n)_{\alpha_n}$

where $\alpha = (\alpha_1, \alpha_2 ... \alpha_n)$ and $\odot$ is the time-indexed tensor product

Boolean combination of histories

Example: Spin: $P_{z^+} \odot P_{x^+} \odot P_{z^+}$ or $[z^+] \odot [x^+] \odot [z^+]$

Fine/coarse-graining:

$$
\begin{aligned}
H_1 &= [z^+] \odot [x^+] \\
H_2 &= [z^+] \odot [x^-] \\
H_{1\vee 2} &= [z^+] \odot I
\end{aligned}
$$

Family of histories (framework): $\{H_\alpha\}_\alpha$

Complete family of histories: when adding them all up, one gets: $I \odot I \odot ... \odot I$

$$
\begin{aligned}
H_1 &= [z^+] \odot [x^+] \\
H_2 &= [z^+] \odot [x^-] \\
H_3 &= [z^-] \odot I
\end{aligned}
$$

Chain operator of a history: $C_\alpha = P(n)_{\alpha_n} U(t_n, t_{n-1}) ... U(t_3, t_2) P(2)_{\alpha_2} U(t_2, t_1) P(1)_{\alpha_1}$

C_α is defined on $\mathcal{H}$ and *not* on the tensor product space

Examples:

Spin: for H_1: $C_1 = [x^+][z^+]$ (here $U(t_2, t_1) = 1$)

Unitary history: $P(i)_{\alpha_i} = U^\dagger(t_i, t_1) P(1)_{\alpha_1} U(t_i, t_1)$ (Heisenberg picture; this is used in the Many-Worlds Interpretation)

We want to assign **probabilities** to the Boolean algebra of histories

Idea: $p_\alpha(\rho) = Tr(C_\alpha \, \rho \, C_\alpha^\dagger)$

This is **positive but not necessarily additive** because of the inference terms $Tr(C_\alpha \, \rho \, C_\beta^\dagger)$

Consistent histories: $Re \, Tr(C_\alpha \, \rho \, C_\beta^\dagger) = 0$ for any α and β

or $Tr(C_\alpha \, \rho \, C_\beta^\dagger) = 0$ (see Diósi 2003)

Unitary families are consistent

Consistent family: when the initial or final projectors are orthogonal. Consequently, two-time histories are almost consistent

Inconsistent family: one needs at least three-time histories

$$
\begin{aligned}
H_1 &= [z^+] \odot [x^+] \odot [z^+] \\
H_2 &= [z^+] \odot [x^-] \odot [z^+] \\
H_3 &= [z^+] \odot [x^+] \odot [z^-] \\
H_4 &= [z^+] \odot [x^-] \odot [z^-] \\
H_5 &= [z^-] \odot I \odot I
\end{aligned}
$$

If $U(t_2, t_1) = U(t_3, t_2) = 1$ (there is **no unitary dynamics**):

$$
\begin{aligned}
C_1 &= [z^+][x^+][z^+] = |\langle x^+ | z^+ \rangle|^2 \, |z^+\rangle\langle z^+| \\
C_2 &= [z^+][x^-][z^+] = |\langle x^- | z^+ \rangle|^2 \, |z^+\rangle\langle z^+| \\
C_3 &= [z^-][x^+][z^+] \\
C_4 &= [z^-][x^-][z^+] \\
C_5 &= [z^-]
\end{aligned}
$$

Consistency condition does not hold: $Re \, Tr(C_1 \, [z^+] \, C_2^\dagger) = 1/4$

Probabilities **don't add up**:

$$
\begin{aligned}
H_{1\vee 2} &= [z^+] \odot I \odot [z^+] \\
C_{1\vee 2} &= [z^+] = |z^+\rangle\langle z^+| \\
1 = p_{1\vee 2}([z^+]) &\neq p_1([z^+]) + p_2([z^+]) = 1/2
\end{aligned}
$$

However, if $U(t_2,t_1) = U(t_3,t_2) = R_{\pi/4}$ (rotation by $\pi/4$), the family will be **consistent**:

$$\begin{aligned} C_3 &= [z^-], \quad \text{the other } C_i = 0 \\ p_3([z^-]) &= Tr(C_3[z^-]C_3^\dagger) = 1, \quad \text{the other probabilities are 0} \end{aligned}$$

Consistency of a family **depends on the dynamics**!

Compatibility: Two consistent families are compatible if and only if they have a common refinement which is a consistent family

Incompatible families:

Single framework rule: quantum probabilistic models employ just one sample space and its associated event algebra. Two incompatible frameworks cannot be combined.

Example: Double slit experiment

$\psi = 1/\sqrt{2}(\psi_a + \psi_b)$

$[\psi_a]$: the particle goes through slit a at t_1

$[\psi_b]$: the particle goes through slit b at t_1

$[x]$: the particle arrives at the screen in patch x at t_2

Histories and chain operators:

$$\begin{aligned} H_1 &= [\psi] \odot [\psi_a] \odot [x] \qquad C_1 = [x][\psi_a][\psi] \\ H_2 &= [\psi] \odot [\psi_b] \odot [x] \qquad C_2 = [x][\psi_b][\psi] \end{aligned}$$

Consistency condition does not hold:

$$Tr(C_1[\psi]C_2^\dagger) = Tr([x][\psi_a][\psi][\psi_b][x]) = \langle\psi|\,[x][\psi_a][\psi_b][x]\,|\psi\rangle = \int_x \psi_a\psi_b\,dx \neq 0$$

because there is inference H_1 and H_2 are inconsistent histories

Interpretation

In certain sense, CH is a neo-Copenhagen interpretation, but there are also other interpretations of CH

Principles by Griffith:

(R1) Liberty. The physicist is free to employ as many frameworks as desired when constructing descriptions of a particular quantum system, provided the principle R3 below is strictly observed.

(R2) Equality. No framework is more fundamental than any other; in particular, there is no “true” framework, no framework that is “singled out by nature”.

(R3) Incompatibility. The single framework rule: incompatible frameworks are never to be combined into a single quantum description. The (probabilistic) reasoning process starting from assumptions (or data) and leading to conclusions must be carried out using a single framework.

(R4) Utility. Some frameworks are more useful than others for answering particular questions about a quantum system.

Evaluation:

- Realist about properties within a fixed consistent framework but denies properties to incompatible frameworks
- Avoids nonlocality by sticking the single framework rule and treats collapse as a computational device
- Measurement is not central; projectors stand for properties of the system
- It does not pick out an actual frame ("small measurement problem" Jeffrey Bub) and neither one history as actual ("big measurement problem").
- (How does it relate to decoherence?)

Assignment 9. (Send it to: hoferszabogabor@gmail.com by Tuesday midnight.)

Readings

Diósi, L. "Anomalies of Weakened Decoherence Criteria for Quantum Histories," *Phys. Rev. Lett.*, 92, 170401-1 (2003).

Griffiths, R. B., *Consistent Quantum theory* (Cambridge: Cambridge University Press, 2000).

Griffiths, R. B., "The Consistent History Approach to Quantum Mechanics," *Stanford Encyclopedia of Philosophy* (2019).

Takács, G., *Quantum World*, Lecture 7.

qm12.pdf (?)

10 The Relational Interpretation

The

Most recent interpretation (Carlo Rovelli, 1996)

Refinement of the **Copenhagen interpretation**

Instead of measurement by a classical, macroscopic observer: **interaction with another quantum system**

The wave function is **not part of the ontology**

ψ is like the **Hamilton-Jacobi function** in classical mechanics

Quantum theory is **about events**

Event: a variable of a system acquires a certain value ("the particle is at x at time t")

Events happen only at **interactions**

An events happens/a variable A acquires a value: when the system S interacts with a second system S´ and the effect of the interaction on S´ depends on the variable A

Events are discrete: physical variables acquire values only at certain times, contrary to classical physics

All contingent physical variables are **relational** (contingent = represented by phase space functions)

Relative variables: velocity, position, electric potential

Don't mix relative and **subjective**! (Although agents often describe the world with the respect to the physical system they are)

The **actualization** of an event is always relative to another system

Perspective of the "observer" S´: the ensemble of all events relative to a system S´, together with the probabilistic predictions

The world is an evolving **network of sparse relative events**

The world is fully quantum: no hidden variables, many worlds, physical collapse mechanisms, or a special role for mind, consciousness, subjectivity, agents, or similar

The observer itself behaves as a **quantum system** when acting on other systems

Central assumption: The probability distribution for (future) values of variables relative to S´ depend on (past) values of variables relative to S´ but not on (past) values of variables relative to another system S″

The interference observed by a system S´ is not erased by the actualization of variables relative to a different system S″

Price to pay: a weakening of the conventional ("strong") realism (existence of non-relational properties)

Some remarks:

Measurement:

- RQM doesn't utilize concepts like decoherence, irreversibility, registration of information...
- In RQM, any interaction counts as a measurement, to the extent one system affects the other and this influence depends on a variable of the first system

Wave function:

- According to RQM, quantum mechanics is **not a theory of the dynamics of an entity** ψ, from which the world of our experience somehow emerges
- It is instead a theory **about the standard world of our experience**, described by values that conventional physical variables take at interactions, and about the transition probabilities that determine which values are likely to be realized, given that others were
- **RQM circumvents the PBR theorem** for the reality of the wave because it is not a strongly realist theory which is an implicit assumption of these theorems
- **The state** ψ **is a compendium of information** of the interactions between the system and a second 'observing' system
- **Quantum state a relative state**, similarly to the Many-Worlds Interpretations, but not realistic

Comparison with other interpretations:

- **Copenhagen:** instead of measurements by a classical observers, RQM describes the quantum system with respect to other quantum systems ("democratized" Copenhagen Interpretation)
- **Many Worlds**: instead of indexing values of variables by branching worlds, RQM indexes by other systems. In neither interpretations there is an a priori special

role for measurement, or observers. The Many Worlds Interpretation is realist with regarding the wave function.

- **Bohmian mechanics:** instead of introducing unobservable entities, RQM has a sparse ontology
- **Zeilinger and Bruckner:** both interpretations inspire the derivation of the formalism of quantum theory from information theoretical postulates
- **QBism:** instead of putting the emphasis on the information acquired by a single subject, RQM regards information as a correlation between the two systems that can be observed by a third system
- **Healey's pragmatist approach:** instead of ascribing the quantum state to the perspective of an actual or potential agent, according to RQM, the values are objective and relative to any physical system

Entanglement: "the external perspective on the very relations that weave reality: the manifestation of one object to another, in the course of an interaction, in which the properties of the objects become actual"

Consistency: If a system S is measured by an observer O, the joint wave function will be the superposition

$$\frac{1}{\sqrt{2}}\big(|0\rangle \otimes |a_0\rangle + |1\rangle) \otimes |a_1\rangle\big)$$

Upon a second measurement by observer O', the wave function collapses to

$$\text{either } \ |0\rangle \otimes |a_0\rangle \ \text{ or } \ |1\rangle) \otimes |a_1\rangle$$

In both cases, the O and O' will detect the same outcome

Locality: RQM claims that one can reduce the failure of locality to the existence of a common cause (Martin-Dussaud, Rovelli, and Zalamea, 2019)

Assignment 10. (Send it to: hoferszabogabor@gmail.com by Tuesday midnight.)

Readings

Laudisa, F., "Relational Quantum Mechanics," *Stanford Encyclopedia of Philosophy* (2019).

qm12.pdf (?)

11 Quantum Bayesianism

The wave function encodes an observer's state of knowledge about a quantum system. Collapse is the change of the observer's knowledge. QBism is an information-theoretic approach to QM

Quantum Bayesianism (Qbism) was born in the early 2000s as an information-theoretic approach to QM elaborated mainly in the Perimeter Institute in Canada.

Main representatives: Carlton Caves, Christopher Fuchs, Rüdiger Schack, Rob Spekkens, David Mermin, Jeffrey Bub

Main ideas:

> The notions of **"observer"** and **"measurement"** are taken as primitive
>
> Quantum states are not something in the external world but are expressions of **information**.
>
> The world may be full of stuff and things of all kinds, but among all the stuff and all the things, there is **no unique, observer-independent, quantum-state kind of stuff**
>
>> "A quantum-mechanical state being a summary of the observers' information about an individual physical system changes both by dynamical laws, and whenever the observer acquires new information about the system through the process of measurement. The existence of two laws for the evolution of the state vector becomes problematical only if it is believed that the state vector is an objective property of the system. If, however, the state of a system is defined as a list of [experimental] propositions together with their [probabilities of occurrence], it is not surprising that after a measurement the state must be changed to be in accord with [any] new information. The "reduction of the wave packet" does take place in the consciousness of the observer, not because of any unique physical process which takes place there, but only because the state is a construct of the observer and not an objective property of the physical system." (Hartle, 1968)

Interpretations of probability: What does it mean the **probability of a coin coming up head is one half**?

> **Classical** (Laplace): the **ratio of favorable cases** (head) and the **equally possible cases** (head, tail)
>
> **Frequentist** (von Mises): the **relative frequency** of heads in a long run of tosses
>
> **Logical** (Carnap): **measure of confirmation** between the statements "The coin is tossed" and the statement "It comes up head"
>
> **Propensity** (Popper): the causal disposition or **propensity** of the coin (and the table, the throwing, etc) to produce heads
>
> **Bayesian (subjectivist):**the **partial belief** of a rational agent in this event

Bayesianism:

> **Thomas Bayes** (1701-1760): English statistician, philosopher and Presbyterian minister
>
> 20. century: **Frank Ramsey, Bruno de Finetti:** "Probability does not exist"
>
> Probability theory is like **formal logic**: a set of criteria for **testing consistency** between truth values of propositions
>
> Bayesian probability theory is a **calculus of consistency** for one's **decision-making degrees of belief**. Probability theory can only say if various degrees of belief are **consistent or inconsistent** with each other. The **actual beliefs** come from another source
>
> **Probability as a guide in life** (Bishop Joseph Butler): A probability assignment is a tool an agent uses **to make gambles and decisions**
>
> There are no **external criteria** for declaring an isolated probability assignment right or wrong. The only basis for a judgment of adequacy comes from **internal coherence**

Quantum Bayesianism:

> Bruno de Finetti: **"Probability does not exist"** $\Longleftrightarrow$ Qbist: **"Quantum states do not exist"**

Quantum information theory:

quantum cryptography, no-cloning theorem, quantum teleportation, quantum key distribution, entanglement monogamy, quantum computation, quantum error correction, superdense coding, etc.

Qbists try to **reconstruct QM from information-theoretic (first) principles:**

Lucian Hardy, "Quantum Theory From Five Reasonable Axioms,"

Giacomo Mauro D'Ariano, Giulio Chiribella, Paolo Perinotti, *Quantum Theory from First Principles: An Informational Approach*

Assignment 11. (Send it to: hoferszabogabor@gmail.com by Tuesday midnight.)

Readings

Fuchs, C. A., "QBism, the Perimeter of Quantum Bayesianism," 2010, URL = https://arxiv.org/abs/1003.5209.

12 The Ensemble Interpretation

The wave function refers to an ensemble of identically prepared systems. Collapse is the change of the ratio of properties in the ensemble

Individual interpretations: ψ refers to an individual systems

Ensemble (statistical) interpretations: ψ refers to an ensemble of systems

Idea: Each individual system possesses a pre-existing property (definite value) with respect to all observables. Measurements reveal this property of the system. The quantum state refers to a heterogeneous ensemble of such systems.

Einstein, 1955:

> "Within the framework of statistical quantum theory there is no such thing as a complete description of the individual system. More cautiously it might be put as follows: The attempt to conceive the quantum-theoretical description as the complete description of the individual systems leads to unnatural theoretical interpretations, which become immediately unnecessary if one accepts the interpretation that the description refers to ensembles of systems and not to individual systems. In that case the whole 'egg-walking' performed in order to avoid the 'physically real' becomes superfluous. There exists, however, a simple psychological reason for the fact that this most nearly obvious interpretation is being shunned. For if the statistical quantum theory does not pretend to describe the individual system (and its development in time) completely, it appears unavoidable to look elsewhere for a complete description of the individual system; in doing so it would be clear from the very beginning that the elements of such a description are not contained within the conceptual scheme of the statistical quantum theory. With this one would admit that, in principle, this scheme could not serve as the basis of theoretical physics. Assuming the success of efforts to accomplish a complete physical description, the statistical quantum theory would, within the framework of future physics, take an approximately analogous position to the statistical mechanics within the framework of classical mechanics. I am rather firmly convinced that the development of theoretical physics will be of this type; but the path will be lengthy and difficult."

I. Ensemble interpretation of (Ballentine, 1970)

The **quantum state** refers to **an ensemble** of individual systems (and not to the preparation procedure of the ensemble, as in the operational interpretation)

Each individual system in the ensemble has a well-defined **(possessed) value or property**

Also **incompatible** observables have well-defined values

The **distribution** of the ith value of the quantity $\mathcal{A}$ in an ensemble is given by $Tr\rho\, P_i^A$

This property uniquely **determines** the measurement outcome of measurement A

The ensemble is **inhomogenous**

Although each individual system has a definite property, only ensembles satisfying **Heisenberg's uncertainty principle** can be prepared (Why?)

That is, individual systems prepared by the same preparations procedure can be **different** (in contrast to the Copenhagen interpretation)

The ensemble interpretation is an **ignorance interpretation**

Advantages:

The ensemble interpretation solves the problem of the **spreading of the wave function**

But QM **has no ensemble interpretation!**

One cannot distribute properties pertaining to incompatible observables in an ensemble. Moreover, there exists no individual system having such properties

Two indirect proofs: Assuming that there is a system with possessed values corresponding to (incompatible) observables, one can derive a **contradiction**

1. State-dependent proof: the GHZ argument

Prepare an ensemble of systems in the GHZ state:

$$
\begin{aligned}
|\Psi_{GHZ}\rangle &= \frac{1}{\sqrt{2}}\big(\,|000\rangle + |111\rangle\,\big) \\
&= \frac{1}{2}\big(\,|x^+x^+x^+\rangle + |x^-x^-x^+\rangle + |x^-x^+x^-\rangle + |x^+x^-x^-\rangle\,\big) \\
&= \frac{1}{2}\big(\,|y^-y^-x^-\rangle + |y^+y^+x^-\rangle + |y^+y^-x^+\rangle + |y^-y^+x^+\rangle\,\big)
\end{aligned}
$$

Measuring $X_1X_2X_3$, the probability of obtaining an **odd** number of $+1$ is 1: the probability is $\langle x^+x^+x^+|x^+x^+x^+\rangle = \frac{1}{4}$ for three $+1$ and $\frac{3}{4}$ for one $+1$

Measuring $Y_1Y_2X_3$, the probability of obtaining an **even** number of $+1$ is 1: the probability is $\langle y^-y^-x^-|y^-y^-x^-\rangle = \frac{1}{4}$ for zero $+1$ and $\frac{3}{4}$ for two $+1$

Similarly, for $Y_1X_2Y_3$ and $X_1Y_2Y_3$

So each system in the ensemble must have the corresponding **properties**

That is, one should **fill in the table**

X_1	X_2	X_3
Y_1	Y_2	X_3
Y_1	X_2	Y_3
X_1	Y_2	Y_3

with ± 1 such that the numbers satisfy the above relations

This is **impossible**! Adding up the $+1$ values along the four orientations yields: odd + even + even + even = an **odd** number of $+1$ outcomes. However, we counted each value twice, so the total number of $+1$ must be **even**. **Contradiction!**

There is **not one single system** with the above properties

2. State-independent proof: the Peres-Mermin square

$\sigma_z \otimes \mathbf{1}$	$\mathbf{1} \otimes \sigma_z$	$\sigma_z \otimes \sigma_z$
$\mathbf{1} \otimes \sigma_x$	$\sigma_x \otimes \mathbf{1}$	$\sigma_x \otimes \sigma_x$
$\sigma_z \otimes \sigma_x$	$\sigma_x \otimes \sigma_z$	$\sigma_y \otimes \sigma_y$

σ_x, σ_y, σ_z are the **Pauli operators** and $\mathbf{1}$ is the **unit operator** on H_2

Each operator in the matrix has **two eigenvalues**: ± 1

Operators are **commuting** if and only if they are in the same row or in the same column

The six commuting triples each have **four common eigenvectors**

It follows from QM that measuring a triple, the three outcomes conform to the **three eigenvalues** of one of the four common eigenvectors

These three outcomes are those for which the **product** is +1 in each row and in the first two columns and −1in the third column

So each system in ensemble must have corresponding **properties**

That is, one should **fill in the** 3×3 **table** with ± 1 such that all six products satisfy the above relations

This is **impossible**! Calculating the product of all nine values by taking the product of the products obtained in the **rows** yields +1. Calculating the product by taking the product of the products obtained in the **columns** yields −1. **Contradiction!**

There is **not one single system** with the above properties

At least some observables must obtain values **during the measurement**

II. Copenhagen ensemble interpretation (von Neumann, 1932)

von Neumann, 1932:

ψ refers to an **individual systems** and hence to a **homogeneous ensemble**

$\rho = \sum_i c_i \, |\psi_i\rangle\langle\psi_i|$ refers to an **ensemble (mixture) of systems** each being in one of the states $|\psi_i\rangle$ and hence ρ refers to an **inhomogeneous ensemble**

This is analogous with the **convex set** of probably measures in classical mechanics

However, the **ignorance interpretation** in the Copenhagen interpretation is **problematic** in two respects:

1. ρ can be expanded in infinitely many ways by non-orthogonal pure states, or if ρ is degenerate, also by orthogonal pure states (**"proper mixtures"**)
2. If ψ_{12} is the state of an (individual) pair of entangled systems, then the state $\rho_1 = Tr_2 \, |\psi_{12}\rangle\langle\psi_{12}|$ of system 1 obtained by the partial trace should also refer to an individual system and not to an ensemble (**"improper mixtures**")

The Copenhagen interpretation assumes that an ensemble of systems in a pure state is homogeneous. But then consider a **homogeneous** ensemble of pairs of particles being in pure entangled state and consider the subsystem taking only system 1 in each pair. This will be described by a density matrix representing an **inhomogeneous** ensemble. How can inhomogenity arise from homogeneity from purely ignoring something?

The difference introduced by the "proper/improper mixtures" is due to the intention of the Copenhagen interpretation to interpret the quantum state as **referring to individual systems**. But it is a kind of abandoning the idea that QM is complete.

Is the subensemble inhomogeneous? One can decide on this by using **von Mises' theory of randomness**. If there is no admissible place selection which alters the relative frequencies in the ensemble, then the the ensemble (sequence) is homogeneous (random). Now, **perform a z-spin measurement on system 2** and register the results. Based on the results, one can obtain a place selection which alters the frequency of the z-spin. Thus, the ensemble will not be homogeneous (improper). Question: Is this place selection admissible? If the measurement on system 2 counts as a **measurement** of the z-spin of system 1, then not. If is counts as a **conditional preparation** of system 1, then it is admissible.

In the **EPR-Bohm experiment**, Ballentine's ensemble interpretation **provides an explanation**: the ensemble is inhomogeneous, half of the pairs have "up" for system 1 and "down" for system 2 for every measurement direction and half of the pairs have opposite possessed values.

Assignment 12. (Send it to: hoferszabogabor@gmail.com by Tuesday midnight.)

Readings

de Muynck, W. M., *Foundations of Quantum Physics, an Empiricist Approach* (Kluwer, 2002).

13 Bohmian Mechanics

The wave function evolves unitarily and never collapses. There are also hidden variables: the position of the particles. Particles move along definite trajectories guided by their pilot waves

History:

Luois de Broglie presented it in 1927 at the Solvay Conference

David Bohm rediscovered it in 1952

Ontology:

We have n **indistinguishable** particles and a **universal wave function**

The configuration space of n **indistinguishable** particles: ${}^{n}\mathbb{R}^{3}$ is different from the configuration space of n **distinguishable** particles: $\mathbb{R}^{3n}$

The Bohmian mechanics is **deterministic**

We specify the motion of the particles with a **velocity field** on the configuration space by taking the **gradient of the complex phase** of the wave function

Two dynamical equations:

1. The wave obeys the **Schrödinger equation:**

$$i\hbar\frac{\partial}{\partial t}\psi \quad = \quad H\psi$$

   where $\psi = \psi(x,t)$ and $x = (x_1, ..., x_n)$ is the coordinates of the n particles

2. The particles obey the **guiding equation**:

$$\frac{dX_k}{dt} \quad = \quad \frac{\hbar}{m_k}\mathrm{Im}\left(\frac{\nabla_k\psi}{\psi}\right)$$

   where X_k is the position of the kth particle

Quantum equilibrium hypothesis: the initial distribution of the position of the particles is given by the wave function:

$$\rho(X = x, 0) \quad = \quad |\psi(x, 0)|^2$$

One can show that then

$$\rho(X = x, t) \quad = \quad |\psi(x, t)|^2$$

also hold for all t. This is called **equivariance**

As a result of the equivariance, the measurement statistics will always conform to the **Born rule**

The Bohmian mechanics is **empirically indistinguishable** from the orthodox QM

Video:

Explanation of the quantum phenomena:

Single slit:

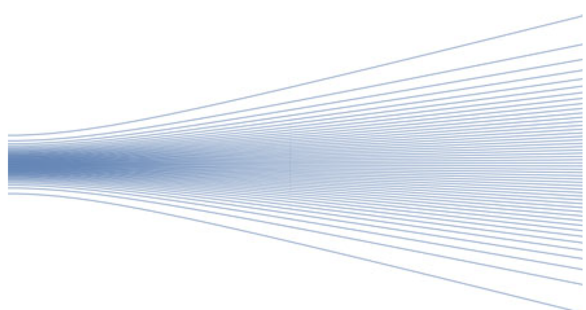

Figure 17: Single slit

Double slit:

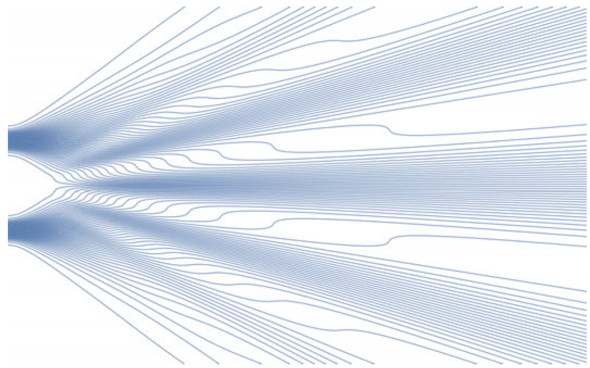

Figure 18: Double slit

Spin:

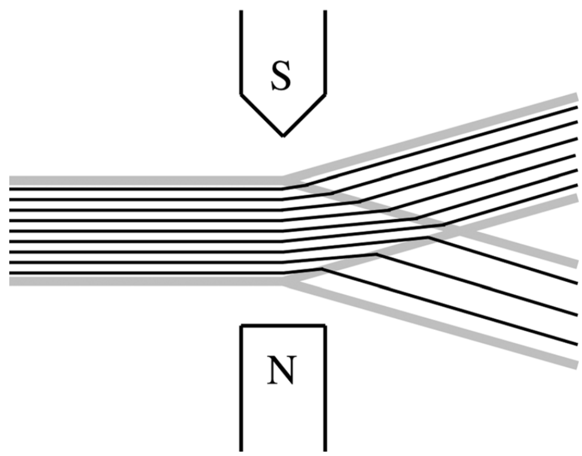

Figure 19: Stern-Gerlach measurement

Spin is **not an intrinsic property** of the particle, it is encoded in the wave function which a spinor field $\begin{pmatrix}\psi_+\\ \psi_-\end{pmatrix}$. The spin measurement is actually a **position measurement**.

In the **Stern-Gerlach measurement**, a particle is represented by a plane wave $e^{iky}\begin{pmatrix}c_+\\ c_-\end{pmatrix}$ which will be deflected upward if $|c_+|^2 > |c_-|^2$, will be deflected downward if $|c_+|^2 < |c_-|^2$ and continue horizontally if $|c_+|^2 = |c_-|^2$ within the (triangular) **overlap region** of the the outgoing wave functions.

Due to the geometry, a **proportion** $|c_+|^2$ of all particles will be deflected upward and $|c_-|^2$ will be deflected downward when leaving the overlap region

Contextuality:

Spin is **a contextual property**: it depends not only on the initial position of the particle and the wave function but also on the measurement apparatus

Suppose we **reverse the polarity** of the Stern-Gerlach magnets. This reverses the **gradient** of the magnetic field responsible for the deflection of the particle

Classically: z**-spin up** particles will be deflected downward and z**-spin down** particles will be deflected upward

Quantum mechanically: the ψ_+ **part of the wavefunction** will be deflected downward and the ψ_- **part of the wavefunction** will be deflected upward.

In both the classical and the quantum mechanics case, the reversion of the polarity counts simply as a **relabeling** of the outcomes: the upper channel counts as z**-spin down** and the upper channel counts as z**-spin up**. In the quantum case the probabilities also flip

Bohmian mechanics: let the ontic state be a **particle in the the upper part** of the slit together with the **symmetric wave function**: $\frac{1}{\sqrt{2}}\big(|\psi_+\rangle + |\psi_-\rangle\big)$

The particle **trajectories cannot cross one another:** a particle starting out in the upper of the slit will be deflected upwards **both in the original and also in the reversed** Stern-Gerlach measurement

Thus, the very same particle with the same initial position and wave function will be assigned **opposite** z**-spin values.** This is **contextuality**

Conditional wave function:

Fundamentally, there it is only the Universe which has a wave function. However, one can assign a **conditional wave function** also to a subsystem of particles by regarding the rest of the particles as the **environment** and **substitute the position of these particles** in the universal wave function. The conditional wave function **does not evolves according to the Schrödinger equation**

Effective collapse:

The wave function has an effect on the particle only **at the place of the particle**. When two part of the wave function separates, the one which is far away **can be thrown away** (until the parts meet in the future)

Nonlocality:

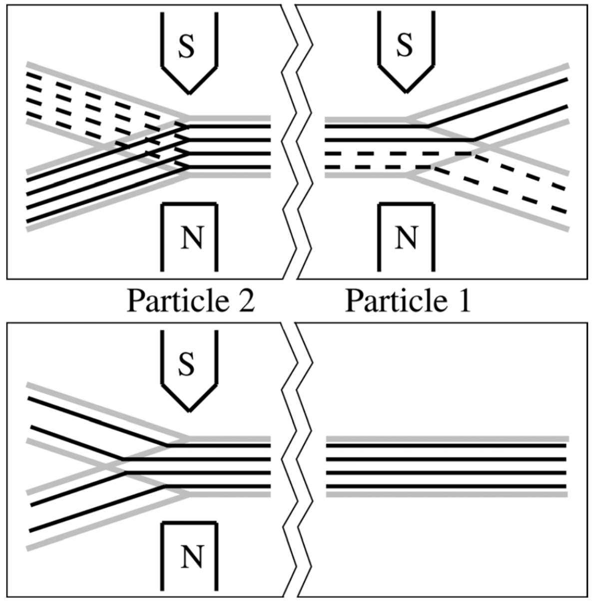

Figure 20: EPR scenario

In the EPR situation, the **deflection of particle 1** (which is first measured—according to a given frame) simply depends on its position between the slit (see above): if the particle is in the upper half it deflects upward, if it is in the lower half it deflects downward

However, the **deflection of particle 2** will already be fixed by the outcome of the previous measurement: even if both particles start out in the upper part of the slit, the second particle will be deflected downward such that it ends up in the opposite channel than particle 1

Suppose both particles are in the upper half and consider **two frames** such that in frame 1 particle 1 is measured first, in frame 2 particle 2 is measured first. That is in frame 1 particle 1 will deflect upward and particle 2 downward and in frame 2 particle 2 will deflect upward and particle 1 downward. **The outcomes** for the very same initial conditions and experimental arrangement **depend on the frame**.

Evaluation:

The wave function as a kind field guides the particles but there is **no back reaction** from the particles to the wave function. Thus, Einstein interpreted the Bohmian wave function as a kind of **"ghost field"** (*Gespensterfeld*), some Bohmians interpret the wave function as a **law** (Callender)

Just as in any ψ-ontic interpretation, the wave function lives in the $3N$ dimension **configuration space** (it is not a real field)

Einstein: "seems too cheap to me"

Pauli: "foolish simplicity" and "beyond all help"

Appendix: Derivation of the guiding equation

de Broglie relation (connecting a particle property and a wave property): $p = \frac{h}{\lambda} = \hbar k$

For a plane wave, $\psi \sim e^{ikx}$, the particle moves with velocity $v = \frac{p}{m} = \frac{\hbar}{m}k$

For general waves, $\psi = R(x,t)e^{iS(x,t)}$, the velocity is $v = \frac{\hbar}{m}\frac{\partial S}{\partial x}$ which is equivalent to $\frac{\hbar}{m_k}\mathrm{Im}\left(\frac{\frac{\partial\psi}{\partial x}}{\psi}\right)$

Assignment 13. Watch the (first 12 minutes of the) video: *PBS Space Time: Pilot Wave Theory and Quantum Realism* and specify in 100-200 words the problems of Bohmian mechanics. (Send it to: hoferszabogabor@gmail.com by Tuesday midnight.)

Readings

Maudlin, T., *Philosophy of Physics: Quantum theory*, (Princeton University Press, 2019), Ch. 5.

de Muynck, W. M., *Foundations of Quantum Physics, an Empiricist Approach* Ch. 7 (Kluwer, 2002).

Norsen, T., *Foundations of Quantum Mechanics*, Springer, 2017, Ch, 7.

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14 The Objective Collapse Interpretations

The wave function evolves unitarily but occasionally collapses. This collapse is a spontaneous, stochastic and dynamical process

In orthodox QM the measurement problem is avoided by the **collapse** occurring during measurements and interrupting the Schrödinger evolution. But this introduces a **fundamental distinction** between measurements and other physical processes. Spontaneous collapse models do the **two jobs together**

GRW, 1986: Spontaneous localization models

The Schrödinger evolution of the wave function is interrupted by so-called **hits** occurring **in every** $\tau = 100$ **Million year** (roughly 100 hits during the history of the Universe) which **localize** or **collapse** the wave function. The hits follow a **Poisson distribution**

The spreading of the wave function and the collapse have **opposite effects**

The occurrence of these hits is not explained, but rather **postulated** as a new fundamental physical mechanism

Technically: Non-linear terms are added to the Schrödinger equation to break superposition

The hits are represented by **multiplying** the the N-particle wave function $\psi(\mathbf{x}_1, ..., \mathbf{x}_N)$ by a **Gaussian** of the form

$$\frac{1}{(2\pi\sigma^2)^{\frac{1}{4}}} e^{-\frac{(\mathbf{X}-\mathbf{x}_k)^2}{4\sigma^2}}$$

localized in the **physical space** (not configuration space) with width $\sigma = 10^{-7}$ meter. σ is roughly 100 atomic radius: **large** compared to the atom, **small** compared to the macroscopic scale

Both the coordinate $\mathbf{x}_k$ and the center of the hit $\mathbf{X}$ are chosen at random and the probability for a specific $\mathbf{X}$ is proportional to $|\psi(\mathbf{x}_k)|^2$, the squared amplitude of the wave function at that point viewed as a function of $\mathbf{x}_k$ only

GRW selects position as the **preferred basis** (just like Bohmian mechanics)

Due to entanglement, **macroscopic systems** will collapse **in every** 10^{-7} **second**

Examples:

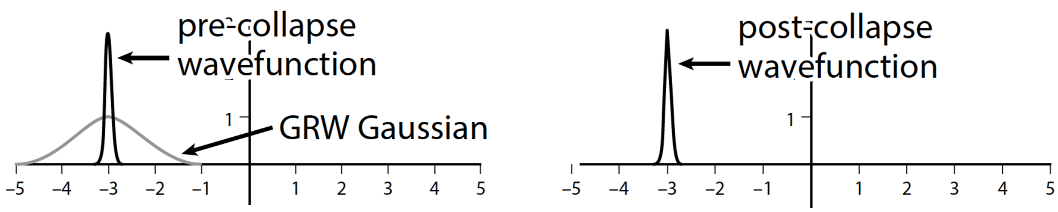

Figure 21: Collapse of a localized wave function

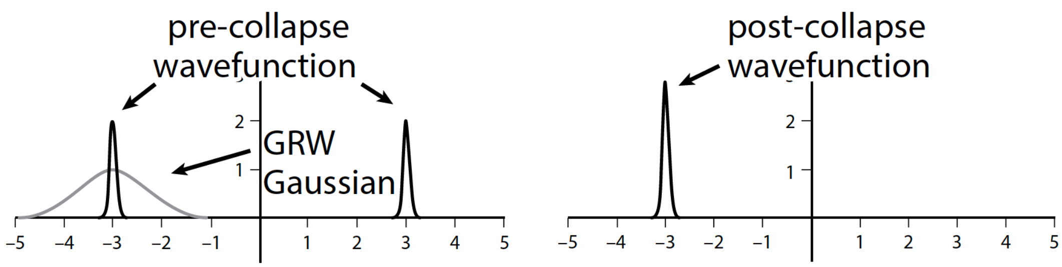

Figure 22: Collapse of an extended wave function

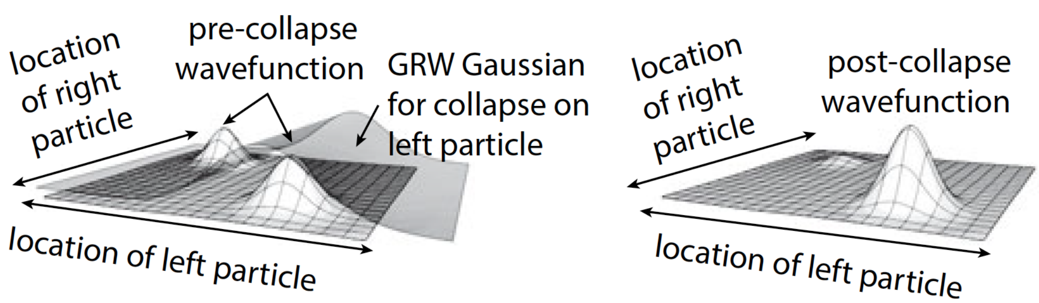

Figure 23: Collapse of an entangled wave function

Continuous spontaneous localization (CSL) models:

The Schrödinger dynamics and the collapse is merged in a continuous common process

Localization is solved by a non-unitary, nonlinear stochastic differential equation

Diósi–Penrose model: the wave function collapse is related to gravity

Ontology:

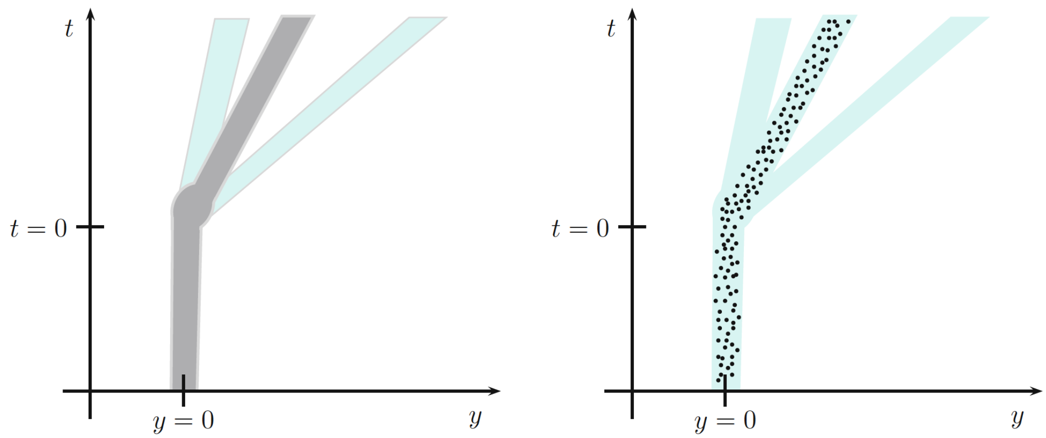

Figure 24: Mass ontology and flash ontology

Mass density ontology (GRWm):

The squared amplitude of the wave function shows how much of each particle is in each configuration

$$\rho_i = m_i \int |\Psi(x_1, x_2, ...x_N)|^2 \, \delta(x_i - x) \, dx_1 dx_2 ... dx_N$$

The mass density would not function **in the orthodox QM** because the mass density corresponding to the different macroscopic worlds would be just **superimposed on top of one another**. It would be a complete mess. But the collapse in the GRW theory ensures that there is **only one macroscopic world**

The tails problem: in any region in space and at any time, the wave function will remain **nonzero**

Flash ontology (GRWf):

This ontology has been proposed by **Bell**

The localized material content of spacetime is not particles with continuous trajectories, nor continuously distributed field-like entities, nor vibrating strings, but rather **point events**

Only a few collapses occur in cells in a second. But if we look at the cells through a microscope, they get localized via entanglement

Most of the time, there is **literally nothing** at all material that is localized in spacetime.

Spontaneous collapse theories are **non-local** (just like any theory which agrees with the predictions of QM)

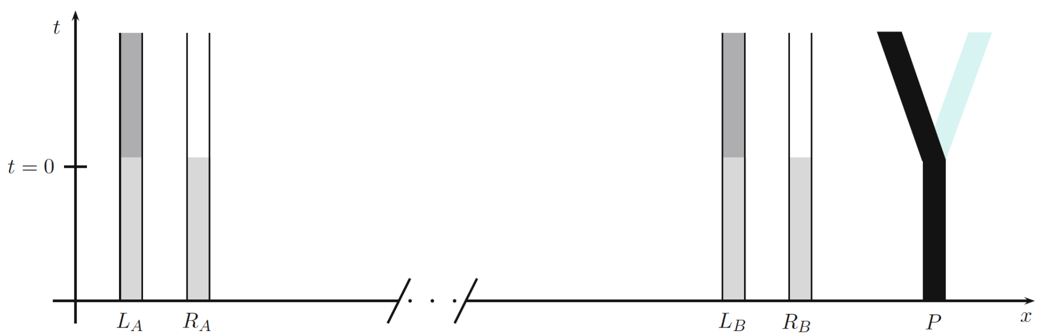

Figure 25: GRW is nonlocal

Experimental tests:

According to the collapse theories, **interference will break down** if the the wave function exceeds the width σ for a time period greater than τ. This allows for an **experimental test** of the collapse theories against ordinary QM

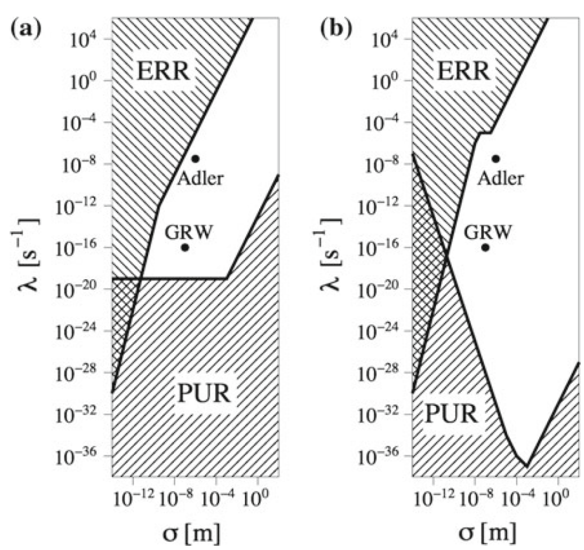

Figure 26: Empirically Refuted Regions (EPR) and Perceptually Unsatisfactory Regions (PUR) for GRW and CSL ($\tau = \frac{1}{\lambda}$)

Empirically Refuted Regions (EPR): Since we observe interference for mesoscopic systems (C_{60} : "buckyball") and for a system composed of N particles $\tau' = \tau/N$, therefore τ **cannot be too small**

Perceptually Unsatisfactory Regions (PUR): Since for visible macroscopic objects, we don't see blurriness, therefore τ **cannot be too big**

The two regions will **close up** in a couple of decades

Assignment 14. Watch the (first 13 minutes of the) video: *PBS Space Time: Is The Wave Function The Building Block of Reality?* and explain in 100-200 words how gravity might cause collapse. (Send it to: hoferszabogabor@gmail.com by Tuesday midnight.)

Ghirardi, G., "Collapse Theories," *Stanford Encyclopedia of Philosophy* (2020).

Maudlin, T., *Philosophy of Physics: Quantum theory*, (Princeton University Press, 2019), Ch. 4.

Norsen, T., *Foundations of Quantum Mechanics*, Springer, 2017, Ch, 9.

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15 The Many-Worlds Interpretation

1. The many-world interpretation solves the measurement problem by rejecting both collapse and hidden variables
2. The many-world interpretation of quantum mechanics is just quantum mechanics itself, taken literally
3. "Worlds" are mutually dynamically isolated structures instantiated within the quantum state, which are structurally and dynamically quasi-classical
4. The existence of these "worlds" is established by decoherence theory

Storyline

Figure 27: Hugh Everett III (1930-1982)

1953: Enters graduate school at Princeton

1954: Starts working on the many-worlds interpretation under the supervision of J. A. Wheeler

1956: Wheeler discusses Everett's ideas with Bohr in Copenhagen

1957: Dissertation ("On the Foundations of QM")

1957: Truncated version (to turn down a clash with the Copenhagen interpretation) published in *Reviews of Modern Physics* (" 'Relative state' Formulation of QM")

1957: Leaves Academia and works for the Pentagon

1959: Visits Bohr (a complete disaster)

1973: Unedited thesis published ("The theory of the Universal Wavefunction") by de Witt and Graham

Everett's questions:

Collapse happen upon observation. But what distinguishes observation from the other interaction?

Can we apply QM to the Universe (where there is no outside observer?)

Everett's solution:

There is only one dynamical process: the Schrödinger dynamics

Relative states:

1. System + Apparatus: $\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_A$

$$\frac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big) \otimes |a_R\rangle \xrightarrow{\text{Schrödinger}} \Psi = \frac{1}{\sqrt{2}}\big(|0\rangle \otimes |a_0\rangle + |1\rangle) \otimes |a_1\rangle\big)$$

The system and the apparatus get entangled

Does it mean that the pointer of the apparatus is blurred among several different positions?

2. System + Apparatus + Observer: $\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_A \otimes \mathcal{H}_O$

$$\frac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big) \otimes |a_R\rangle \otimes |o_R\rangle \xrightarrow{\text{Schrödinger}} \Psi = \frac{1}{\sqrt{2}}\big(|0\rangle \otimes |a_0\rangle \otimes |o_0\rangle + |1\rangle) \otimes |a_1\rangle \otimes |o_1\rangle\big)$$

The entangled state gets bigger

Neither of the entangled components have a definite state; but we can define a **relative state** for each component

There are correlation built into the state

3. System + Apparatus + Observer + Environment: $\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_A \otimes \mathcal{H}_O \otimes \mathcal{H}_E$

$$\frac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big) \otimes |a_R\rangle \otimes |o_R\rangle \otimes |e_R\rangle \quad \xrightarrow{\text{Schrödinger}} \quad \Psi = \frac{1}{\sqrt{2}}\big(|0\rangle \otimes |a_0\rangle \otimes |o_0\rangle \otimes |e_0\rangle + |1\rangle\big) \otimes |a_1\rangle \otimes |o$$

If air molecules or photons (or Wigner's friend) are also around, the entangled state gets even bigger enclosing also the environment (Schrödinger's tiger)

The whole world gets entangled

Remarks:

There is just **one World or Universe** (energy conserves across the branches)

The branches live on top of one another: **they do not interact** (just like two light beams—a light beam emitted from the sun and a light beam emitted from the headlights of a car—do not interact)

The branches do not interact due to the **decoherence:** as more and more components enter the entangled state, the configuration gets bigger and bigger and the decohered branches will be separated in this **extremely high-dimensional space**. (If for two wave packets each composed of 10^{23}particles, each particle is separated from its corresponding other particle by distance d, the two wave packets will be separated by $\sqrt{10^{23}}d$, due to the **Pythagorean theorem**. So it is **practically impossible** that they meet again.

Decoherence

System + environment: $\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_A$

$|\psi_i\rangle \otimes |a_R\rangle \longrightarrow |\psi_i\rangle \otimes |a_i\rangle$

$\langle a_i|a_j\rangle \approx \delta_{ij}$: record states are **sufficiently distinguishable**

The density operator is **diagnosable** in the basis state $|\psi_i\rangle$

Decoherence is diachronic: because the environment is constantly measuring the system in the basis $|\phi_i\rangle$, and **interference** between distinct terms in this basis will be washed away

The system's dynamics is **quasi-classical**: the evolution of the system can be calculated as the weighted sum of the evolution of the basis vectors

Decoherence allows us to extract from the unitary dynamics a **space of histories** ("consistent histories") and to assign classical probabilities to each history

Decoherence is a **dynamical process** by which two components of a complex entity (the quantum state) come to evolve independently of one another

That decoherence is **approximate**, effective, for-all-practical purposes, occurs on short timescales (not instantaneously), causes interference effects to become negligible (not zero), approximately diagonalizes the density operator (not exactly), approximately selects a preferred basis (not precisely)

Many **concrete models** (qubit toy model, microwave background radiation, residual degrees of freedom of a fluid)

Coherent states:

The basis picked out by decoherence is approximately a **coherent state basis**: a basis of wave-packets approximately localized in both position and momentum. The dynamics is **quasi-classical**: the behavior of those wave-packets approximates the behavior predicted by classical mechanics.

> $|q, p\rangle$: **state** of the system localized around phase-space point (q, p).
>
> $f(q, p)$: **probability density** that the system is localized around (q, p).
>
> Then, $f(q, p)$ evolves, to a good approximation, like a classical probability density on phase space; it evolves, approximately, under the Poisson equations

If the system is classically **non-chaotic**, then it follows a classical trajectory on phase space; each wave-packet is structurally the same as the behavior of a macroscopic classical system

Decoherence process is an absolutely standard feature of **emergence**. Each decoherent history is an emergent structure within the underlying quantum

At the fundamental level there is **no collapse** of the quantum state. There is just a dynamical process – decoherence – whereby certain components of that state become dynamically autonomous of one another

A unitary quantum theory with emergent, decoherence-defined quasi-classical histories is a **many-worlds theory**

Emergence

Try to **demystify** the talk about "existence" and "reality"

QM, taken literally, claims that we are living in a **multiverse**.

According to our best current physics, **branches are real**.

How many branches?

> "Our world is filled with things that are neither mysterious and ghostly nor simply constructed out of the building blocks of physics. Do you believe in voices? How about haircuts? Are there such things? What are they? What, in the language of the physicist, is a hole - not an exotic black hole, but just

a hole in a piece of cheese, for instance? Is it a physical thing? What is a symphony?" (Hofstadter and Dennett, 1981)

Dennett's criterion: A macro-object is a pattern, and the existence of a pattern as a real thing depends on the usefulness – in particular, the explanatory power and predictive reliability – of theories which admit that pattern in their ontology

Macro-objects:

chess program

Quasi-particles in solid-state physics: phonons, magnons, plasmons. They can be created, annihilated, be scattered off one another, their time of fight can be measured

Elementary particles turn out to be emergent from an underlying quantum field

In certain quantum gravity theories, **spacetime** is emergent

In string theory, **spacetime** is fundamentally high-dimensional and only emergently four-dimensional

(The absence of a precise boundary to a mountain does not undermine the existence of mountains.)

Ontology:

Figure 28: NGC 1300 spiral galaxy (61 million light years from Earth)

How many worlds?

Branching is caused by any process which magnifies microscopic superpositions up to the level where decoherence kicks in, and there are basically **three such processes**:

1. **Deliberate human experiments:** Schrödinger's cat, the two-slit experiment, Geiger counters, and the like.
2. **Natural quantum measurements**, such as occur when radiation causes cell mutation.
3. **Classically chaotic processes**, which cause quantum states by small variations in initial conditions to spread over macroscopically large regions.

Branching has no natural "grain": a finer or coarser choice also gives branching. There is **no finest choice of branching** structure: as we fine-grain our decoherent history space, we will eventually reach a point where interference between branches ceases to be negligible, but there is no precise point where this occurs

A metaphor:

1. A world consisting of a very thin, in infinitely long and wide, slab of matter, in which various complex internal processes are occurring
2. Stacking many thousands of these slabs one atop the other, but without allowing them to interact at all (parallel worlds).
3. Introduce a weak force normal to the plane of the slabs – a force with an effective range of 2-3 slabs, perhaps and small compared to the intra-slab force.
4. Turn up the interaction sharply: let it have an effective range of several thousand slabs, and let it be comparable in strength with characteristic short-range interaction strengths within a slab

Conclusion

The claims of the Everett interpretation are:

1. The many-world interpretation solves the measurement problem by rejecting both collapse and hidden variables
2. The many-world interpretation of quantum mechanics is just quantum mechanics itself, taken literally
3. "Worlds" are mutually dynamically isolated structures instantiated within the quantum state, which are structurally and dynamically quasi-classical
4. The existence of these "worlds" is established by decoherence theory

Problems:

Preferred basis problem: something needs to be added to QM

Probabilities

Simple branch counting: Suppose we measure the z-spin of n particle one at a time each in state

$$|+\rangle = \frac{1}{\sqrt{2}}\big(\,|0\rangle - |1\rangle\,\big)$$

Now, the in the **overwhelming majority** of the branches the number of "up" outcome will be $\frac{n}{2}$ or very close to it: **the Born rule will be typical**

Unfortunately, the same holds if we start from the state

$$|\psi\rangle = c_0\,|0\rangle - c_1\,|1\rangle$$

and use the branch counting technique. We get the right result only if we introduce **branch weights**, that is, count the "up" result with $|c_0|^2$weight and the "down" result with $|c_1|^2$weight. But this is to smuggle in the Born rule (*cf.* Lazarovici)

Some try to derive the probabilities from **rationality principles, decision theory and Dutch book arguments**

Ontology

There are **different takes** on the ontology of MWI

Some say that one should consider the **mass distribution** in $3D$ of the different particles in the different non-interacting and causally independent branches

The **"wave function monists**" who take that "the wave function is everything", however, are hostile to the mass distribution idea and think that the $3D$ **structure are only emergent**. This noncommittal position, however, is untenable: the isomorphism of the $3N$-dimensional and the 3-dimensional description is not enough to explain the emergence of the $3D$ world. For Wallace, $3D$ is a kind of illusion

Locality

Whether the MWI is local can be decided only after specifying what is the 3-**dimensional ontology** of the theory. In $3N$- dimension there is no locality problem

If we accept the mass distribution interpretation of MWI then it seems to be **nonlocal**

Assignment 15. Watch the video: *PBS Space Time: The Many Worlds of the Quantum Multiverse* and explain in 100-200 words the Many-Worlds Interpretation. (Send it to: hoferszabogabor@gmail.com by Tuesday midnight.)

Readings

Barrett, J., "Everett's Relative-State Formulation of Quantum Mechanics," *Stanford Encyclopedia of Philosophy* (2018).

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