LOCAL CAUSALITY

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Project

- **Broader project:** How the three concepts of *causality*, *probability* and *locality* relate to one another in our fundamental physical theories?

Project

- **Broader project:** How the three concepts of *causality*, *probability* and *locality* relate to one another in our fundamental physical theories?
- **Narrower question:** How to formulate local causality in local classical and quantum theory?

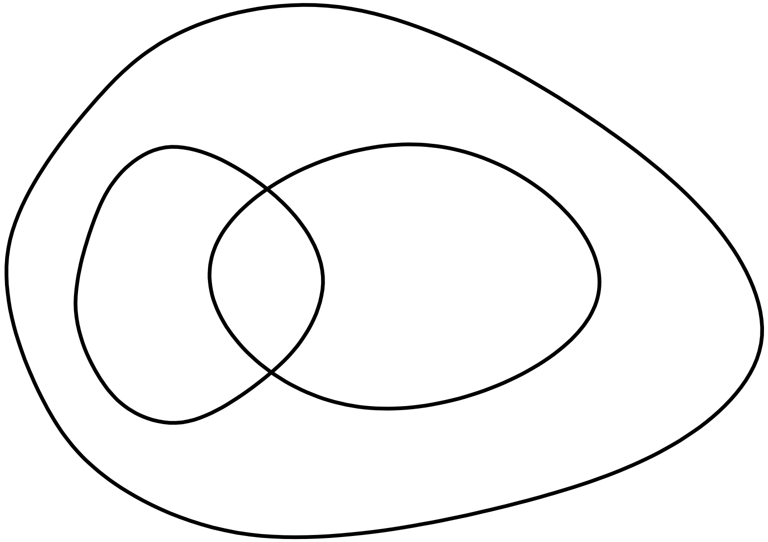
Project

- **I.** What is a local physical theory?
- **II.** Locality concepts
- **III.** Local causality
- **IV.** Classical nets

I. What is a local physical theory?

Local physical theory

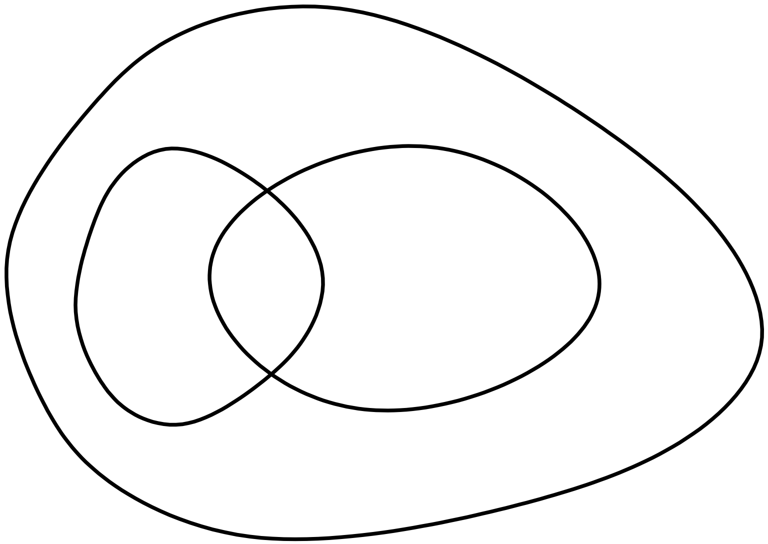
Minkowski spacetime:
Directed poset: $(\mathcal{K}, \subseteq)$



Local physical theory

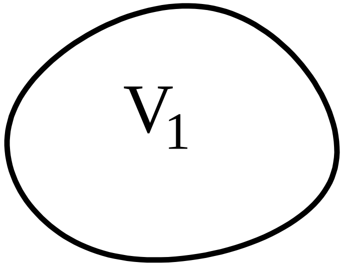
Minkowski spacetime:

Net: $\{\mathcal{N}(V), V \in \mathcal{K}\}$



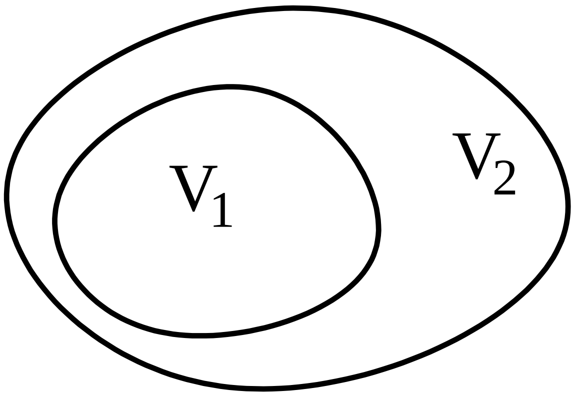
Local physical theory

Isotony:



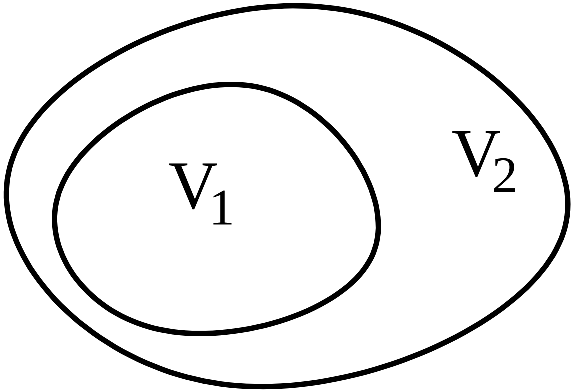
Local physical theory

Isotony: if $V_1 \subset V_2$



Local physical theory

Isotony: if $V_1 \subset V_2$, then $\mathcal{N}(V_1)$ is a subalgebra of $\mathcal{N}(V_2)$



Local physical theory

Microcausality (Einstein causality):

Local physical theory

Microcausality (Einstein causality): $[\mathcal{N}(V_1), \mathcal{N}(V_2)] = 0$

Local physical theory

Covariance: spacetime symmetries are represented on $\mathcal{N}$

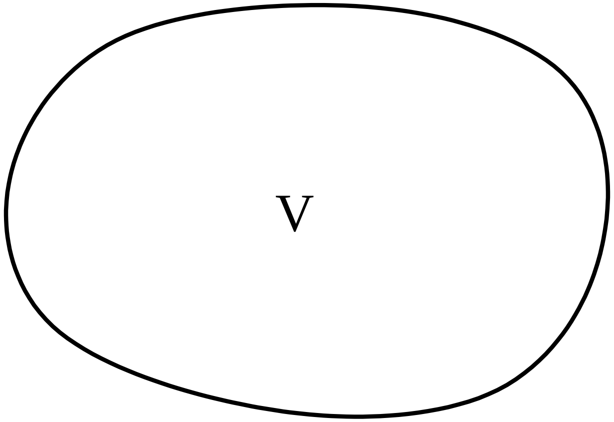
Local physical theory

Local physical theory: an isotone, microcausal and covariant net

- It embraces local classical and quantum theories

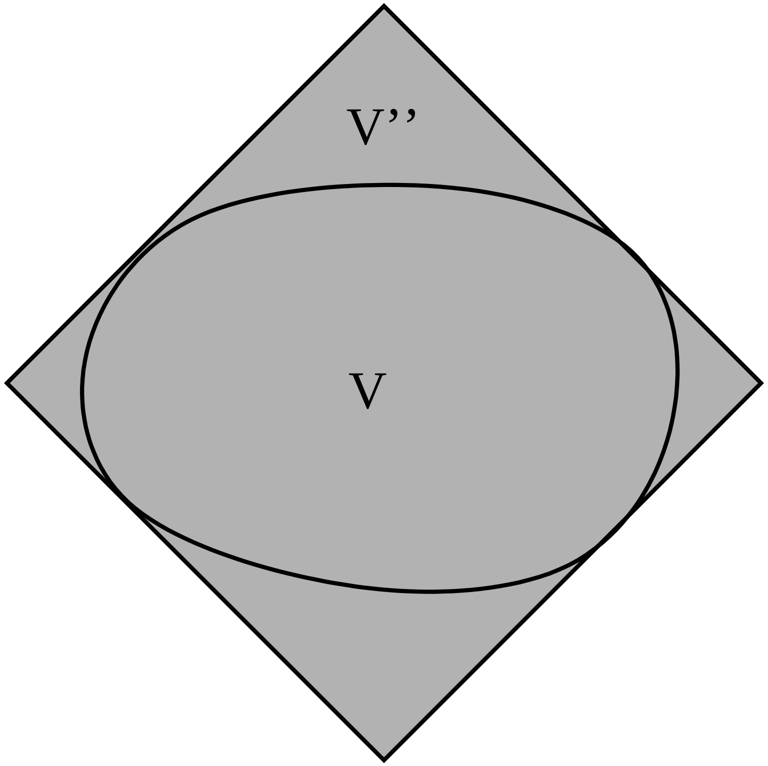
Local physical theory

Local primitive causality:



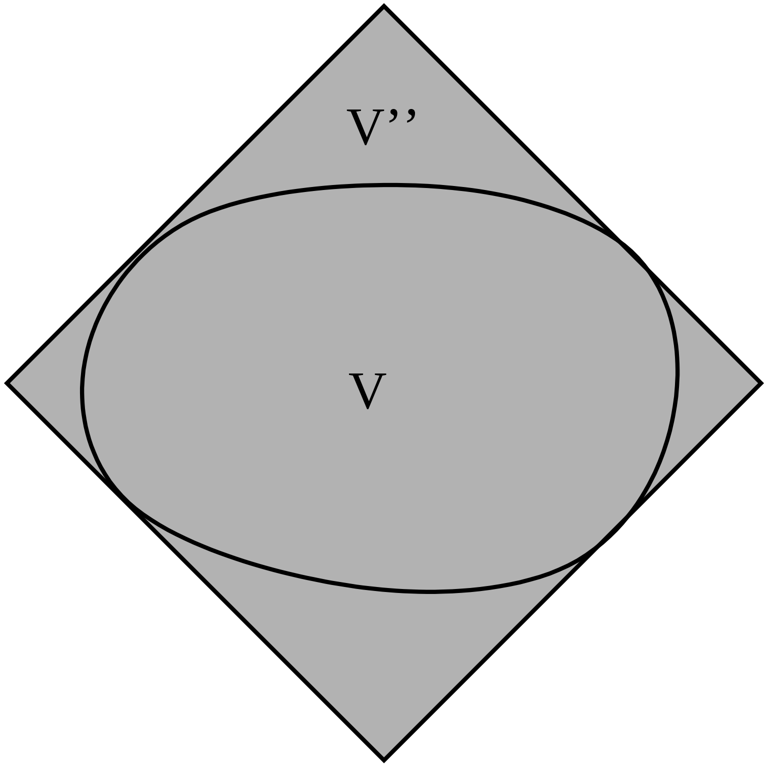
Local physical theory

Local primitive causality:



Local physical theory

Local primitive causality: $\mathcal{N}(V) = \mathcal{N}(V'')$



Local physical theory

Microcausality $\not\Leftrightarrow$ Local primitive causality

- Example: local field algebras

$$\mathcal{F}(V) := \mathcal{N}(V')' \cap \mathcal{F}$$

II. Locality concepts

Locality concepts

- **Microcausality:** no-signalling, parameter independence
- **Local primitive causality:** no superluminal propagation

Locality concepts

No-signalling, parameter independence:

- $\{A_k\}_{k\in K}$: mutually orthogonal projections in $\mathcal{N}(V_A)$
- Non-selective projective measurement:

$$E_{\{A_k\}} : \mathcal{N} \ni X \mapsto \sum_{k\in K} A_k X A_k$$

- **No-signalling:** for any locally faithful state ϕ and for any projection $B \in \mathcal{N}(V_B)$ such that V_A and V_B are spatially separated spacetime regions:

$$(\phi \circ E_{\{A_k\}})(B) = \phi(B)$$

Locality concepts

Outcome independence

- A: projection in $\mathcal{N}(V_A)$
- Selective projective measurement:

$$E_A : \mathcal{N} \ni X \mapsto AXA$$

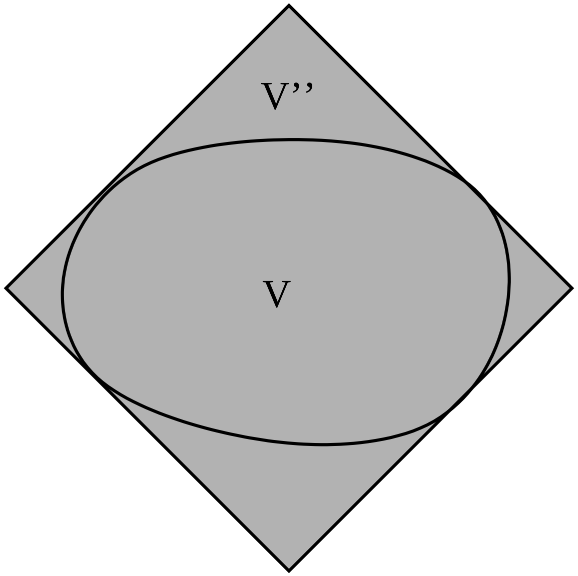
- **Outcome independence:** for any locally faithful state ϕ and for any projection $B \in \mathcal{N}(V_B)$ such that V_A and V_B are spatially separated spacetime regions:

$$\frac{(\phi \circ E_A)(B)}{\phi(A)} = \phi(B)$$

Locality concepts

Local determinism:

- For any two states ϕ and ϕ' and for any nonempty convex spacetime region V, if $\phi|_{\mathcal{N}(V)} = \phi'|_{\mathcal{N}(V)}$ then $\phi|_{\mathcal{N}(V'')} = \phi'|_{\mathcal{N}(V'')}$

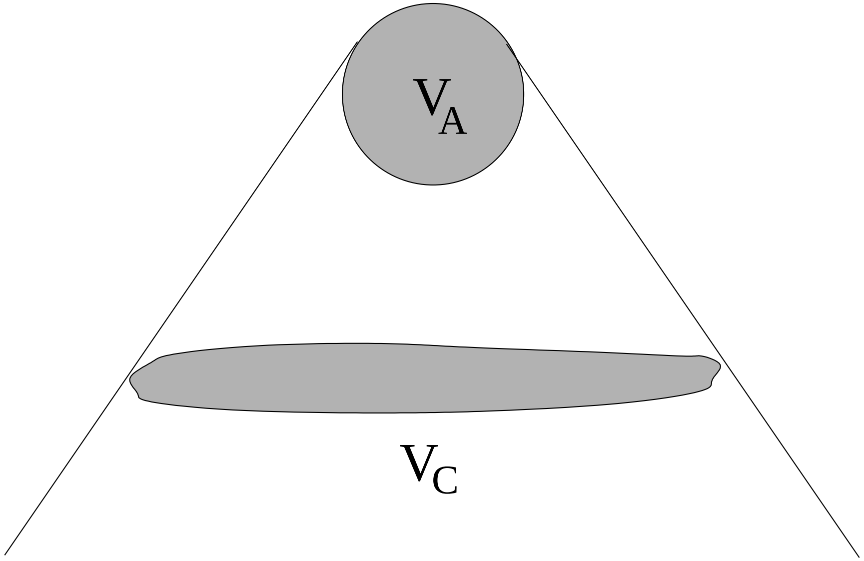


- Local primitive causality $\Longrightarrow$ Local determinism

Locality concepts

Stochastic Einstein locality:

- For any two states ϕ and ϕ', for any V_A spacetime regions, any projection $A \in \mathcal{N}(V_A)$ and any spacetime region V_C such that $V_C \subset I_-(V_A)$ and $V_A \subset V_C''$, if $\phi|_{\mathcal{N}(V_C)} = \phi'|_{\mathcal{N}(V_C)}$ then $\phi(A) = \phi'(A)$



- Local determinism $\Longrightarrow$ Stochastic Einstein locality

Locality concepts

Primitive causality:

- For any Cauchy surface $\mathcal{S}$ and any open neighborhood $\mathcal{O}_{\mathcal{S}}$ of $\mathcal{S}$: $\mathcal{N}(\mathcal{O}_{\mathcal{S}}) = \mathcal{N}$
- Local primitive causality $\Longrightarrow$ Primitive causality

Locality concepts

Determinism:

- For any two states ϕ and ϕ' and for any Cauchy surface $\mathcal{S}$ and any open neighborhood $\mathcal{O}_\mathcal{S}$ of $\mathcal{S}$, if $\phi|_{\mathcal{N}(\mathcal{O}_\mathcal{S})} = \phi'|_{\mathcal{N}(\mathcal{O}_\mathcal{S})}$ then $\phi = \phi'$
- Local determinism $\Longrightarrow$ Determinism

III. Local causality

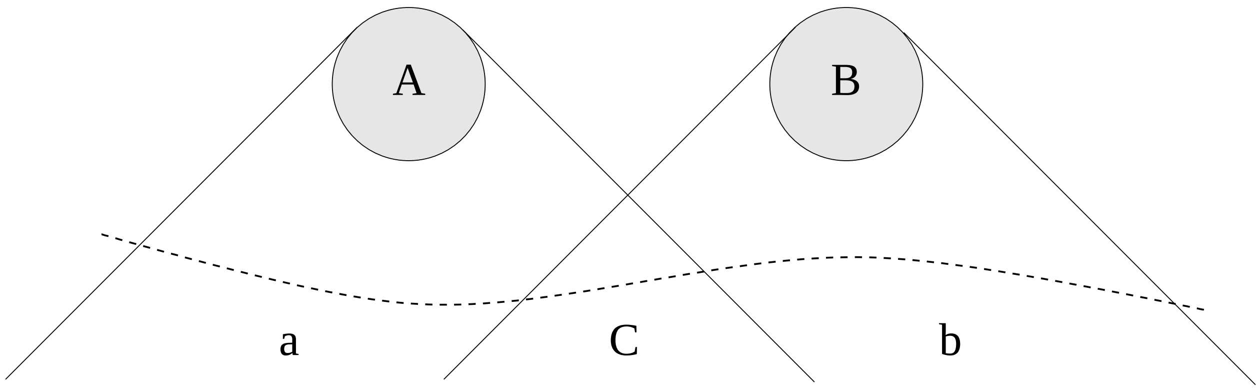
Local causality

Remark:

- Local primitive causality is a dependence relation; local causality is an *in*dependence relation.
- Local primitive causality does *not* rely on the notion of state, it is a property of the net exclusively; local causality *does* depend on the state.

Local causality

”Let C denote a specification of *all* beables, of some theory, belonging to the overlap of the backward light cones of spacelike regions A and B.



Let a be a specification of some beables from the remainder of the backward light cone of A, and B of some beables in the region B. Then in a *locally causal theory*

$$p(A|a, C, B) = p(A|a, C) \tag{1}$$

whenever both probabilities are given by the theory.” (Bell, 1987, p. 54)

Local causality

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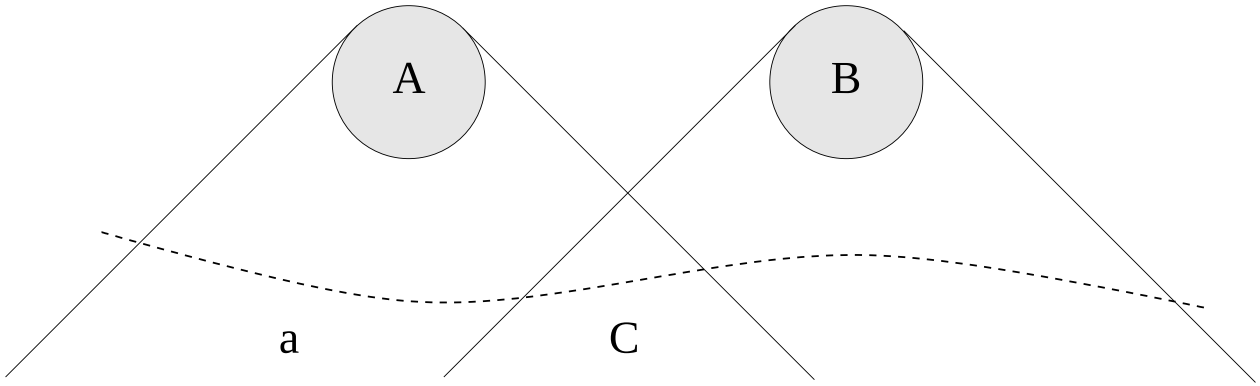
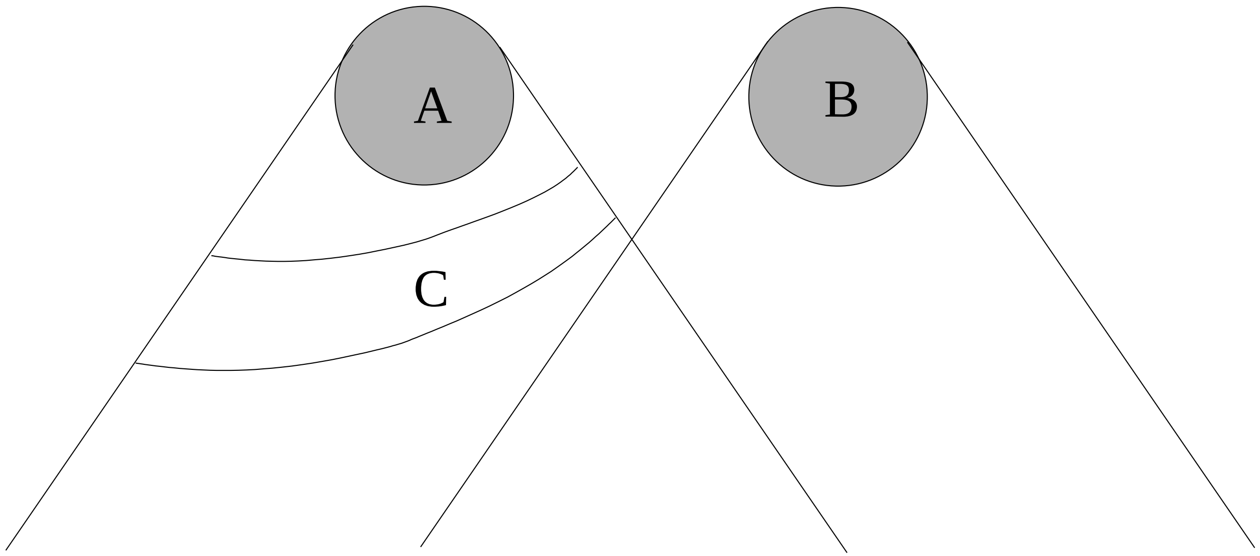


Let a be a specification of some beables from the remainder of the backward light cone of A, and B of some beables in the region B. Then in a *locally causal theory*

$$p(A|a,C,B) = p(A|a,C) \tag{2}$$

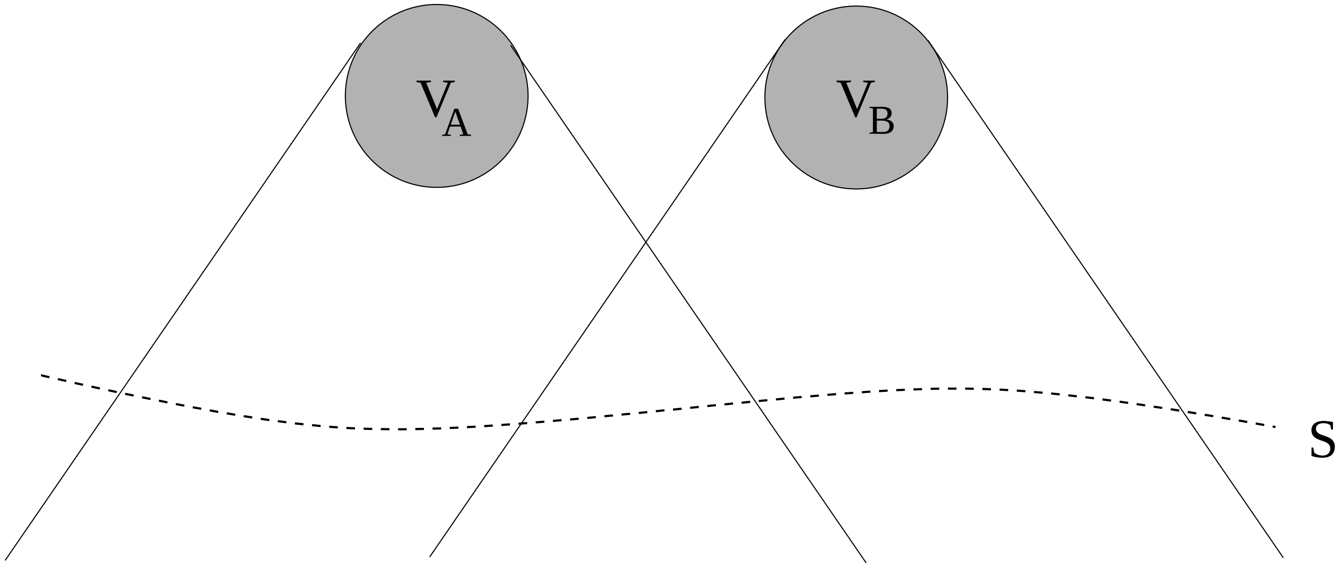
whenever both probabilities are given by the theory.” (Bell, 1987, p. 54)

Local causality



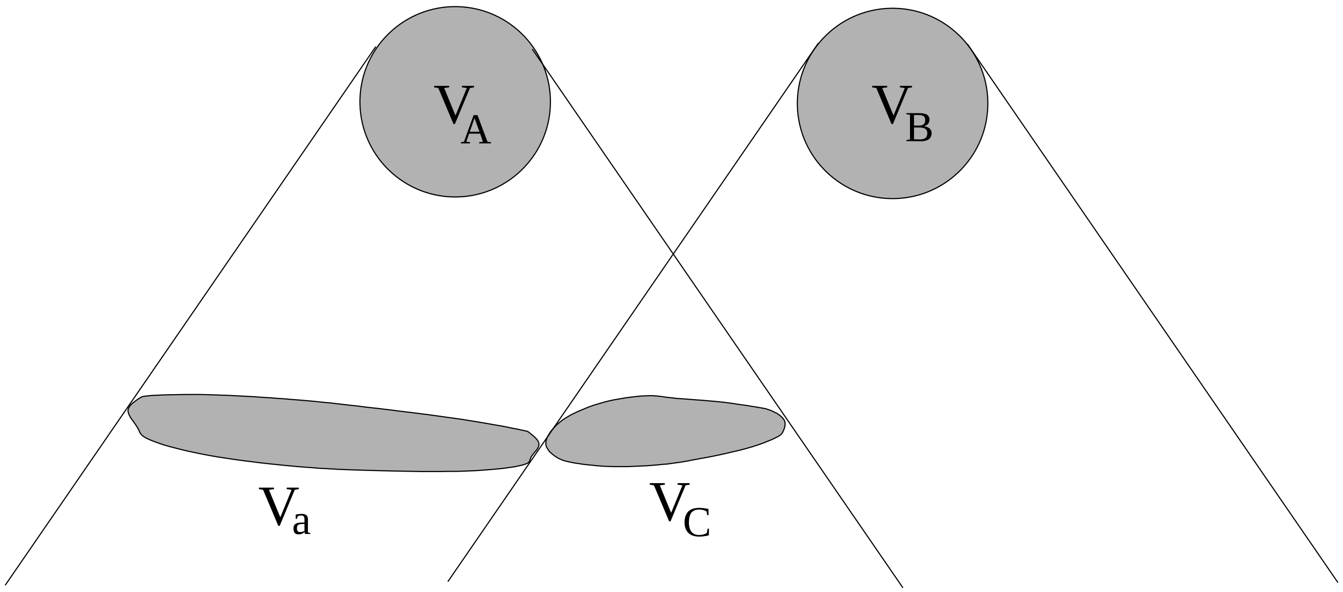
“A theory will be said to be locally causal if the probabilities attached to values of local beables in a space-time region A are unaltered by specification of values of local beables in a space-like separated region B, when what happens in the backward light cone of A is already sufficiently specified, for example by a full specification of local beables in a space-time region C . . . ” (Bell, 1990)

Local causality



Definitions. A local physical theory represented by a net $\{\mathcal{N}(V), V \in \mathcal{K}\}$ is called *locally causal*, if for any pair $A \in \mathcal{N}(V_A)$ and $B \in \mathcal{N}(V_B)$ of projections supported in spacelike separated regions $V_A, V_B \in \mathcal{K}$ and for every locally faithful state ϕ establishing a correlation between A and B, the following is true:

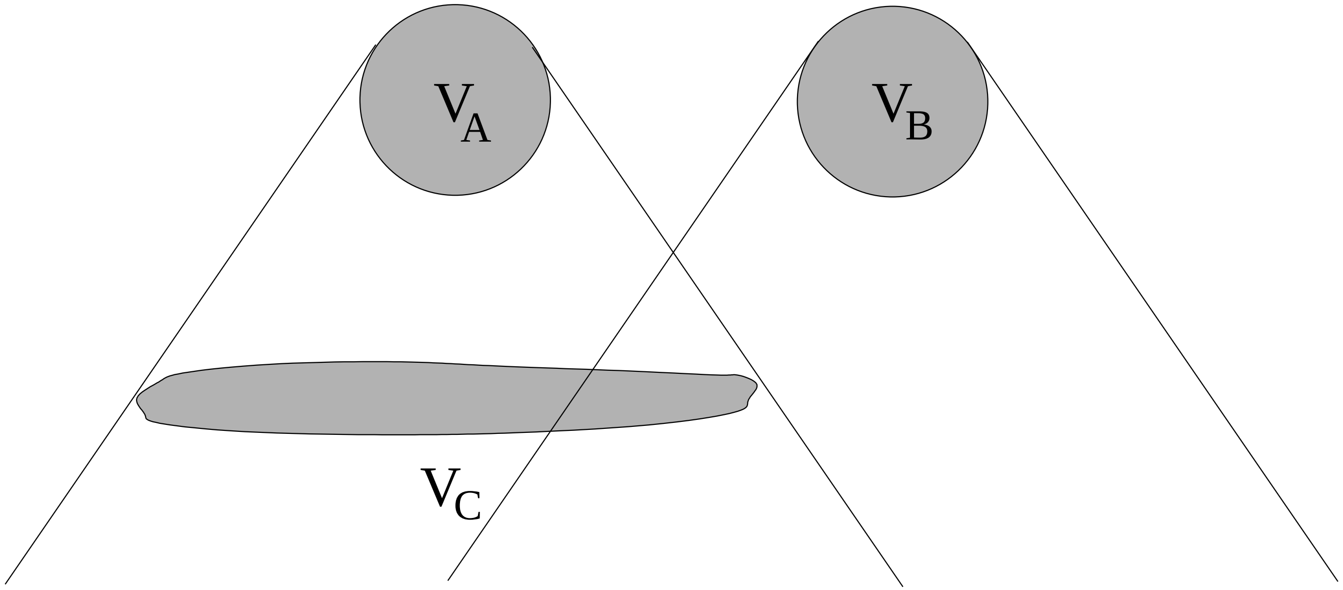
Local causality



Local causality I. For any $a_m \in \mathcal{N}(V_a)$ and *atomic* event $C_k \in \mathcal{N}(V_C)$

$$\phi(A_m B_n | a_m C_k) \;=\; \phi(A_m | a_m C_k)\phi(B_n | a_m C_k)$$

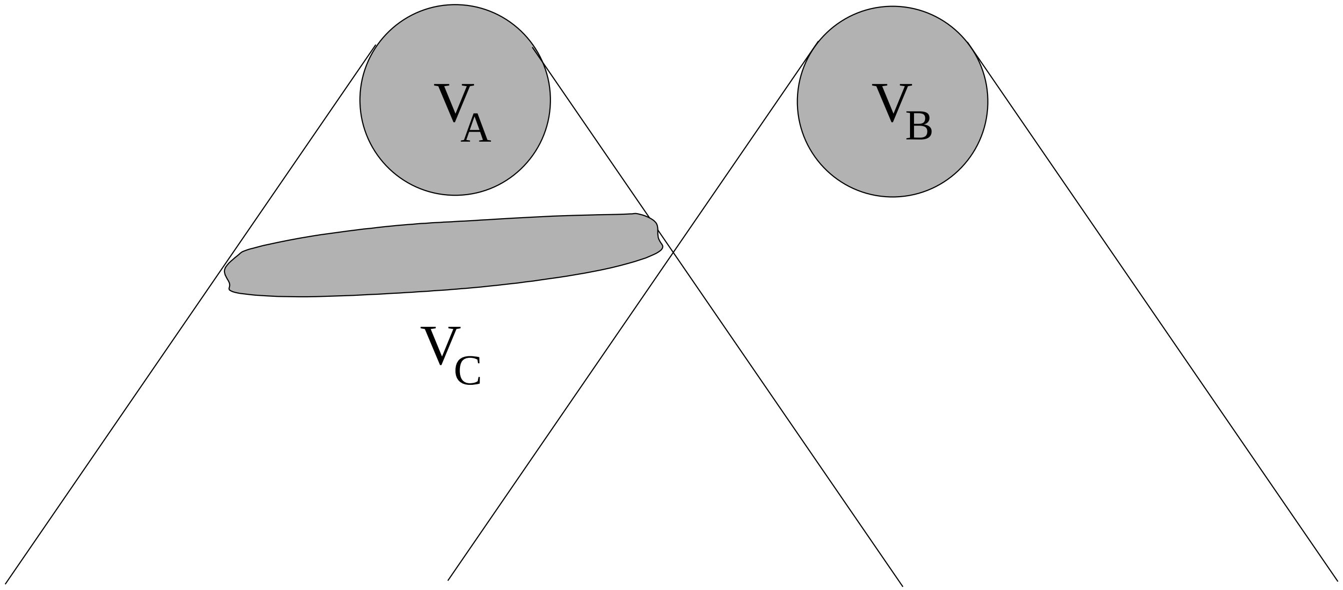
Local causality



Local causality II. For any *atomic* event $C_k \in \mathcal{N}(V_C)$

$$\phi(A_m \wedge B_n | C_k) \;\; = \;\; \phi(A_m | C_k)\phi(B_n | C_k)$$

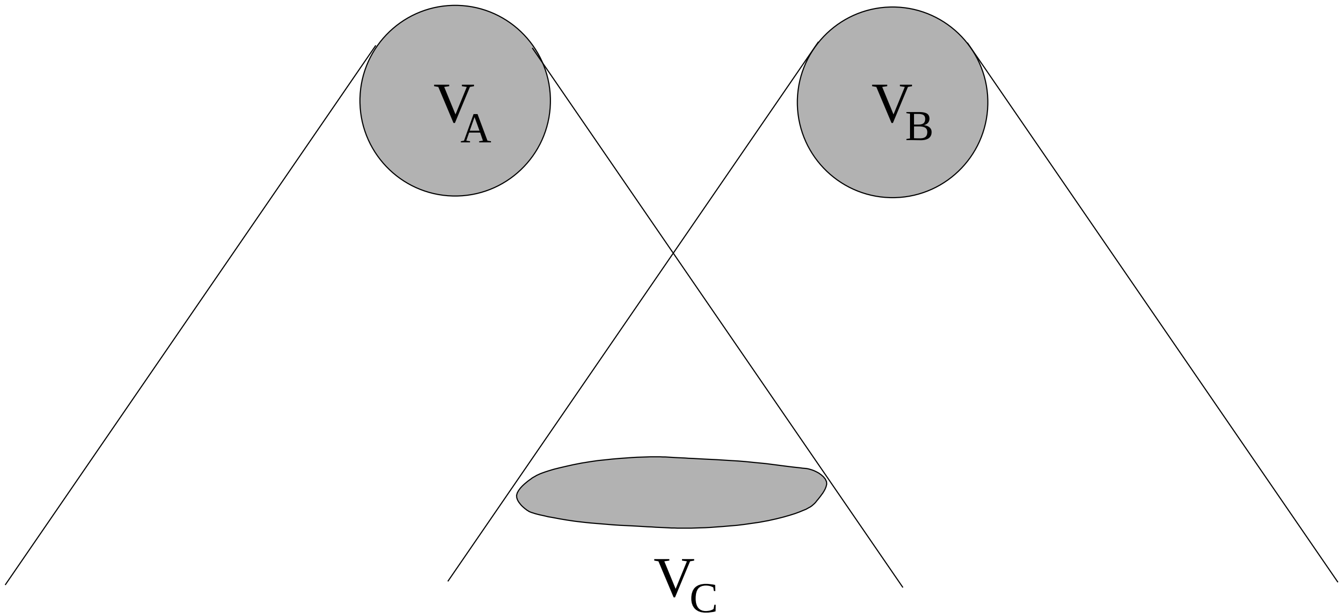
Local causality



Local causality III. For any *atomic* event $C_k \in \mathcal{N}(V_C)$

$$\phi(A_m \wedge B_n | C_k) \;=\; \phi(A_m|C_k)\phi(B_n|C_k)$$

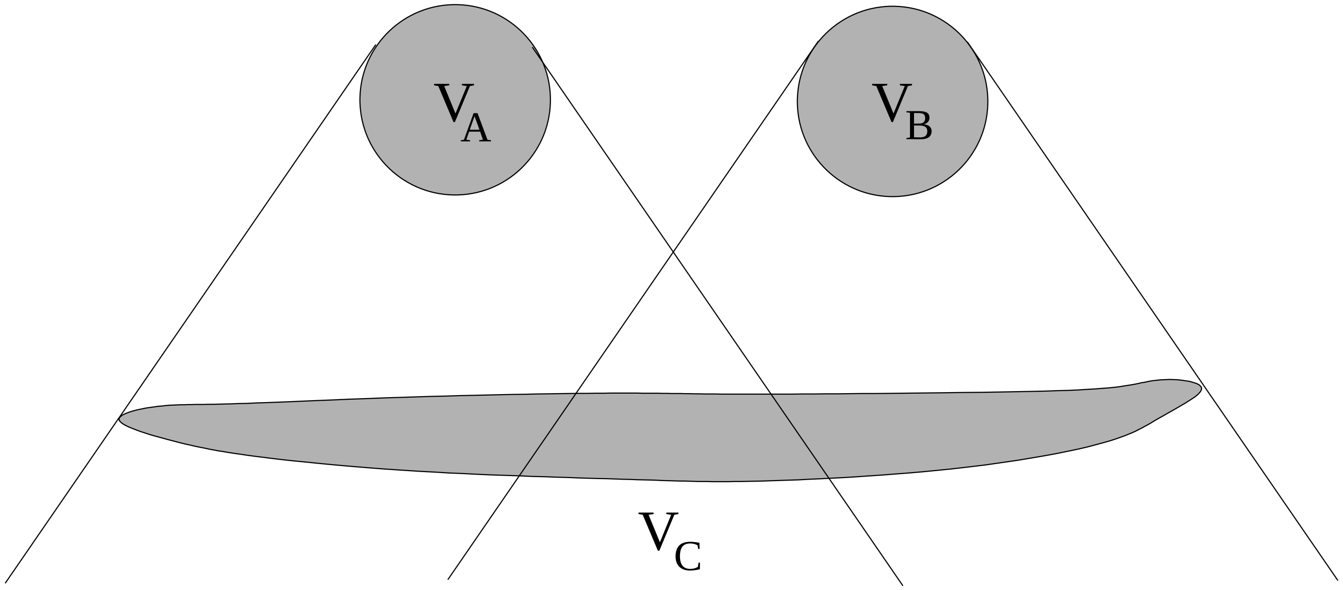
Local causality



Local causality IV. For any *atomic* event $C_k \in \mathcal{N}(V_C)$

$$\phi(A_m \wedge B_n|C_k) \;=\; \phi(A_m|C_k)\phi(B_n|C_k)$$

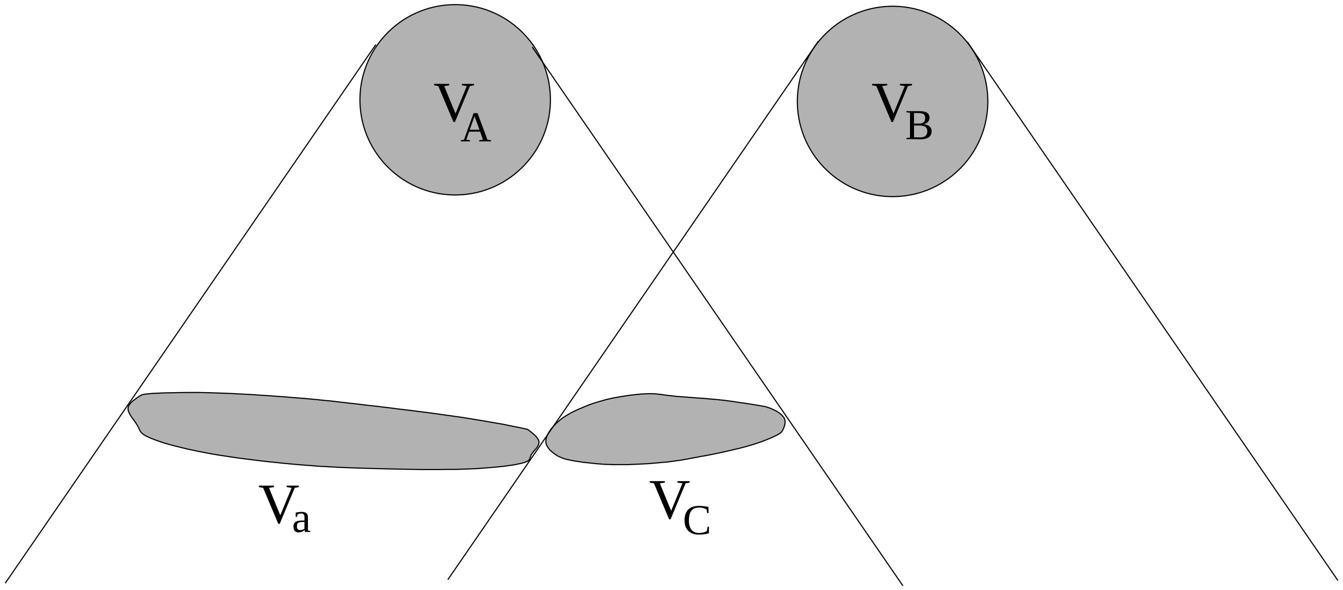
Local causality



Local causality V. For any *atomic* event $C_k \in \mathcal{N}(V_C)$

$$\phi(A_m \wedge B_n | C_k) \;=\; \phi(A_m | C_k)\phi(B_n | C_k)$$

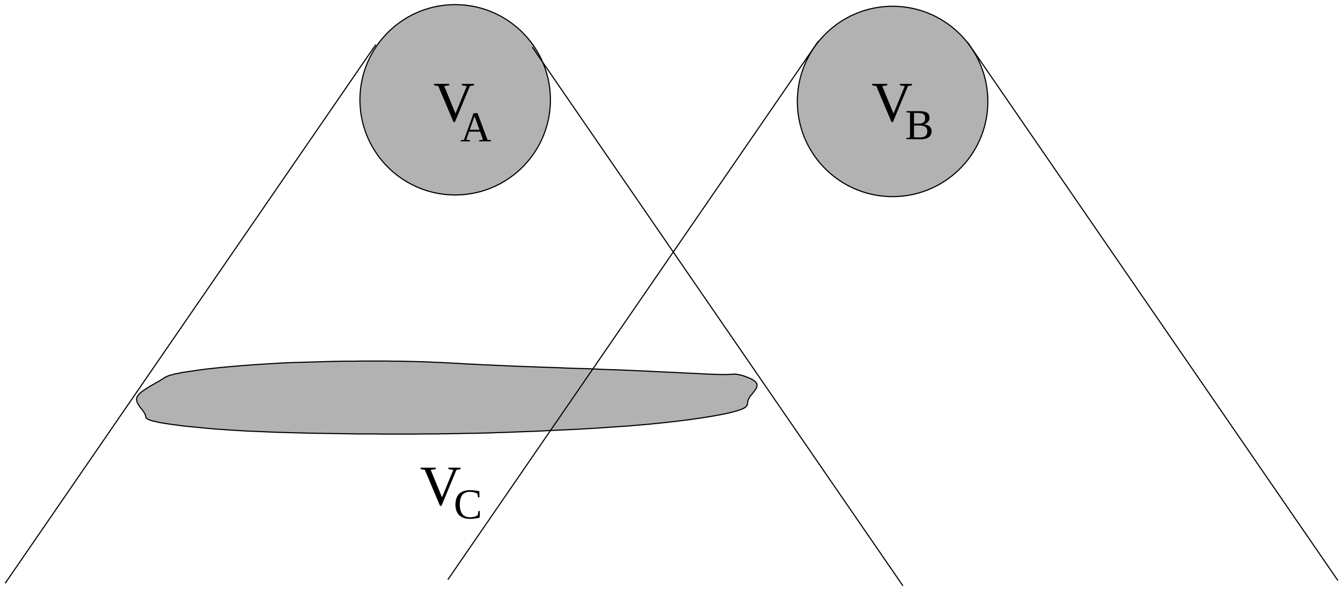
Local causality



Local causality I.

- It implies the standard probabilistic characterization of the common cause (screening-off, locality but not no-conspiracy).

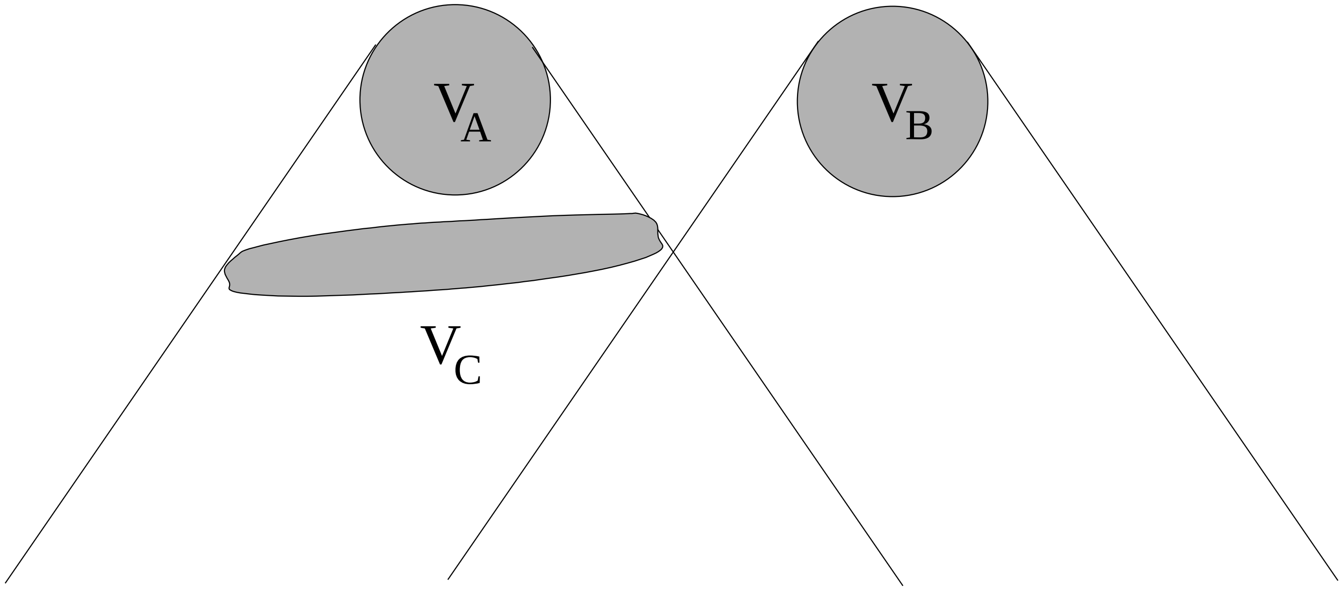
Local causality



Local causality II.

- Weaker than Local causality I.
- Trivially holds for a classical, atomic net satisfying local primitive causality.
- (For non-atomic nets it holds vacuously.)

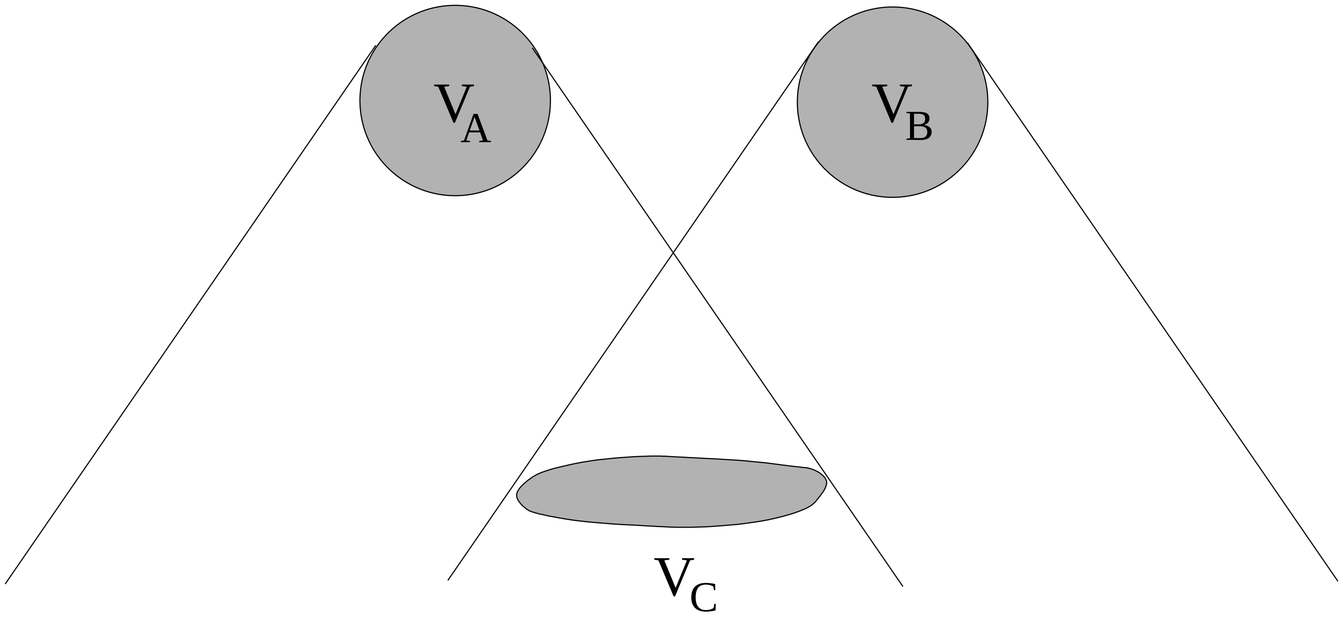
Local causality



Local causality III.

- Is there a difference between Local causality II and III? (Causal Markov Condition)

Local causality

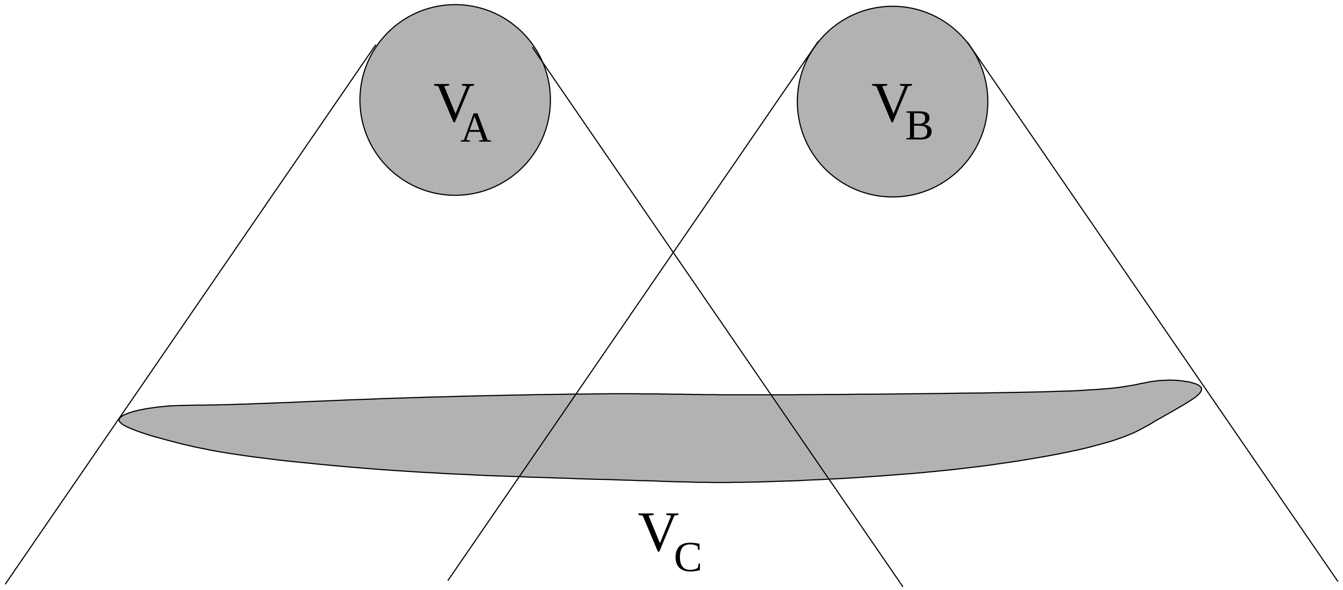


Local causality IV.

- $\neq$ **Strong Common Cause Principle:** there exists a *non-trivial* partition $\{C_k\} \in \mathcal{N}(V_C)$ such that

$$\phi(A_m \wedge B_n | C_k) \;=\; \phi(A_m|C_k)\phi(B_n|C_k)$$

Local causality



Local causality V.

- Trivially holds for a classical, atomic net satisfying local primitive causality.
- $\neq$ **Weak Common Cause Principle**

Local causality

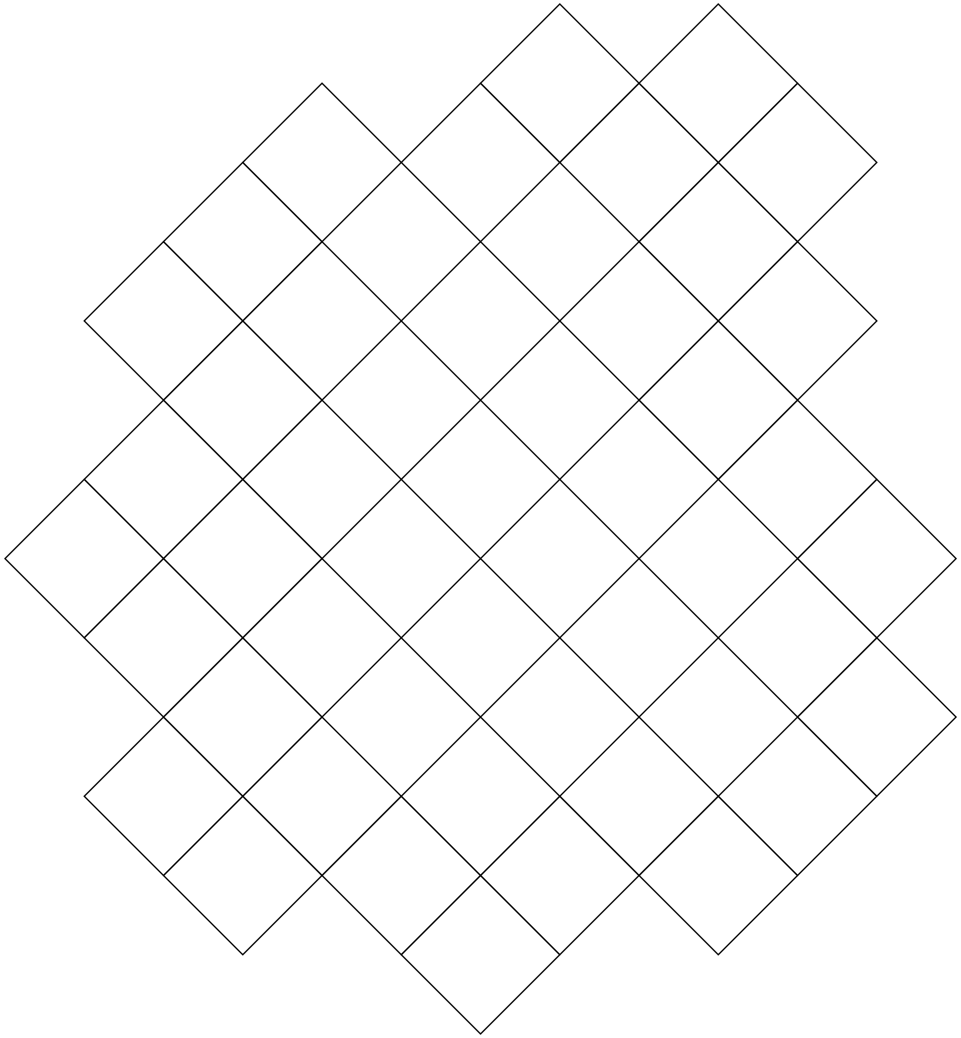
Questions:

1. How Local causality IV and V relate to the Common Cause Principles in classical and non-classical nets?
2. What is the relation between *local primitive causality* and *local causality*?

IV. Classical nets

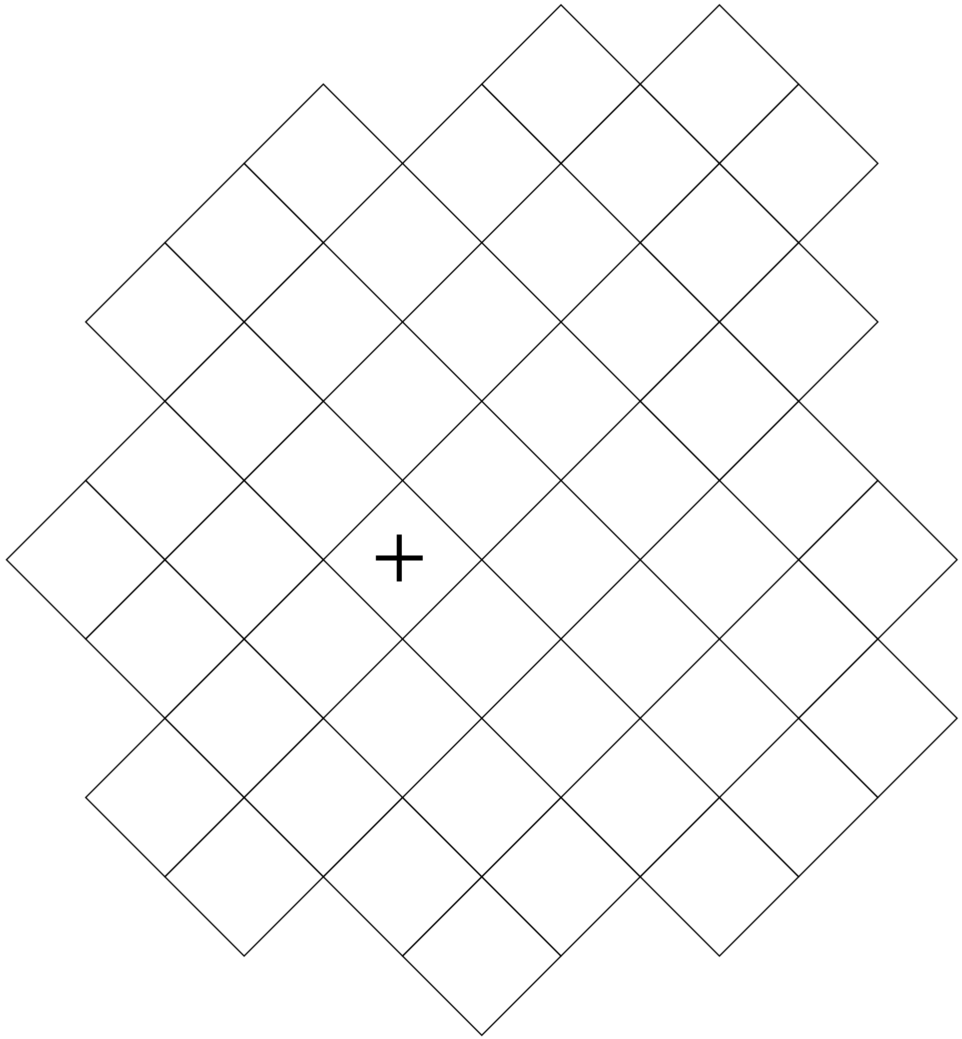
Classical nets

Two dimensional discrete Minkowski spacetime:



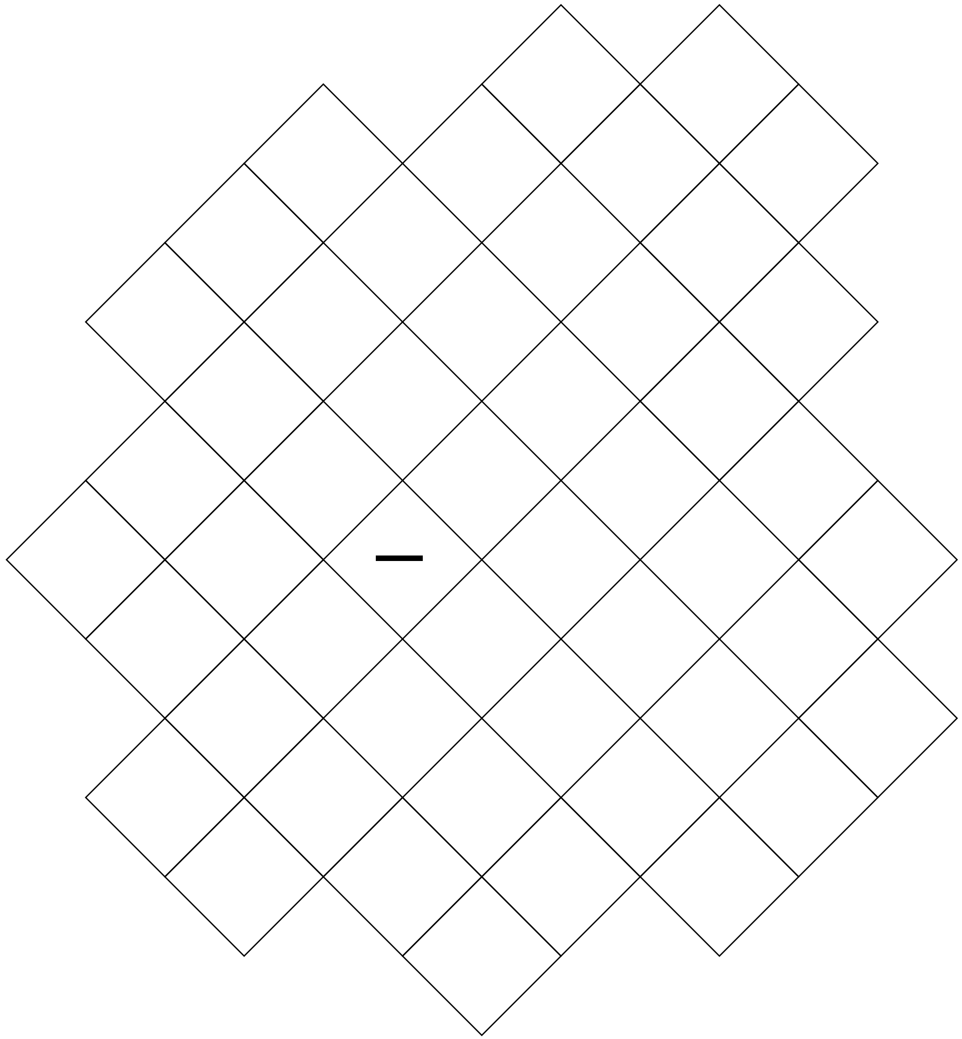
Classical nets

Local algebras:



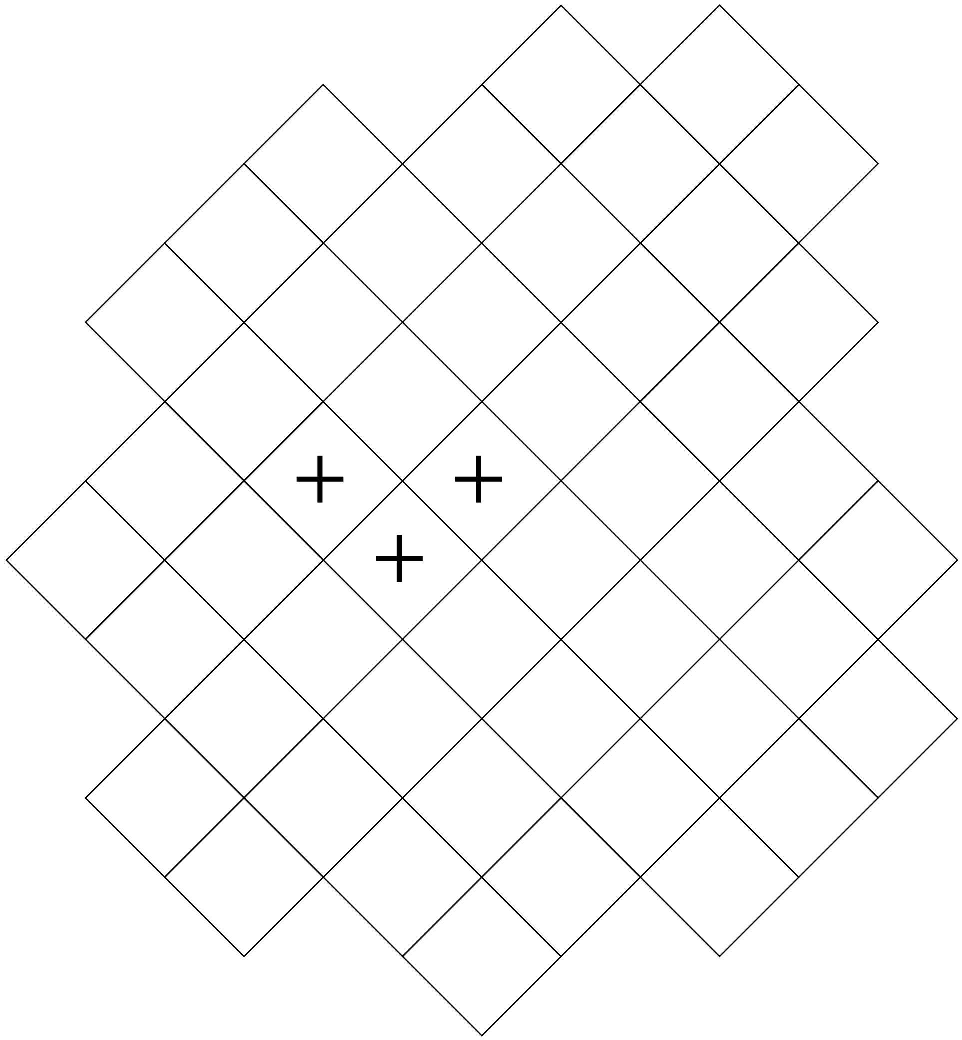
Classical nets

Local algebras:



Classical nets

Deterministic dynamics:



Classical nets

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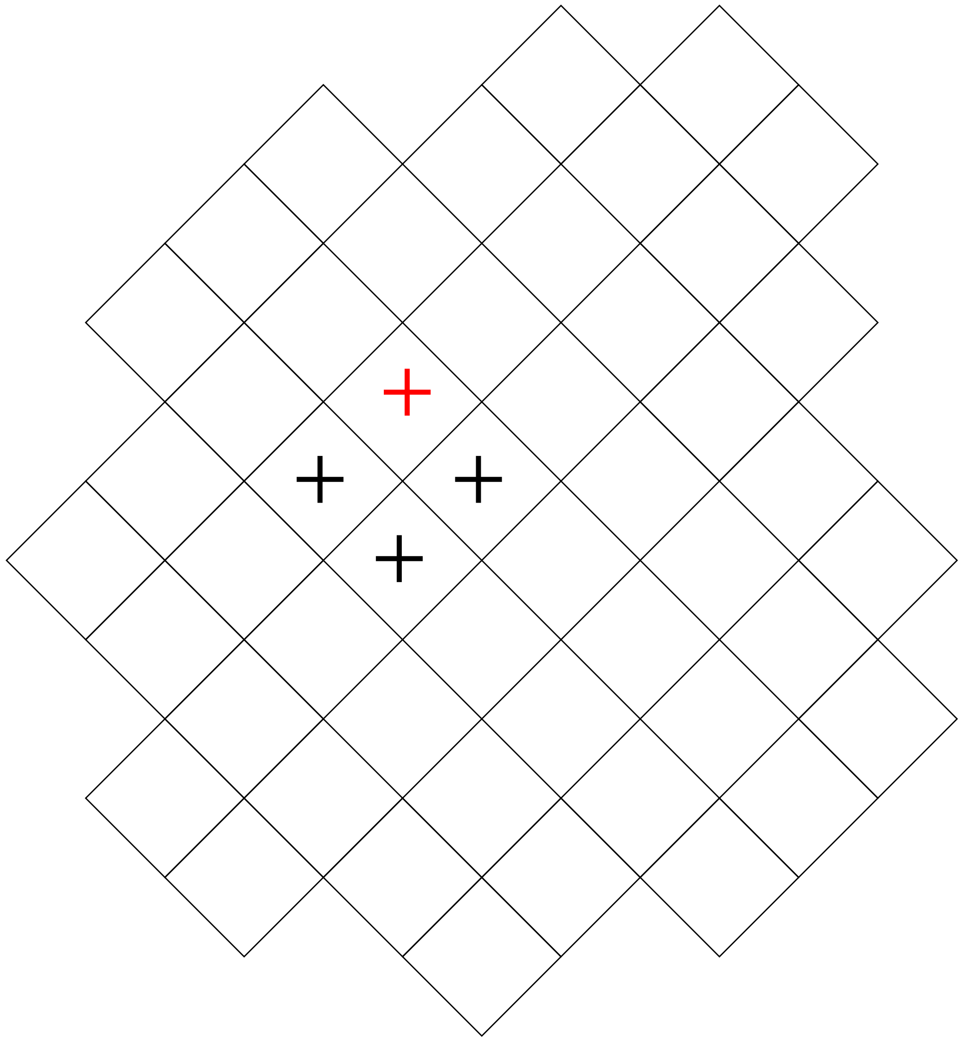
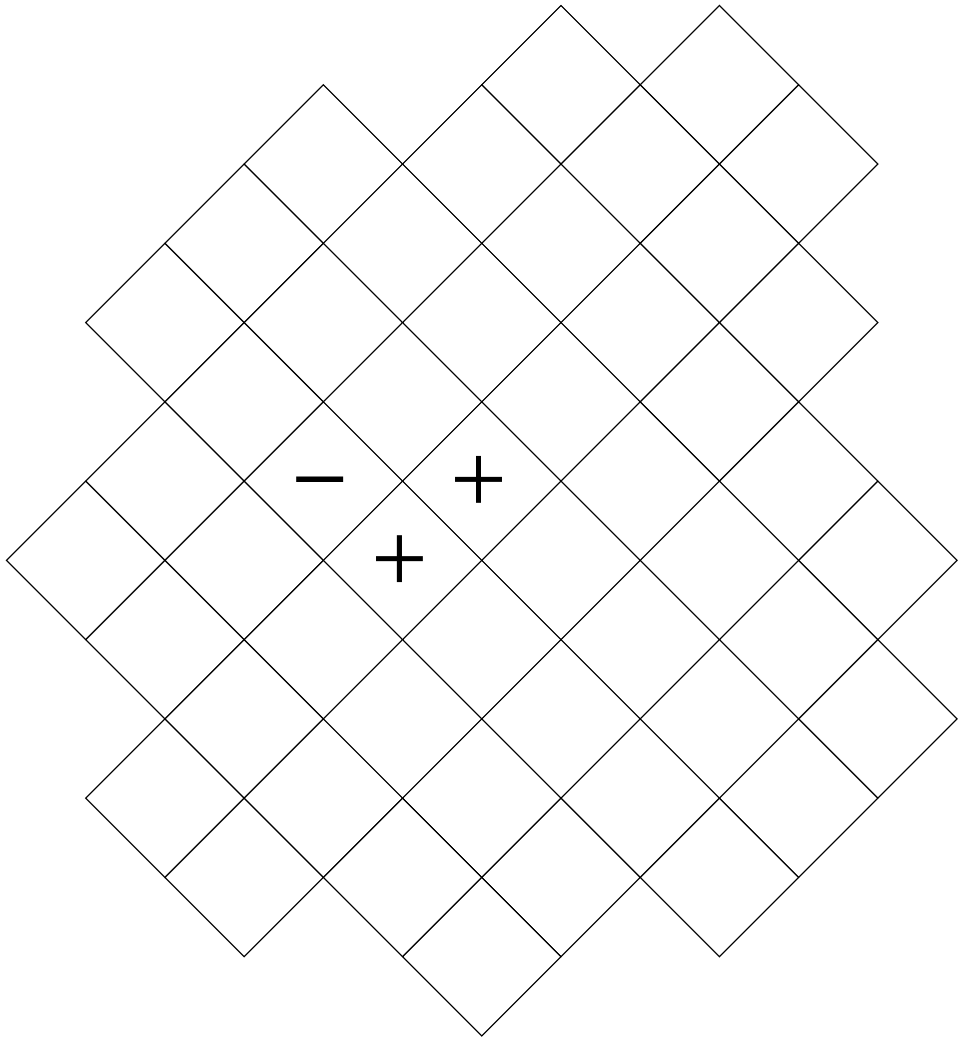


Deterministic dynamics:



Classical nets

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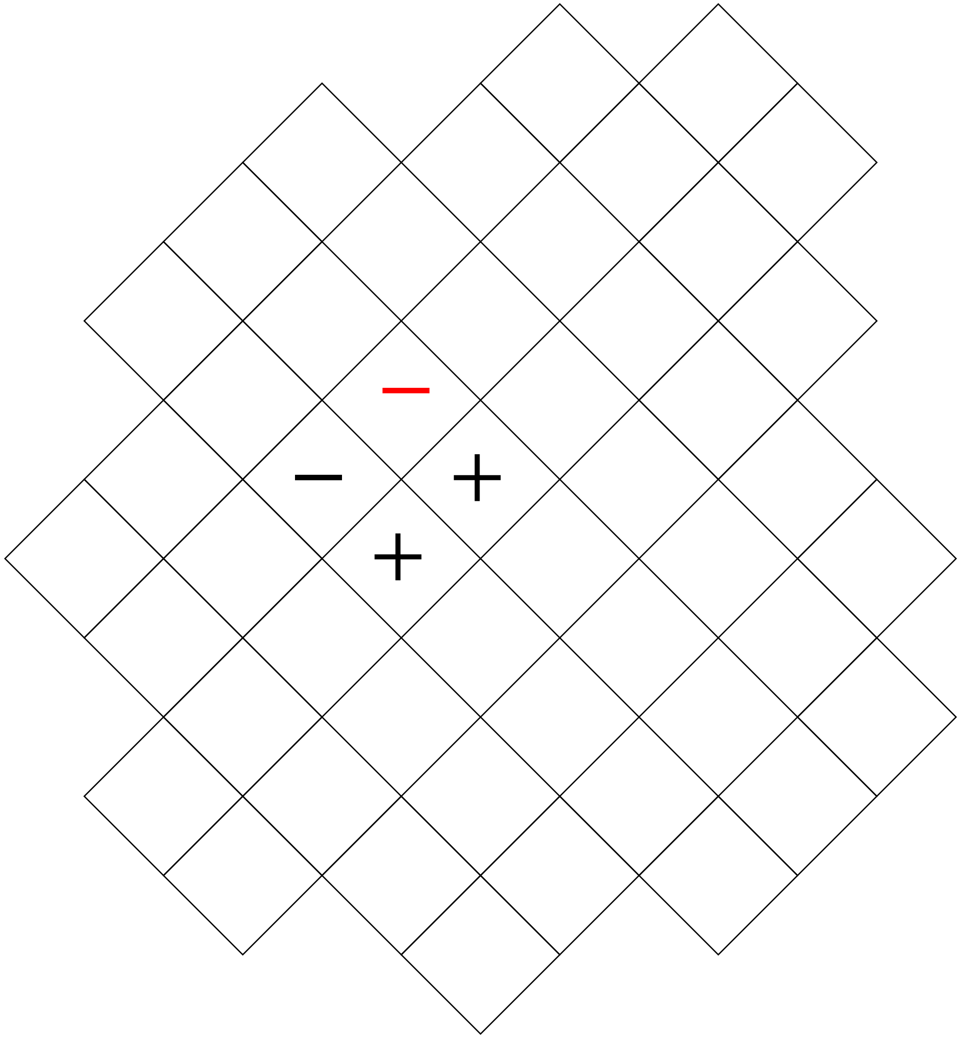
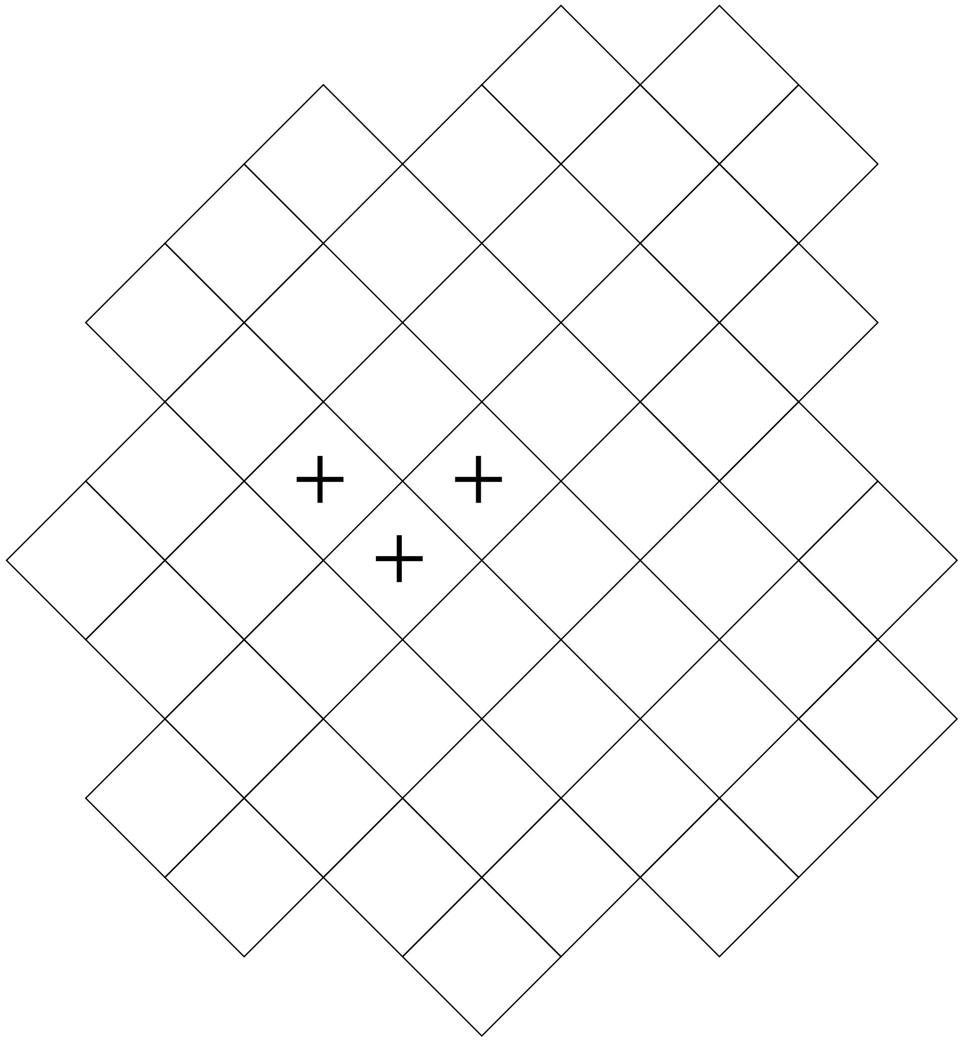


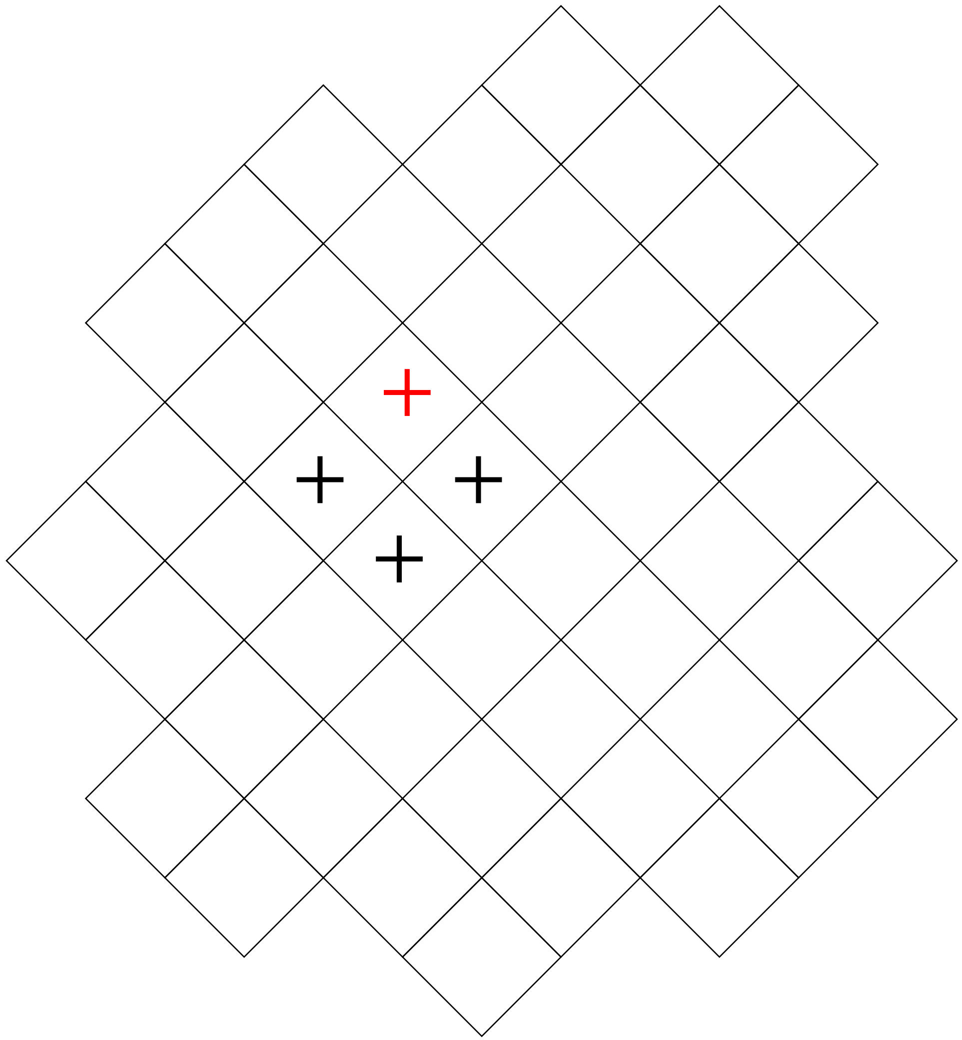
Classical nets

Stochastic dynamics:



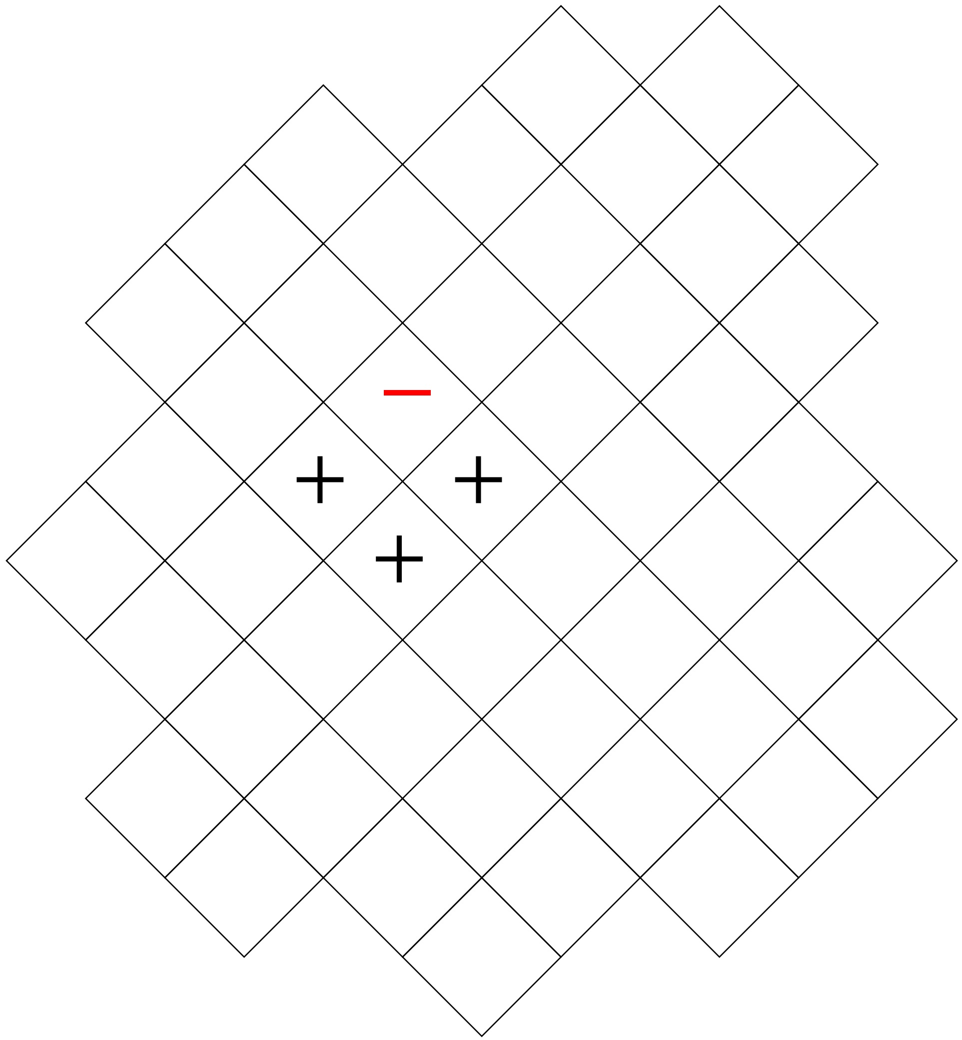
Classical nets

Stochastic dynamics: with probability p



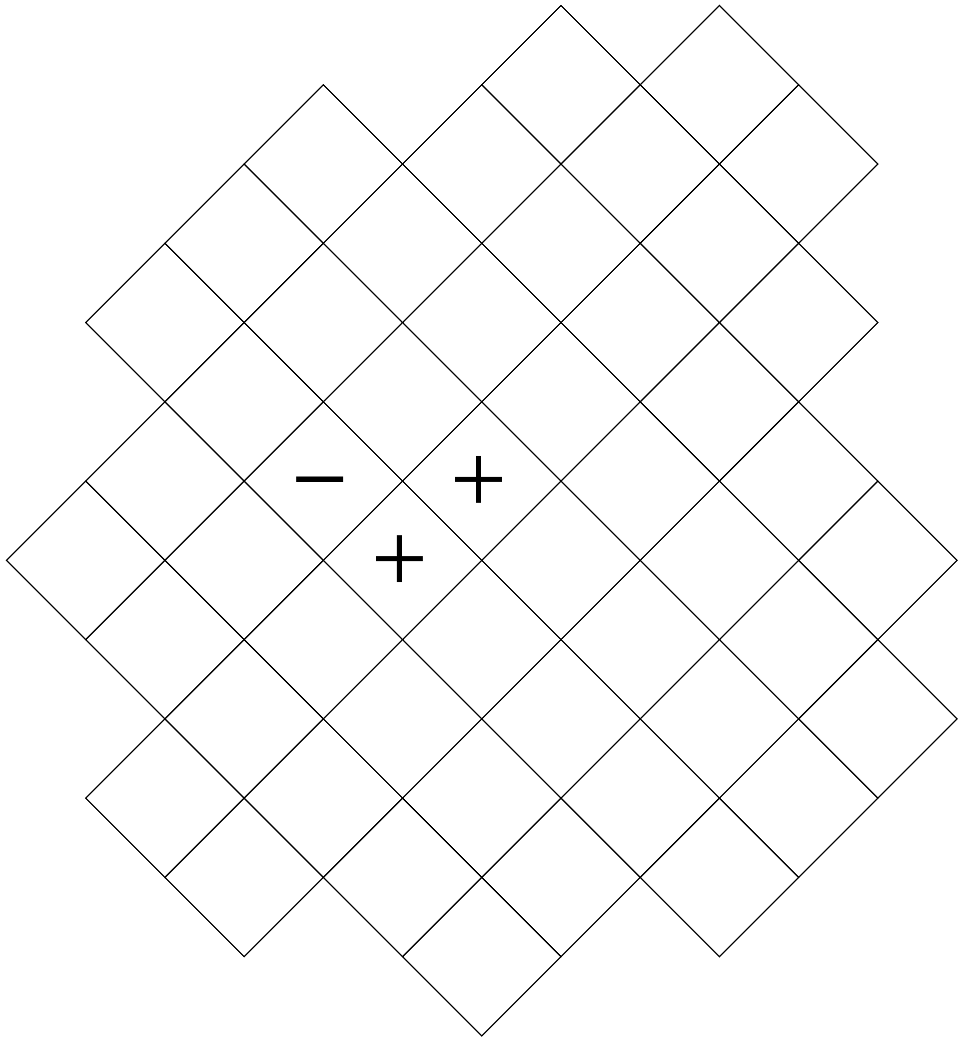
Classical nets

Stochastic dynamics: with probability $1 - p$



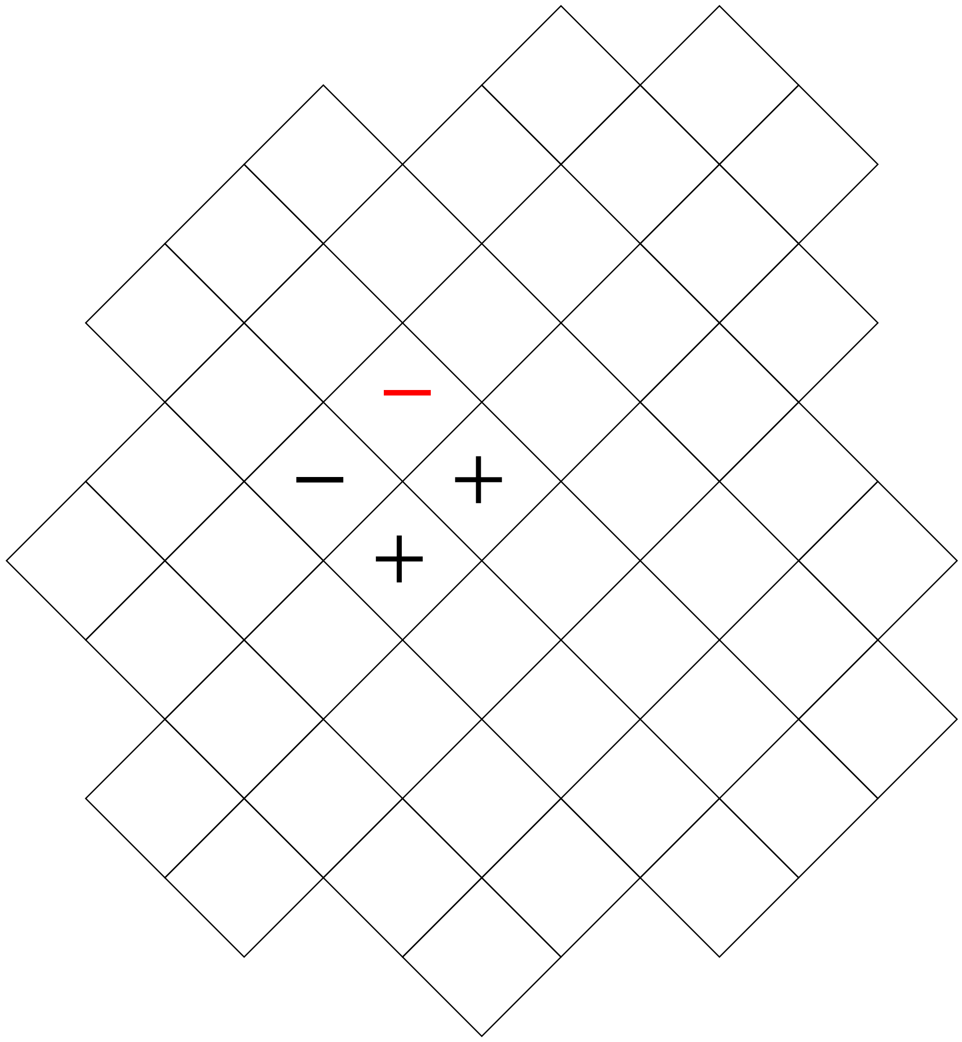
Classical nets

Stochastic dynamics:



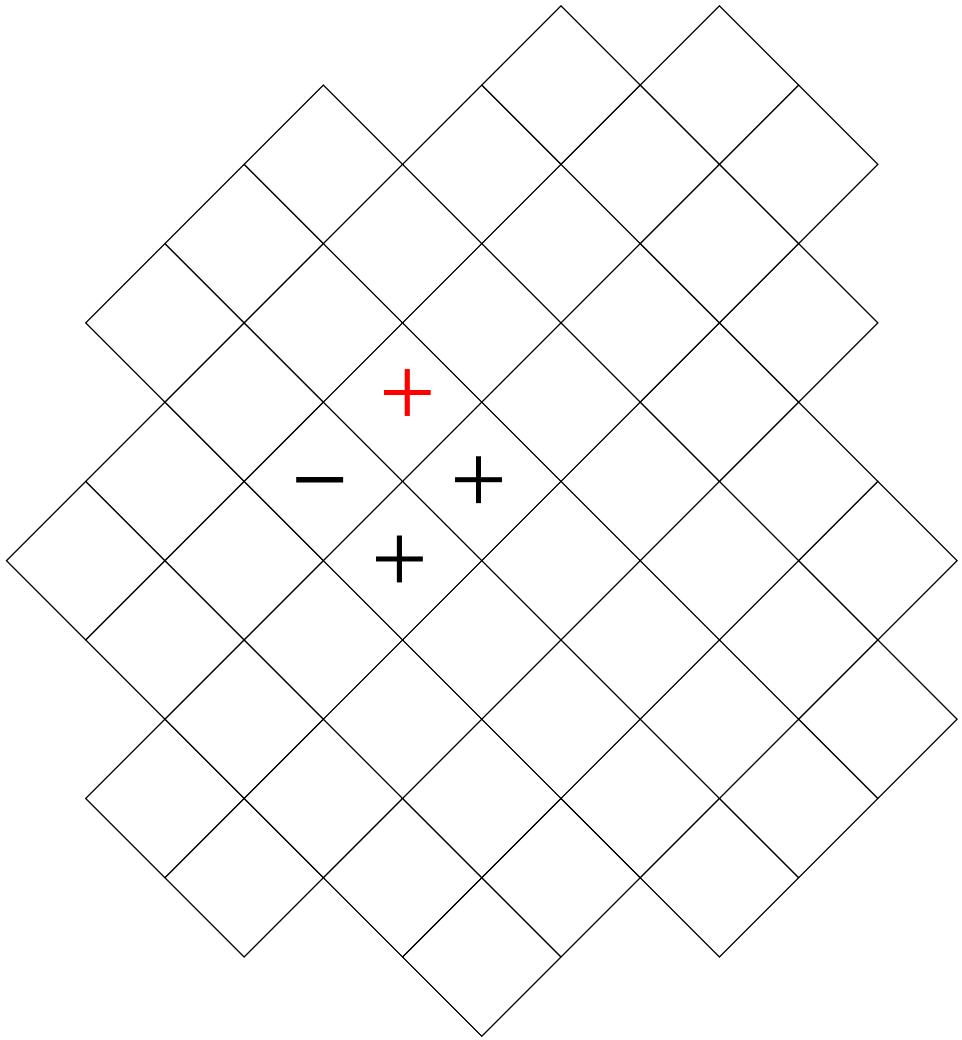
Classical nets

Stochastic dynamics: with probability p



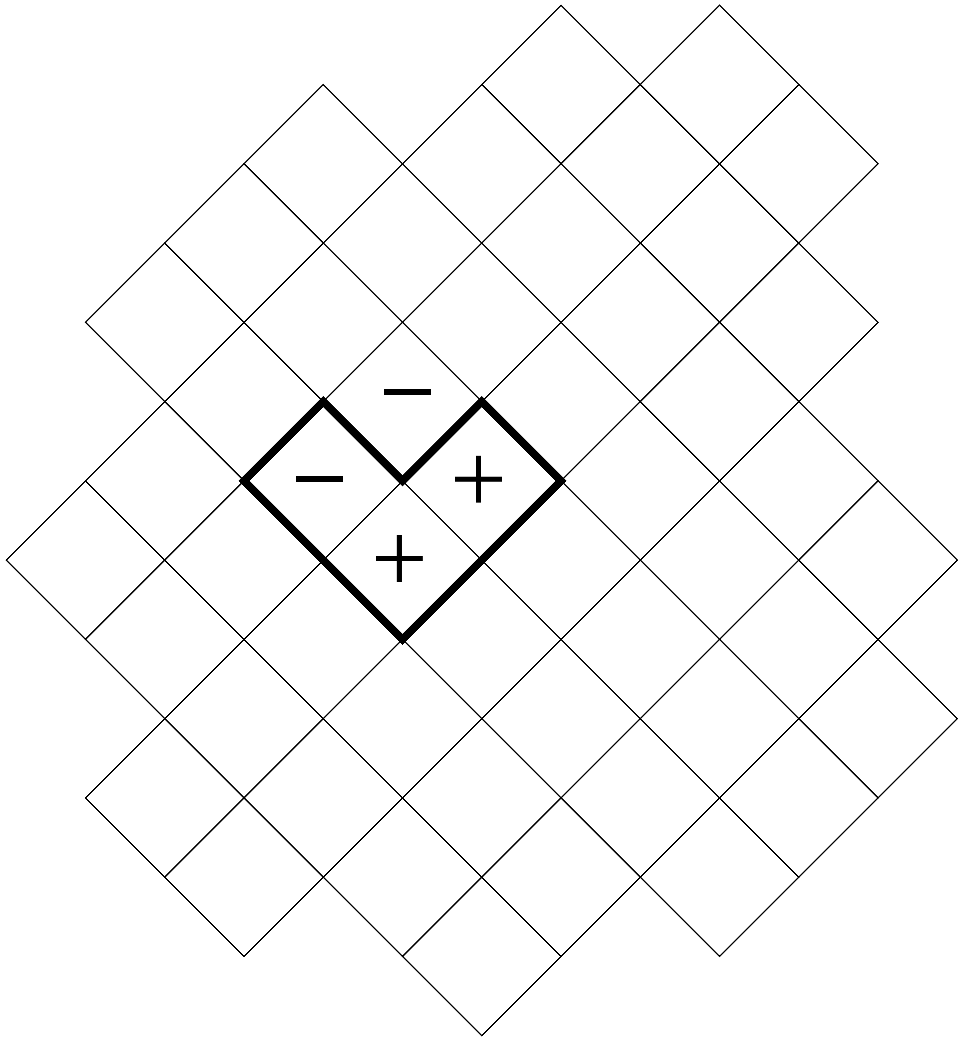
Classical nets

Stochastic dynamics: with probability $1 - p$

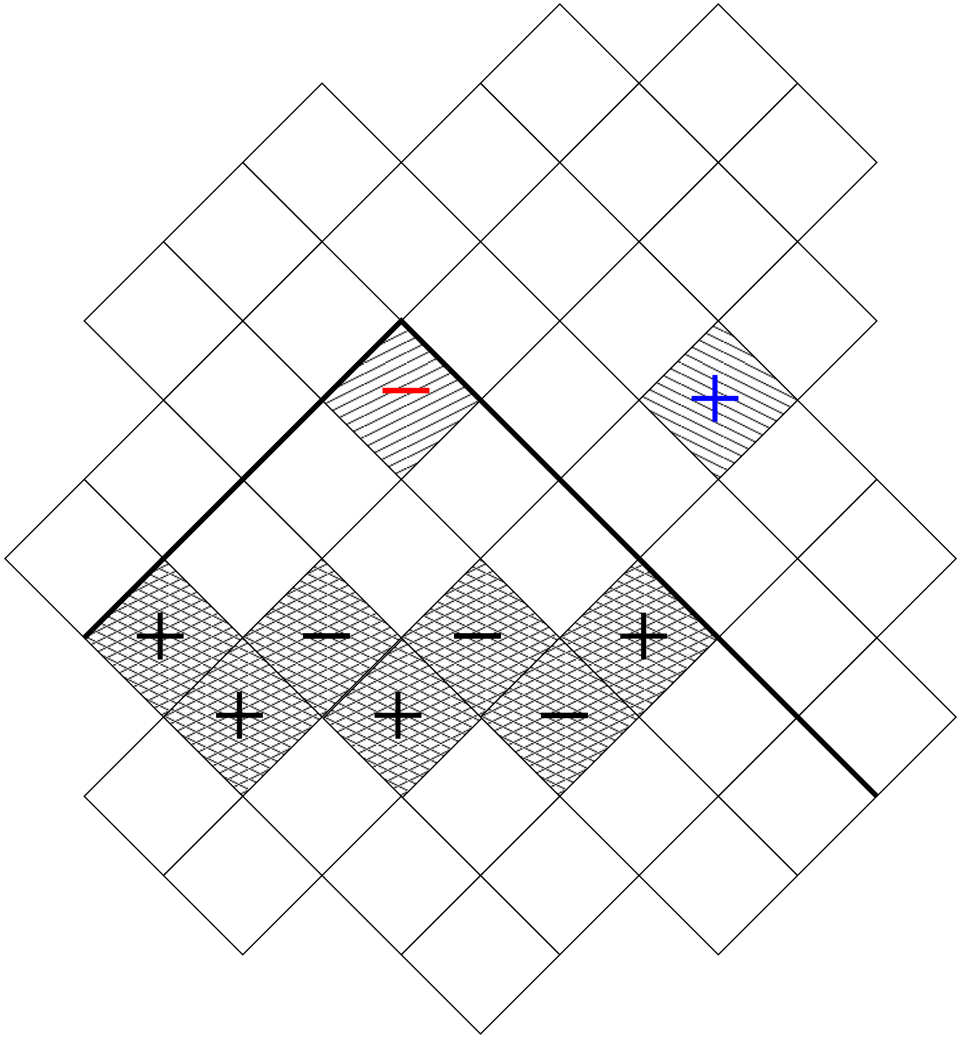


Classical nets

Local primitive causality does *not* hold:



But local causality *does* hold:



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