Bell's local causality in local classical and quantum theory

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Project

I. What is a local physical theory?

II. Bell's local causality in a local physical theory

III. Common Cause Principle

IV. Bell inequalities

V. Causal Markov Condition

I. What is a local physical theory?

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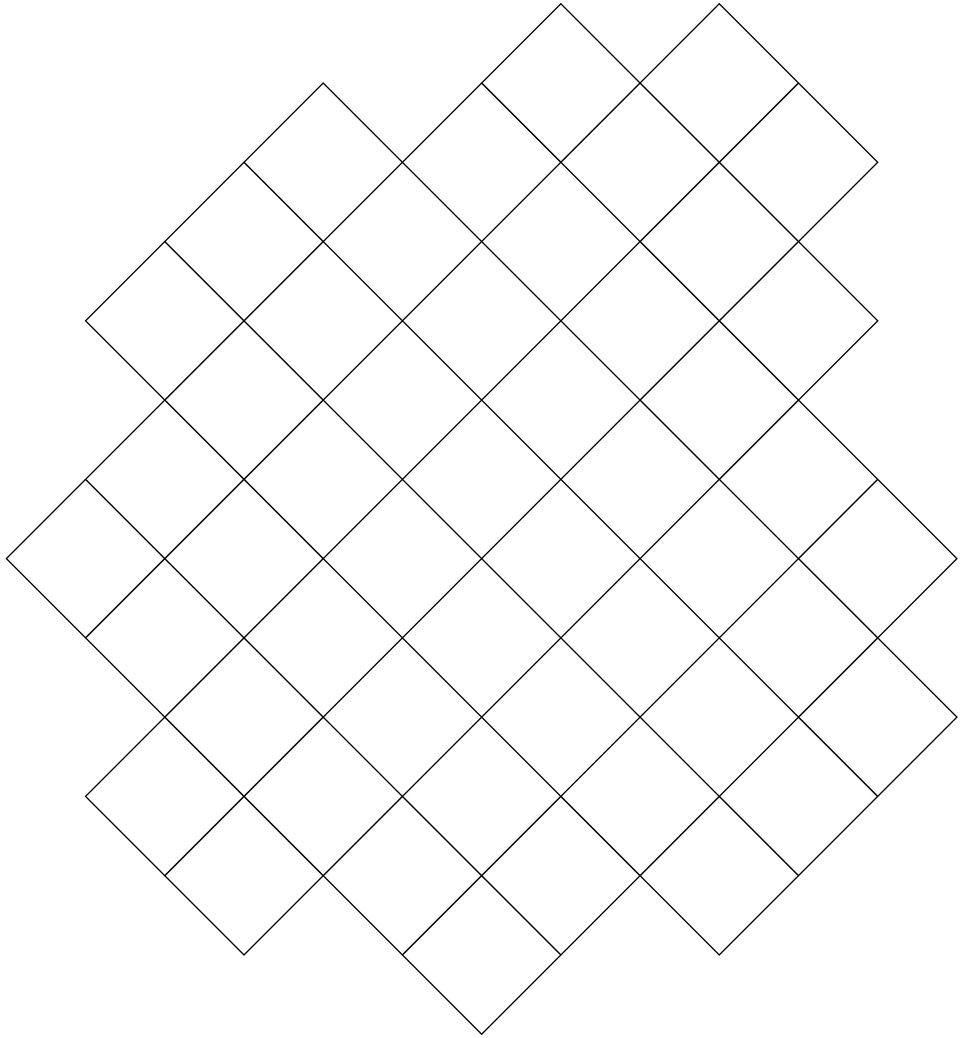
- $\mathcal{M}$: **globally hyperbolic spacetime** with symmetries $\mathcal{P}$
- $\mathcal{K}$: **covering collection** of bounded, globally hyperbolic regions of $\mathcal{M}$
- $(\mathcal{K}, \subseteq)$: **directed poset**
- $\mathcal{P}_{\mathcal{K}}$: **subgroup** of $\mathcal{P}$ leaving $\mathcal{K}$ invariant

I. What is a local physical theory?

- **Discretized two dimensional Minkowski spacetime:**

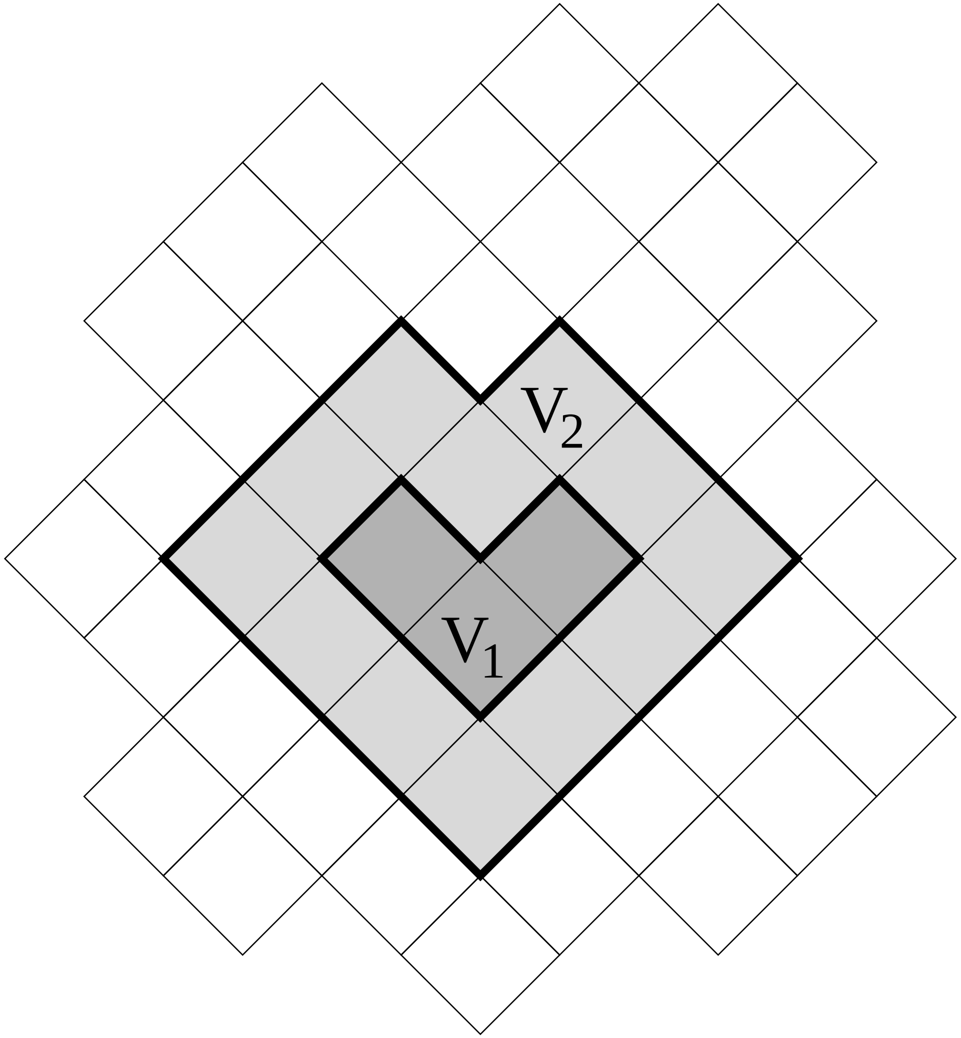


I. What is a local physical theory?

- **Definition.** A $\mathcal{P}_{\mathcal{K}}$-covariant **local physical theory (LPT)** is a net $\mathcal{K} \ni V \mapsto \mathcal{N}(V)$ associating von Neumann algebras to spacetime regions which satisfies
  1. **isotony**,
  2. **microcausality**,
  3. **covariance**.

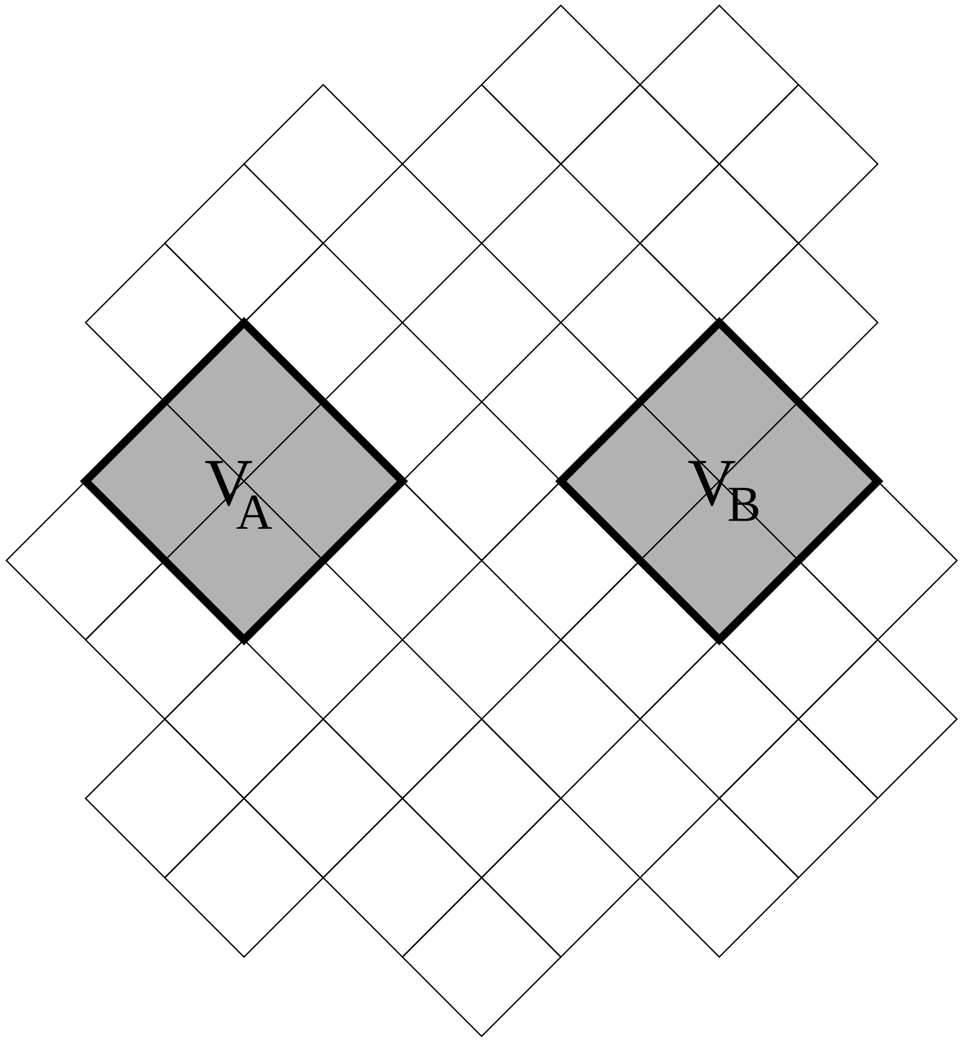
I. What is a local physical theory?

- **Isotony:** if $V_1 \subset V_2$, then $\mathcal{N}(V_1)$ is a unital subalgebra of $\mathcal{N}(V_2)$



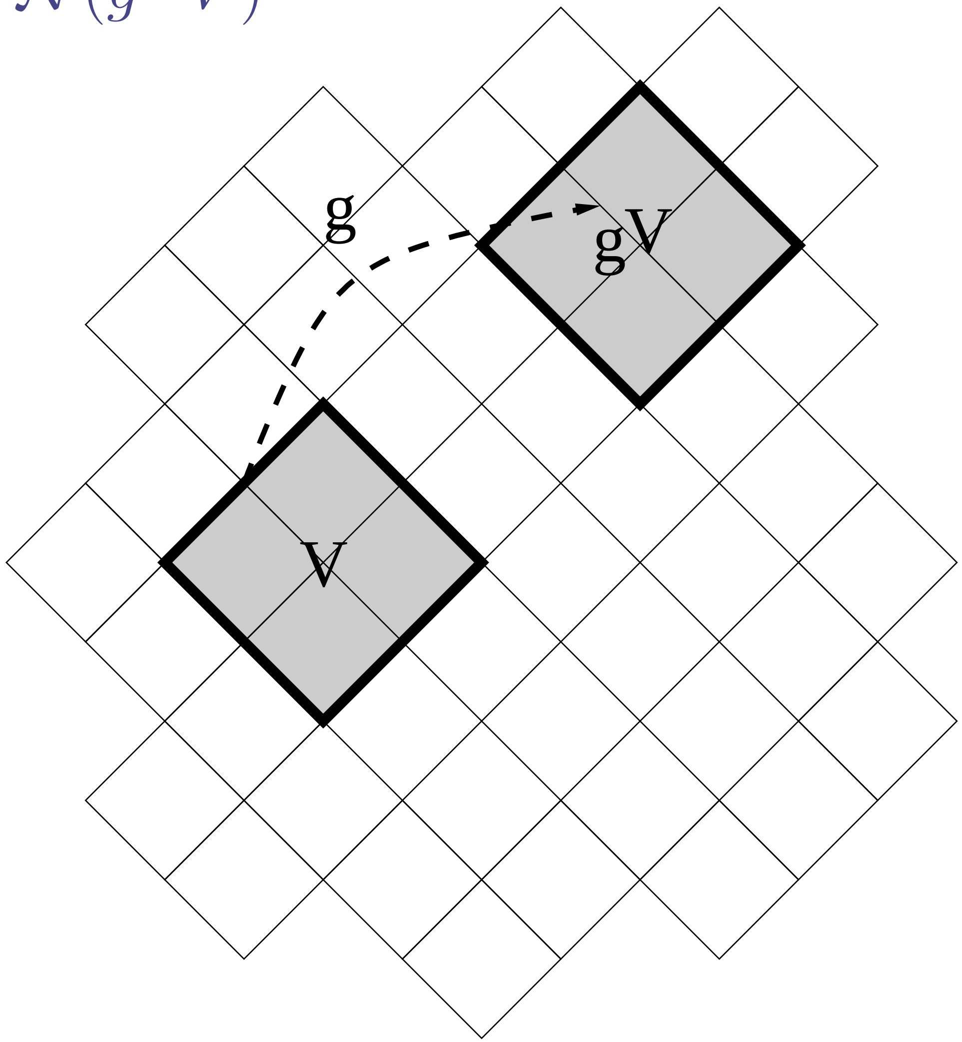
I. What is a local physical theory?

- **Microcausality (Einstein causality):** $[\mathcal{N}(V_A), \mathcal{N}(V_B)] = 0$



I. What is a local physical theory?

- **Covariance:** covariant group homomorphism on the net
  $\alpha_g(\mathcal{N}(V)) = \mathcal{N}(g \cdot V)$



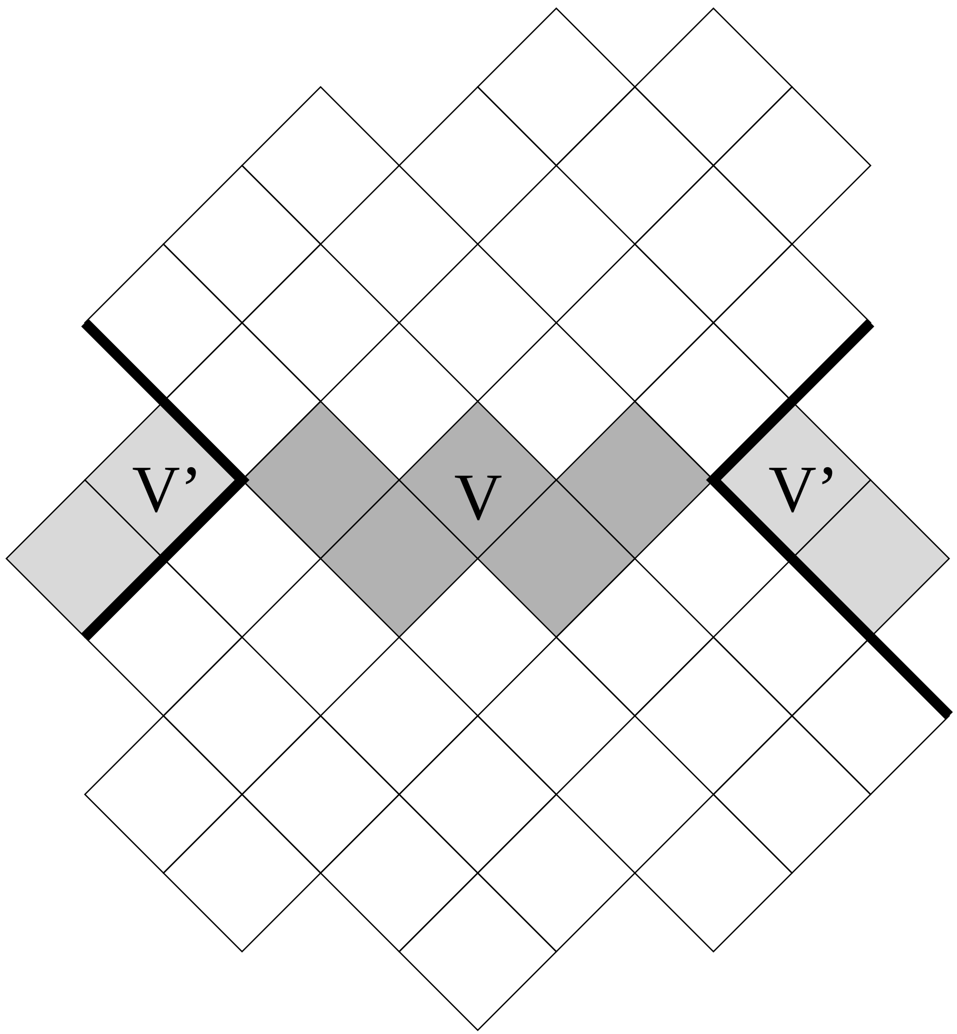
I. What is a local physical theory?

Remarks:

- **Quasilocal algebra** $\mathcal{A}$: the inductive limit C^*-algebra of the net
- $\mathcal{A}$ is commutative: **local classical theory (LCT)**
- $\mathcal{A}$ is noncommutative: **local quantum theory (LQT)**

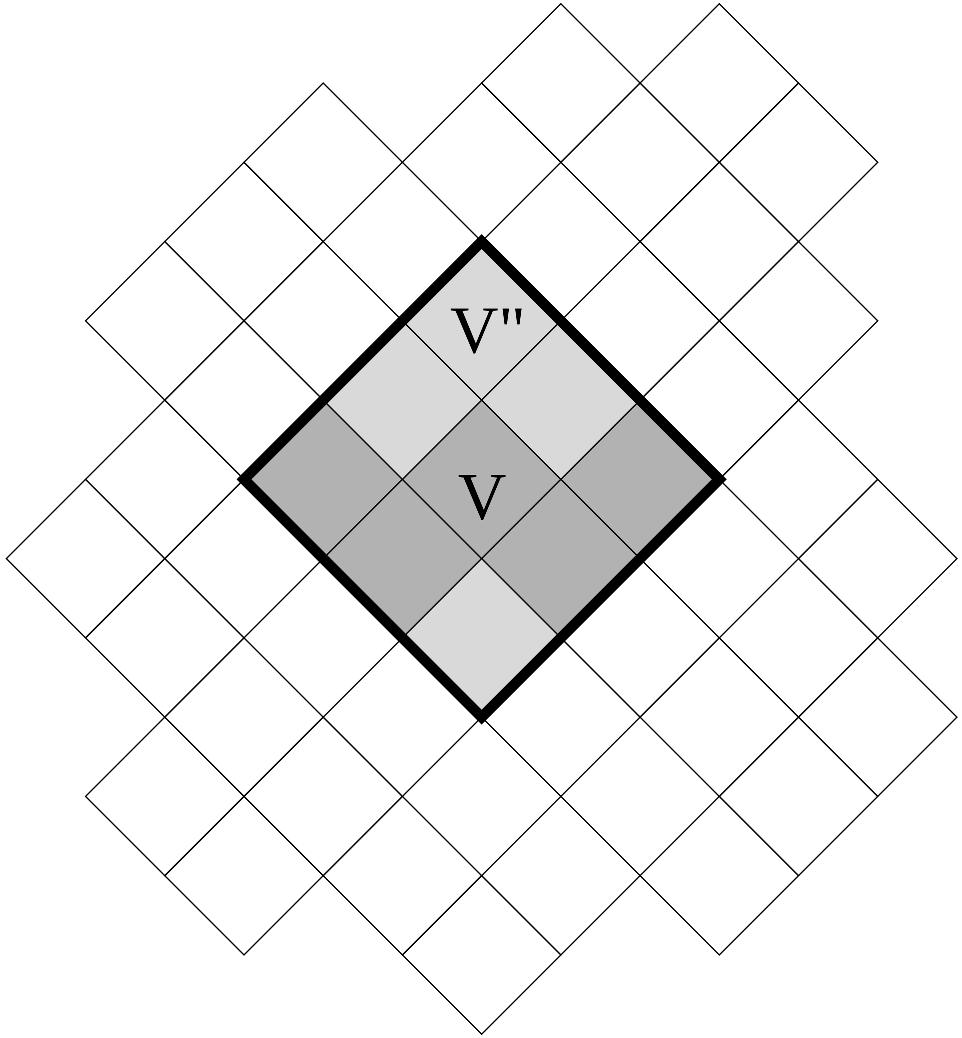
I. What is a local physical theory?

- **Causal complement:** V'



I. What is a local physical theory?

- **Domain of dependence:** V''



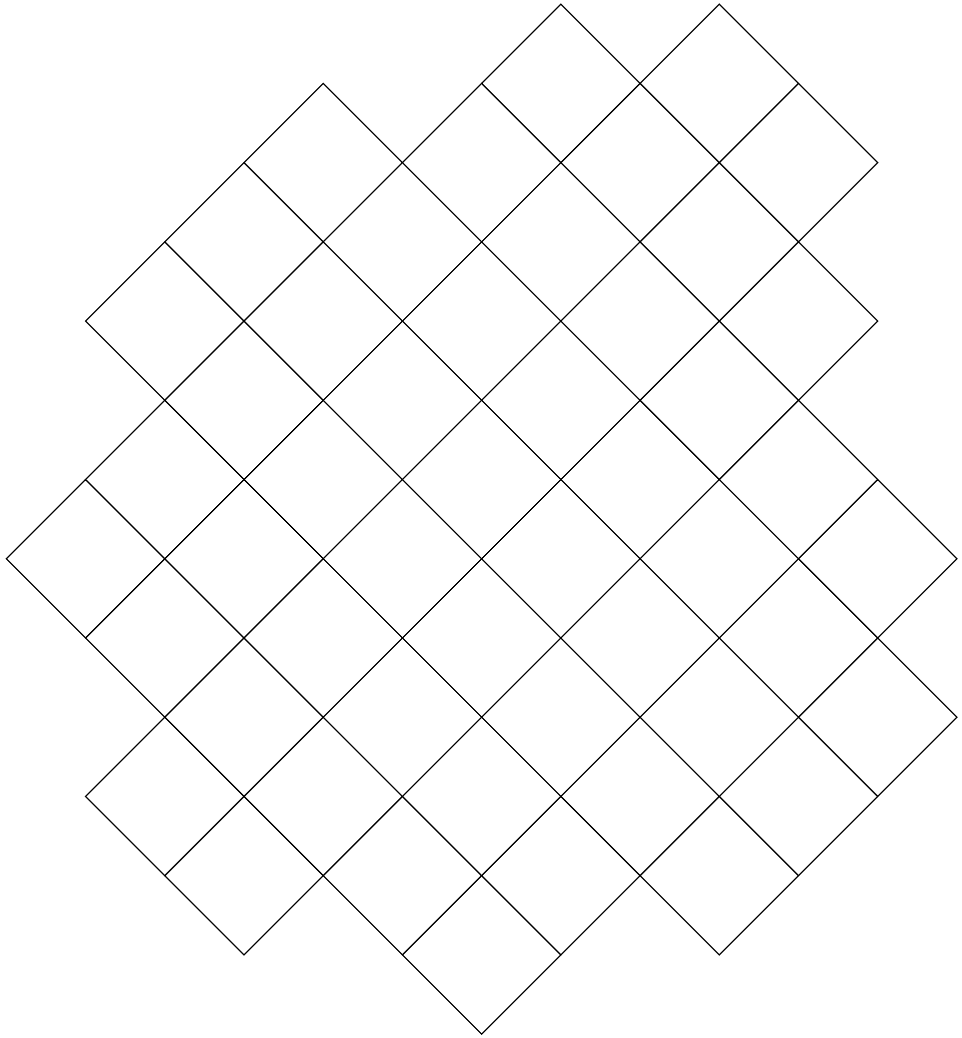
I. What is a local physical theory?

Examples:

1. Deterministic LCT
2. Stochastic LCT
3. Deterministic LQT
4. Stochastic LQT (not known)

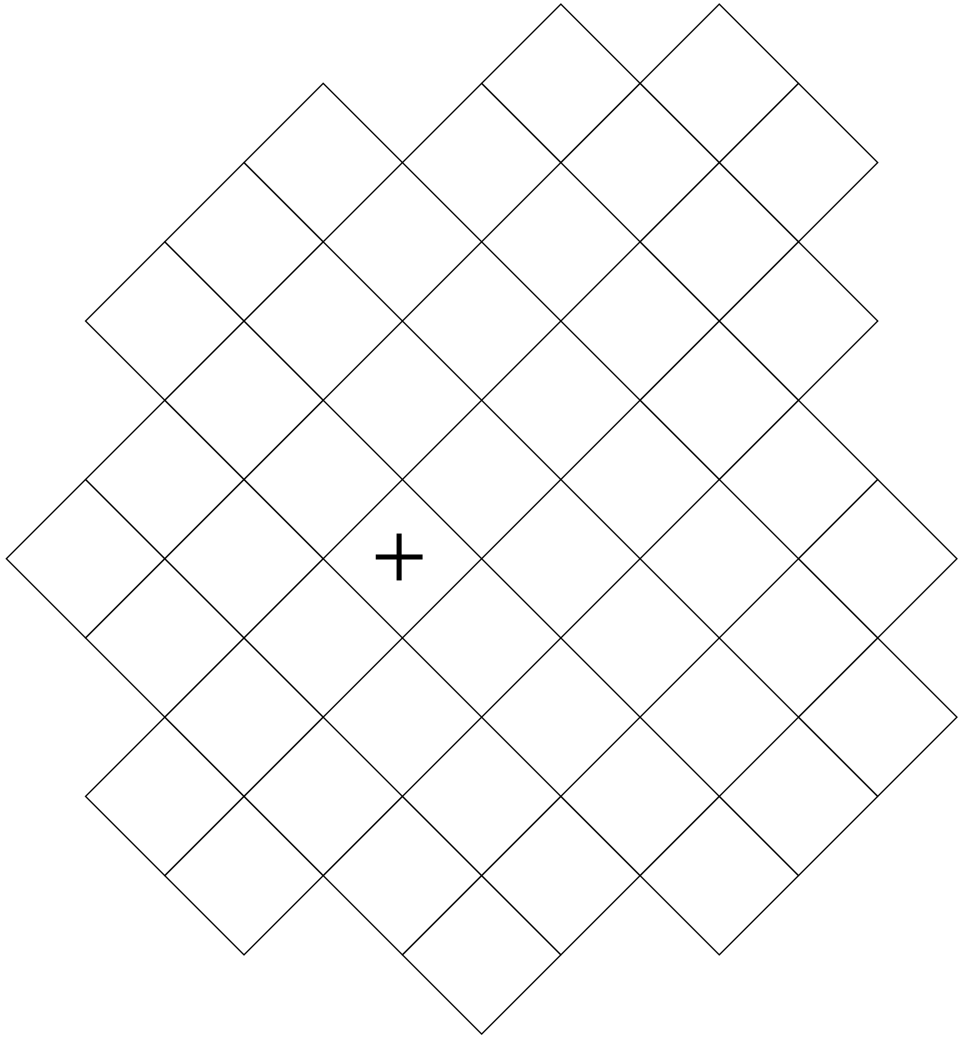
I. What is a local physical theory?

1. **Deterministic LCT**



I. What is a local physical theory?

- **Local algebras:**



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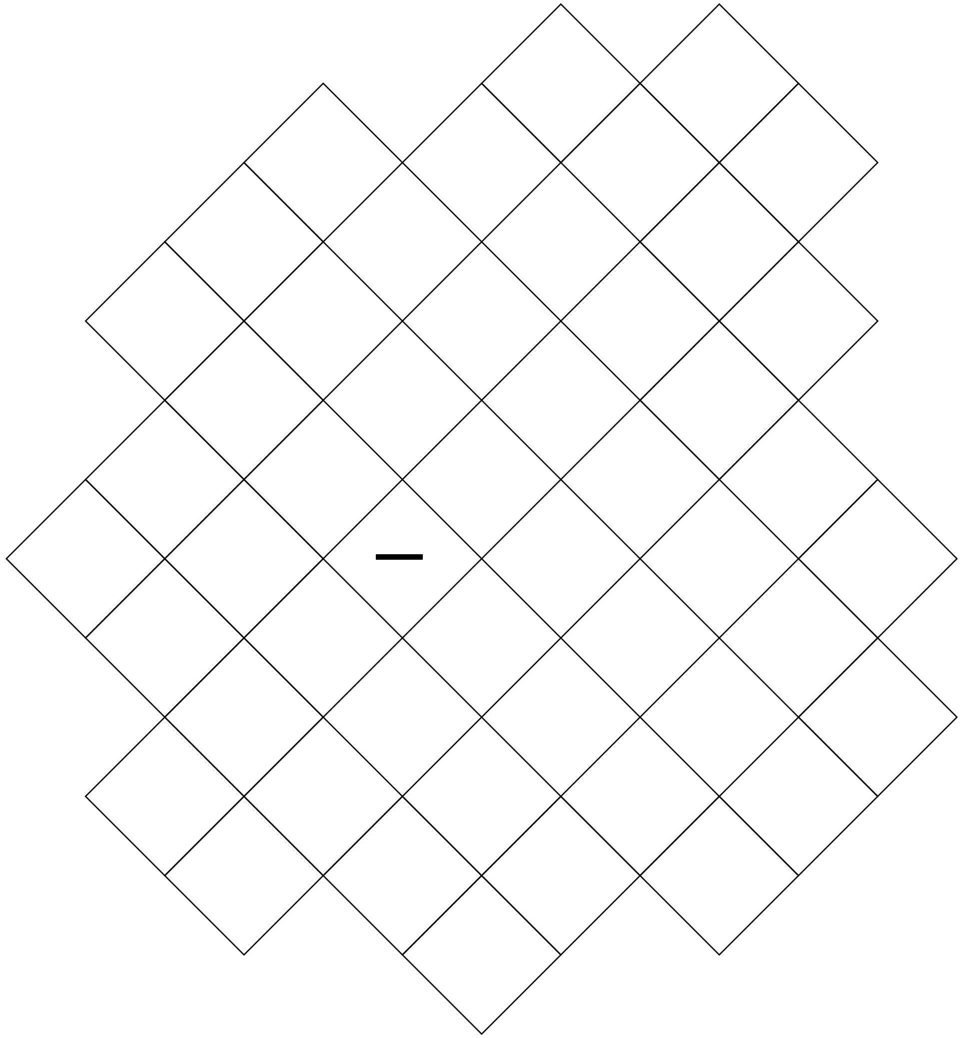
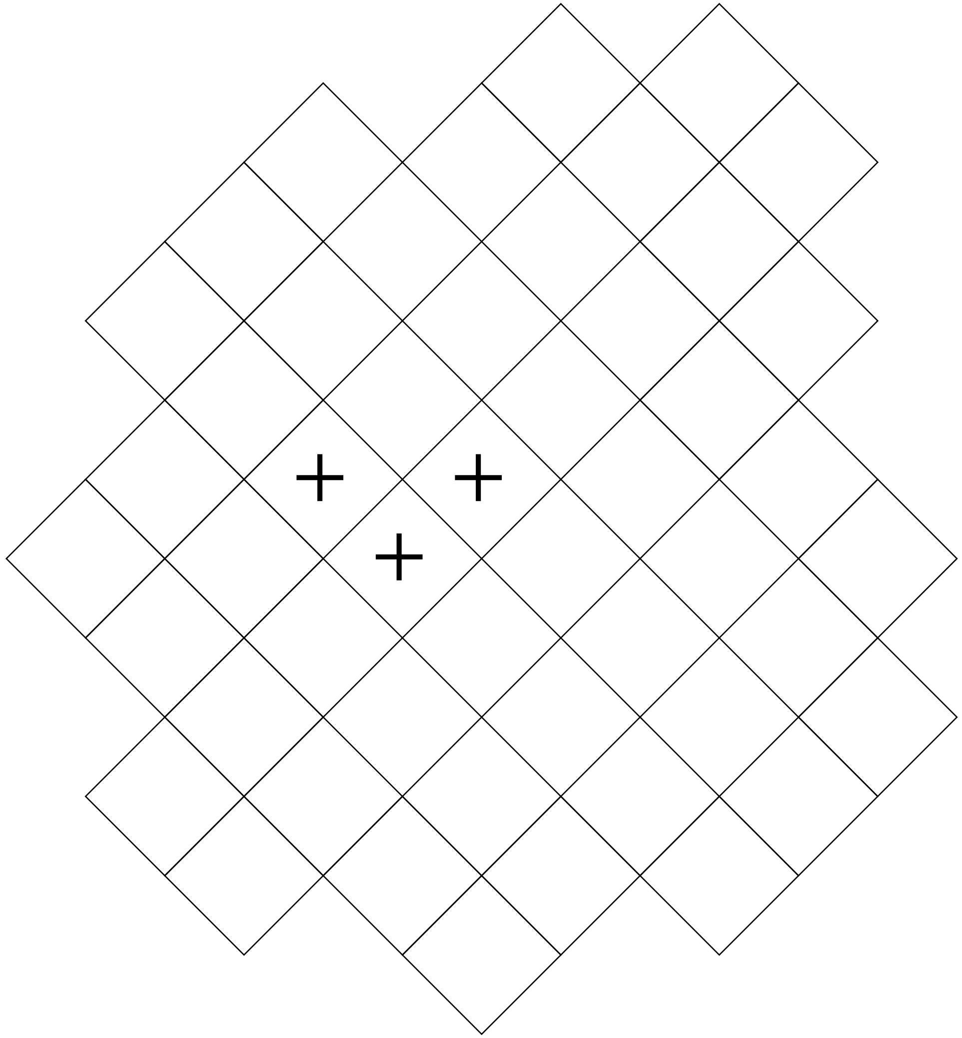


I. What is a local physical theory?

- **Deterministic dynamics:**



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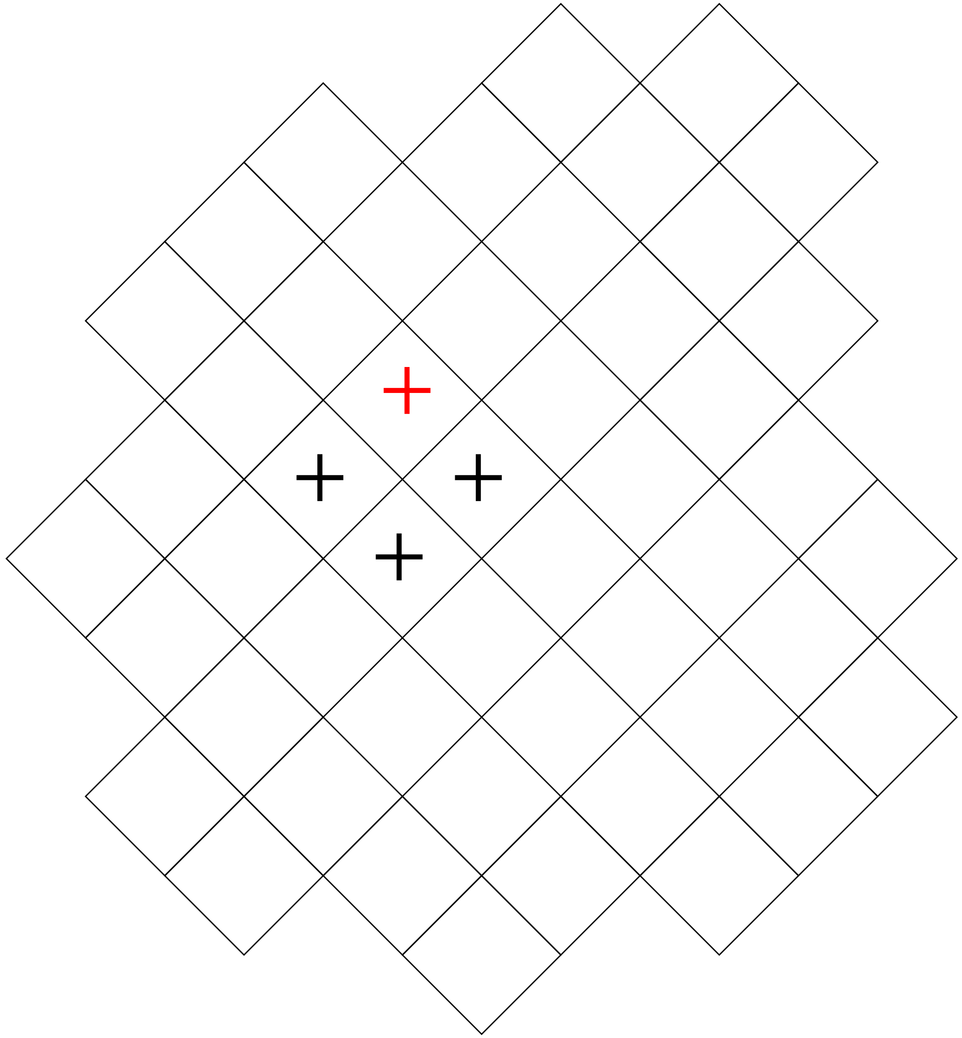

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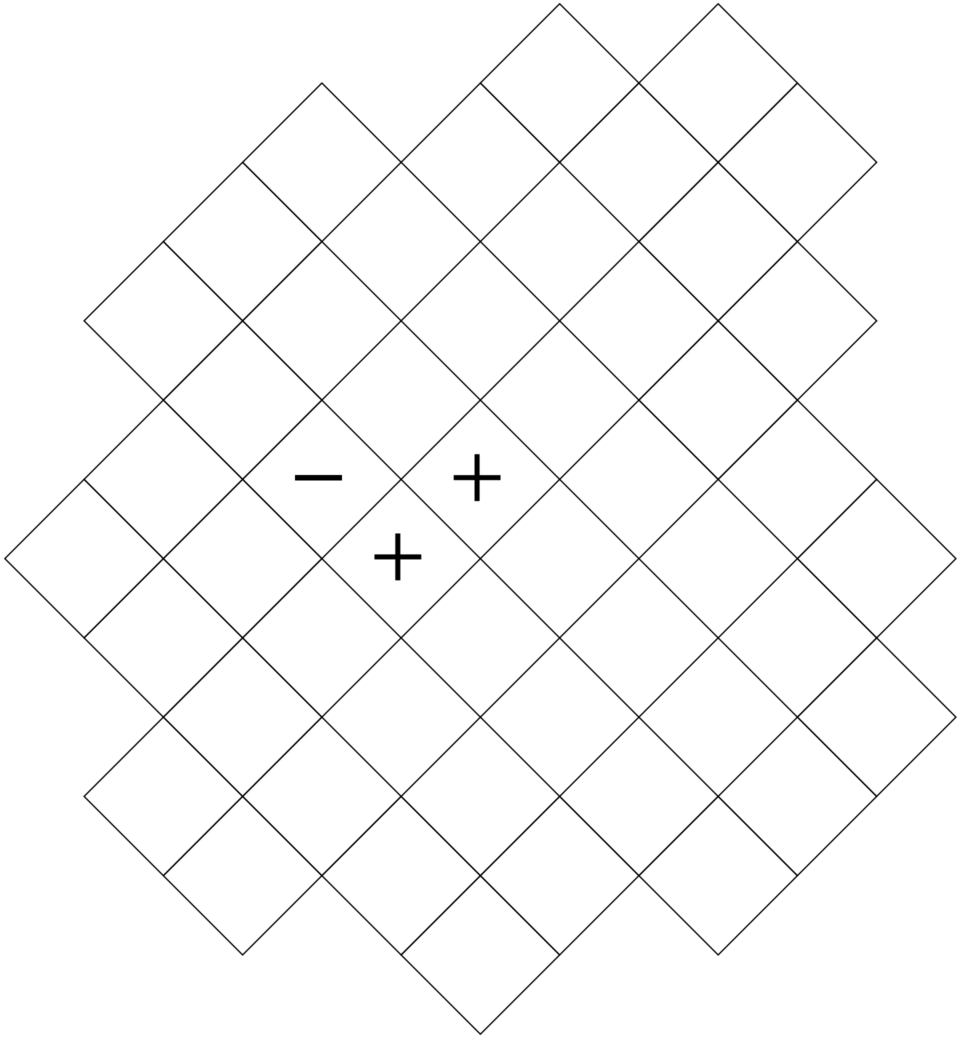

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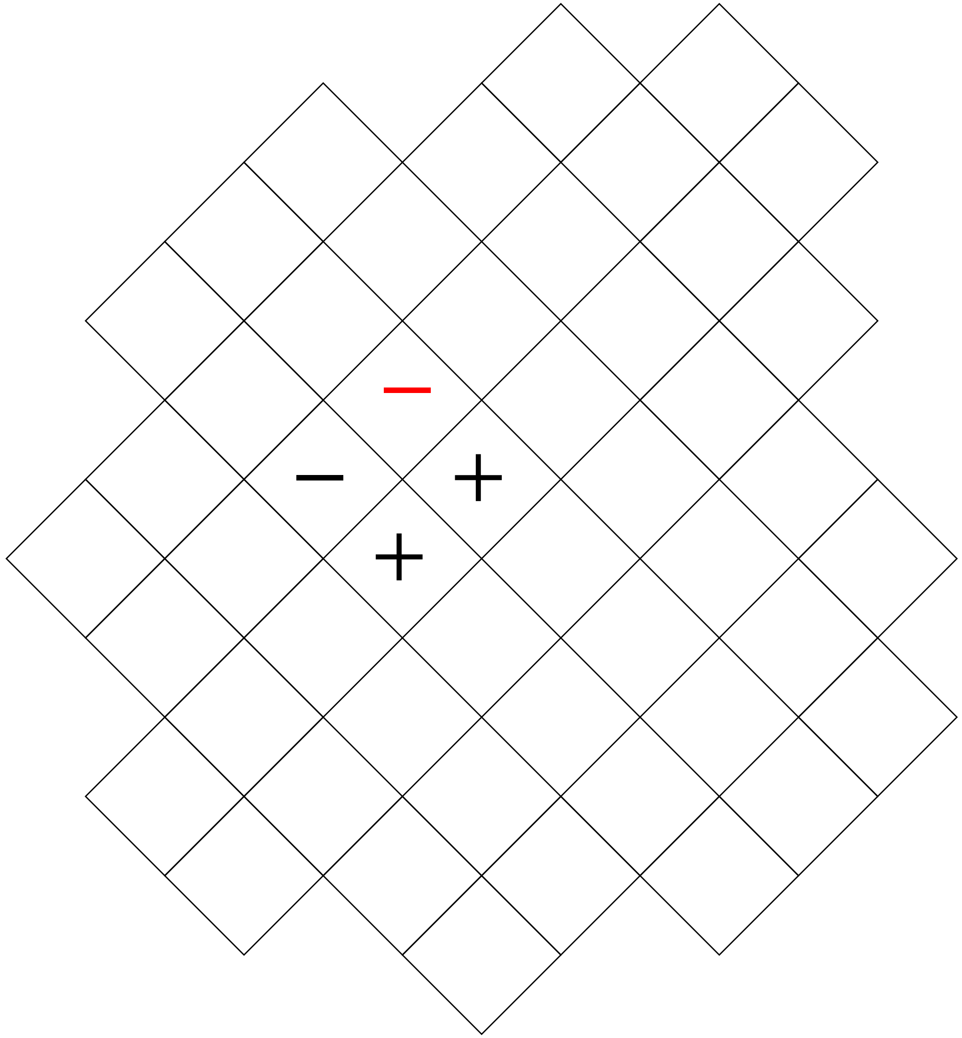
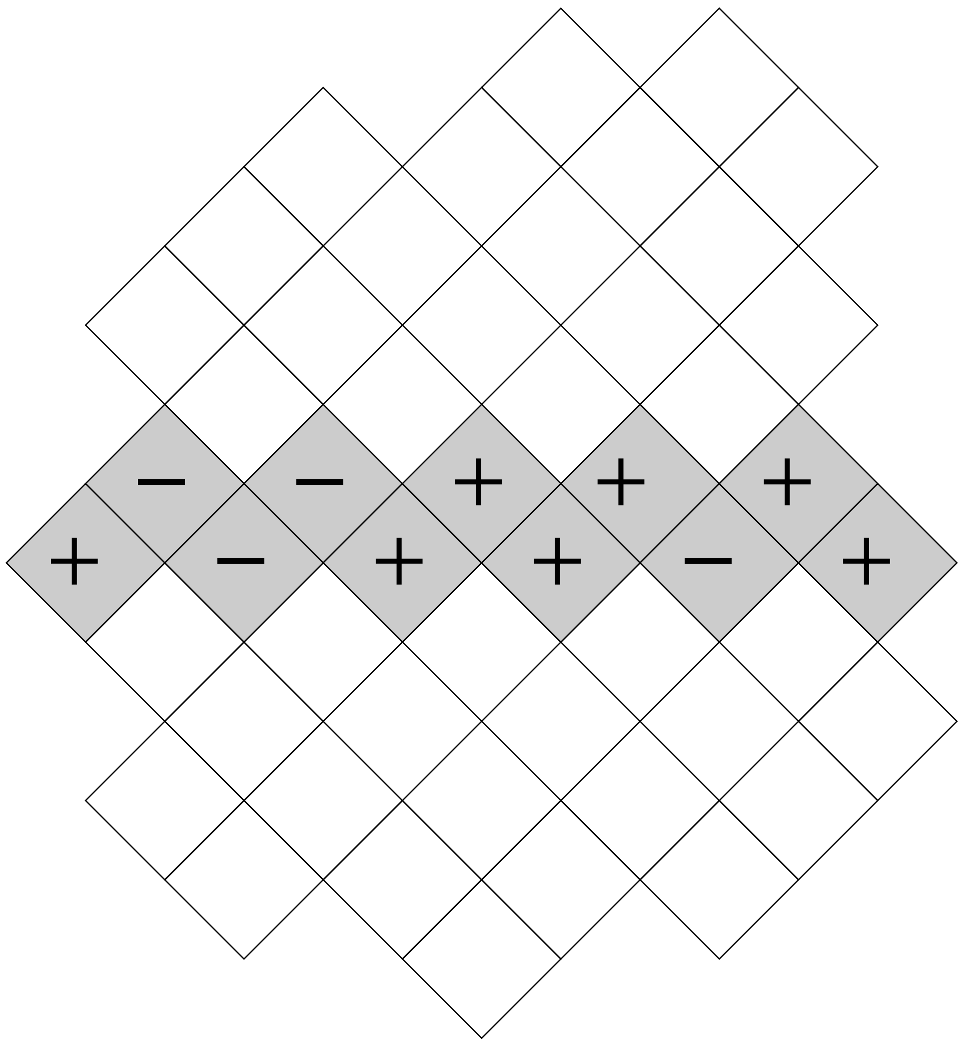


I. What is a local physical theory?

- **Dynamics in action:**



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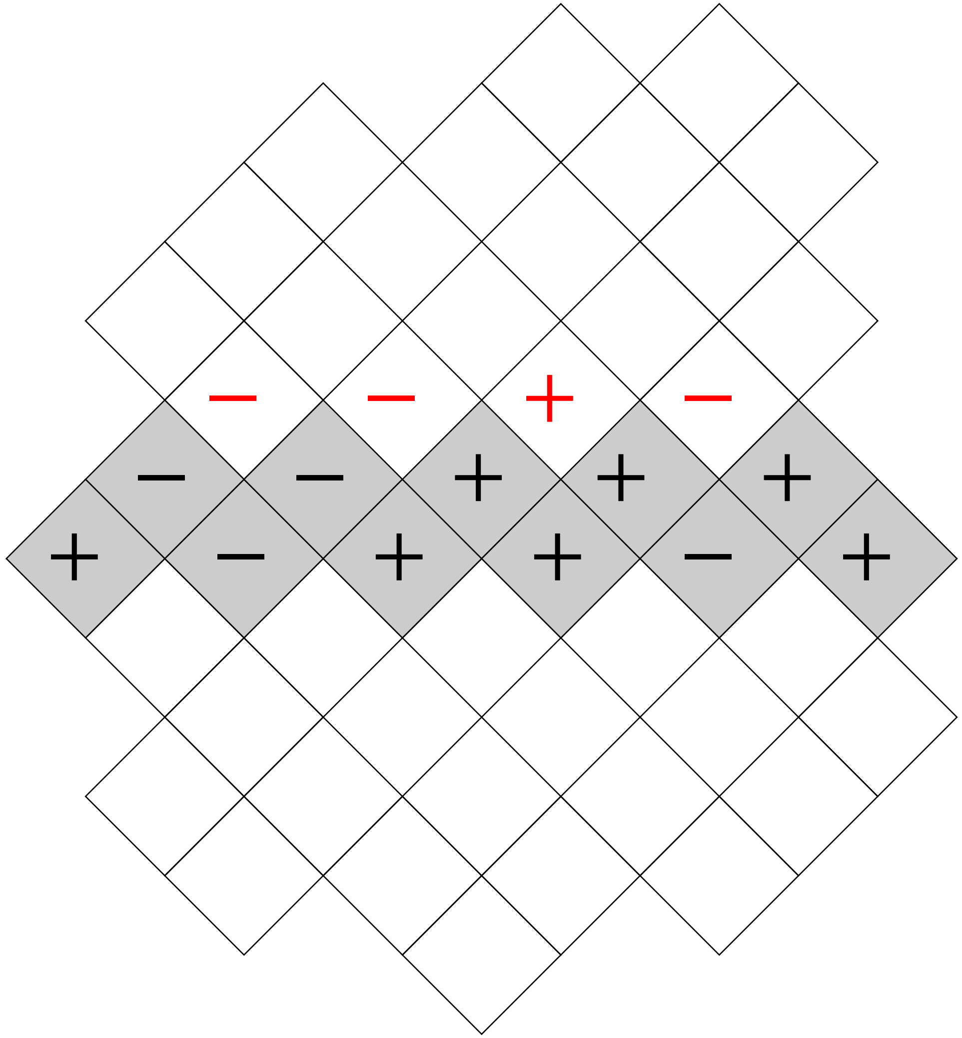

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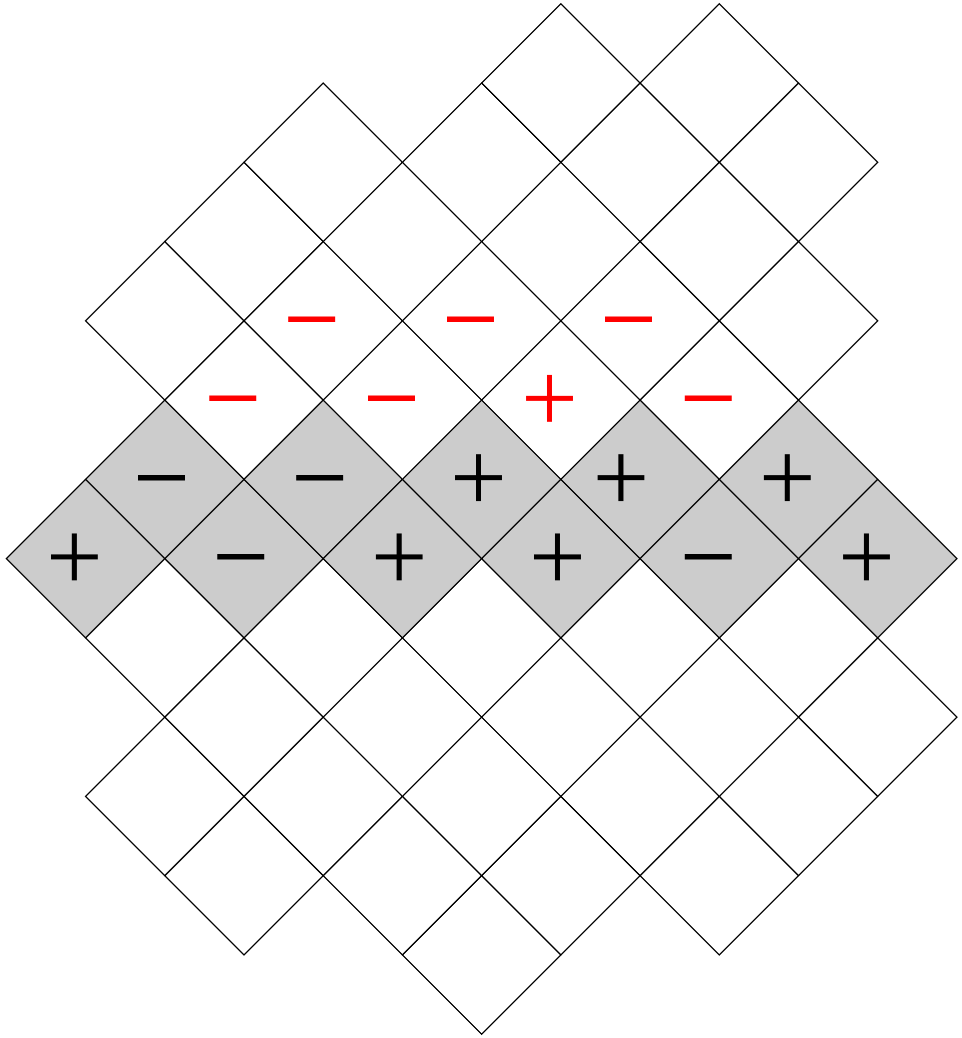

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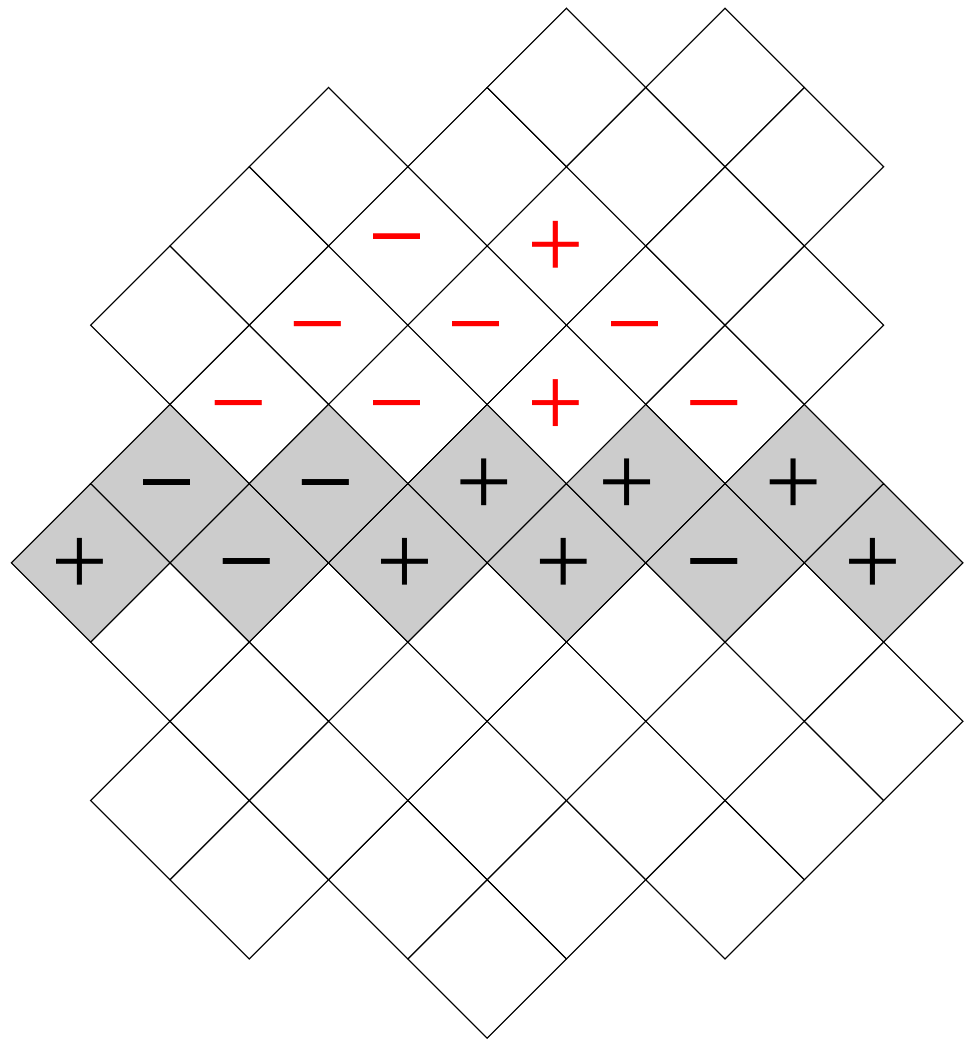

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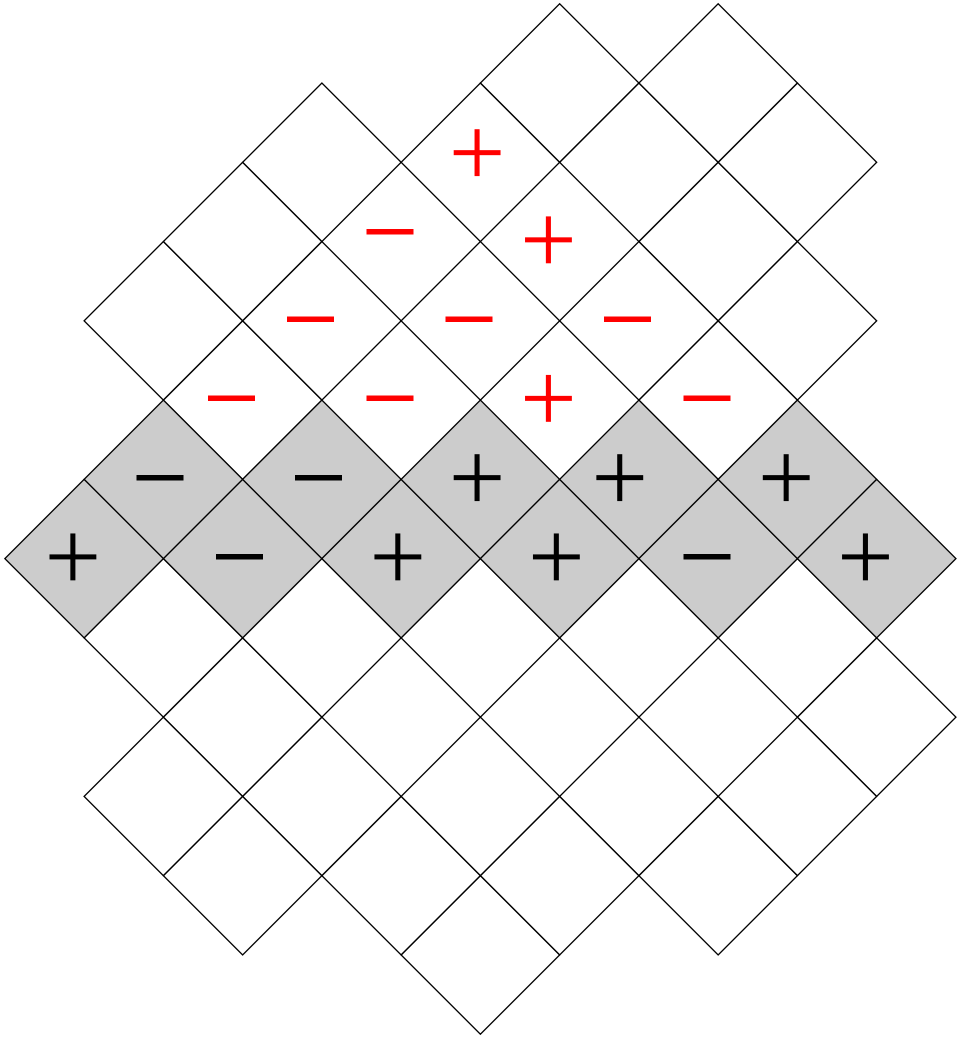
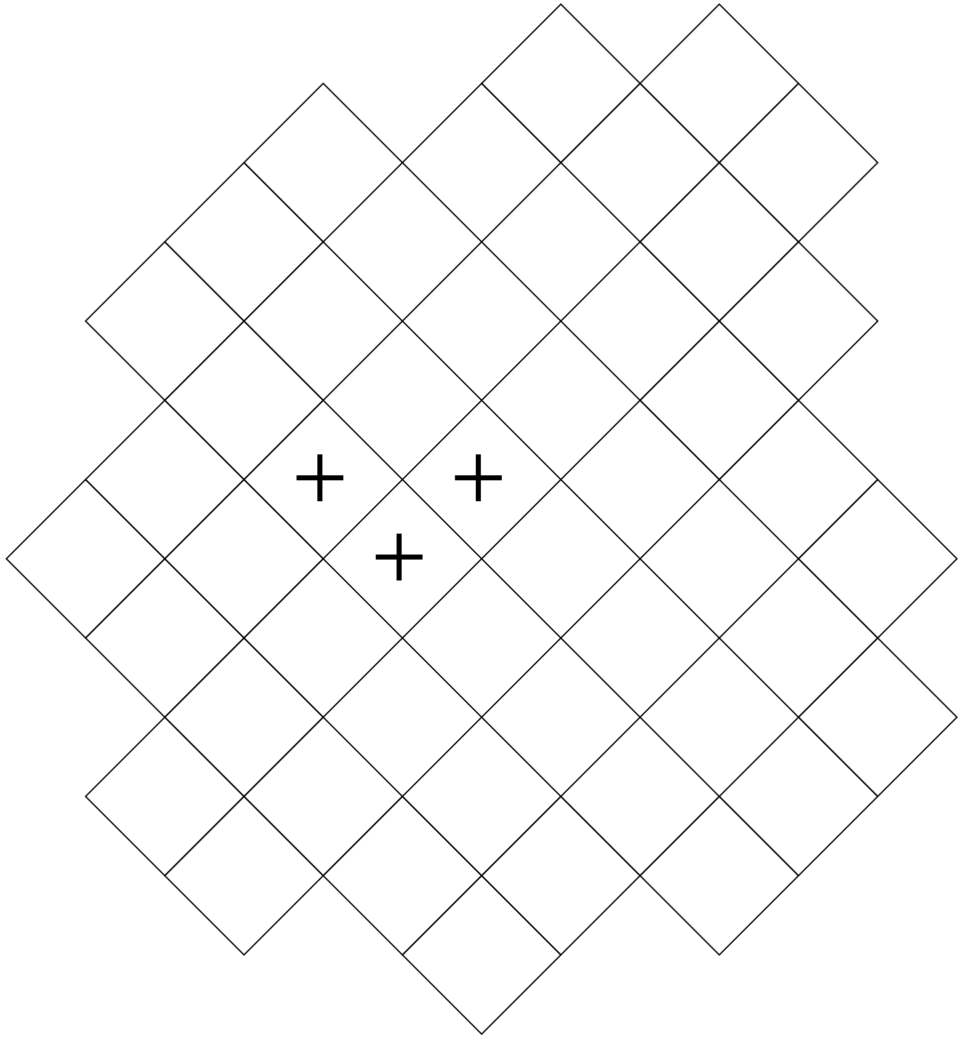


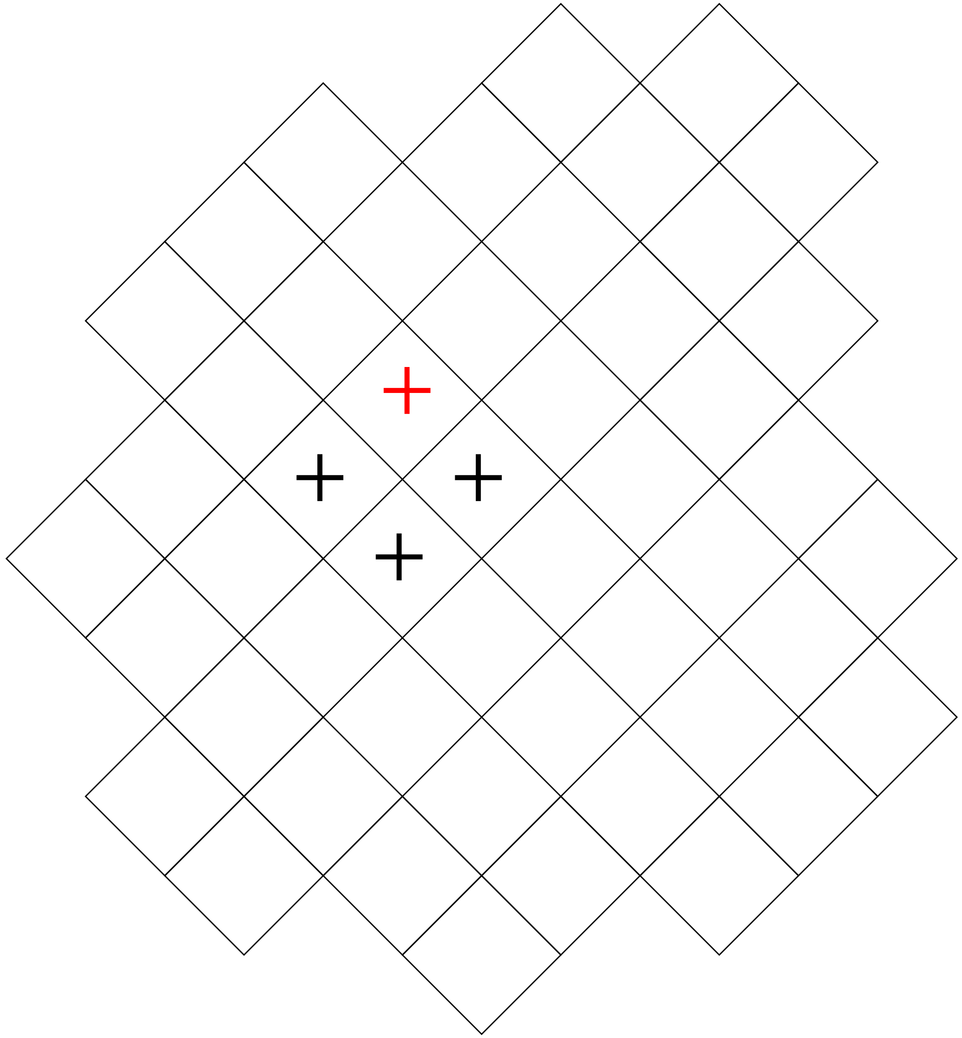
I. What is a local physical theory?

2. **Stochastic LCT**

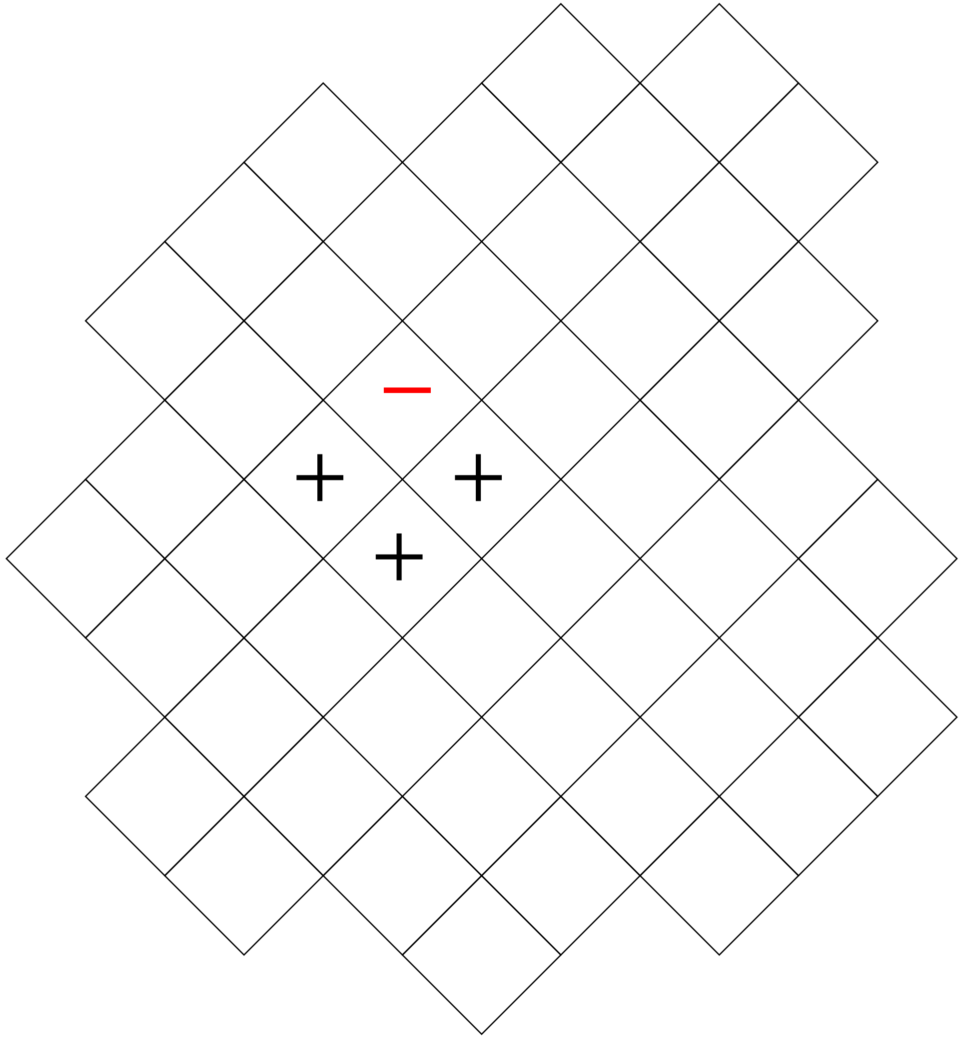


I. What is a local physical theory?

- **Stochastic dynamics:** with probability p



- **Stochastic dynamics:** with probability $1 - p$



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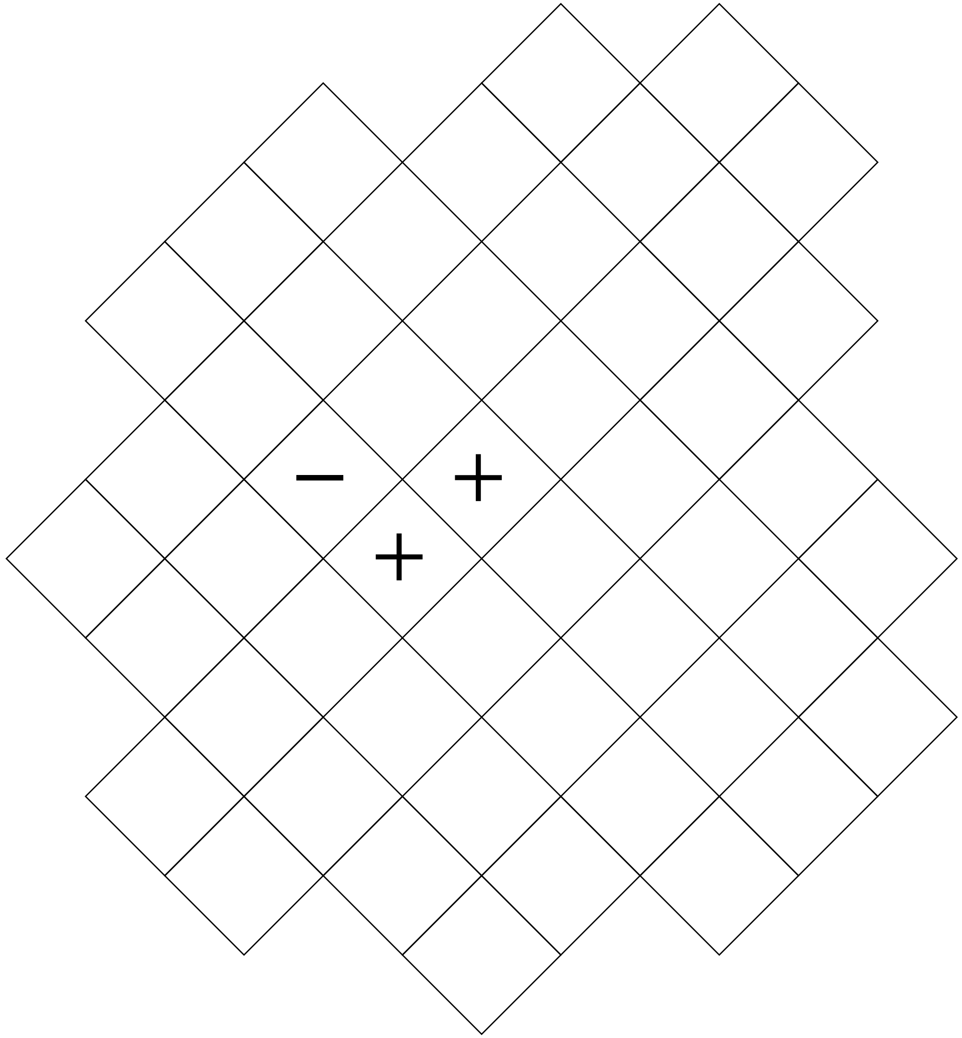

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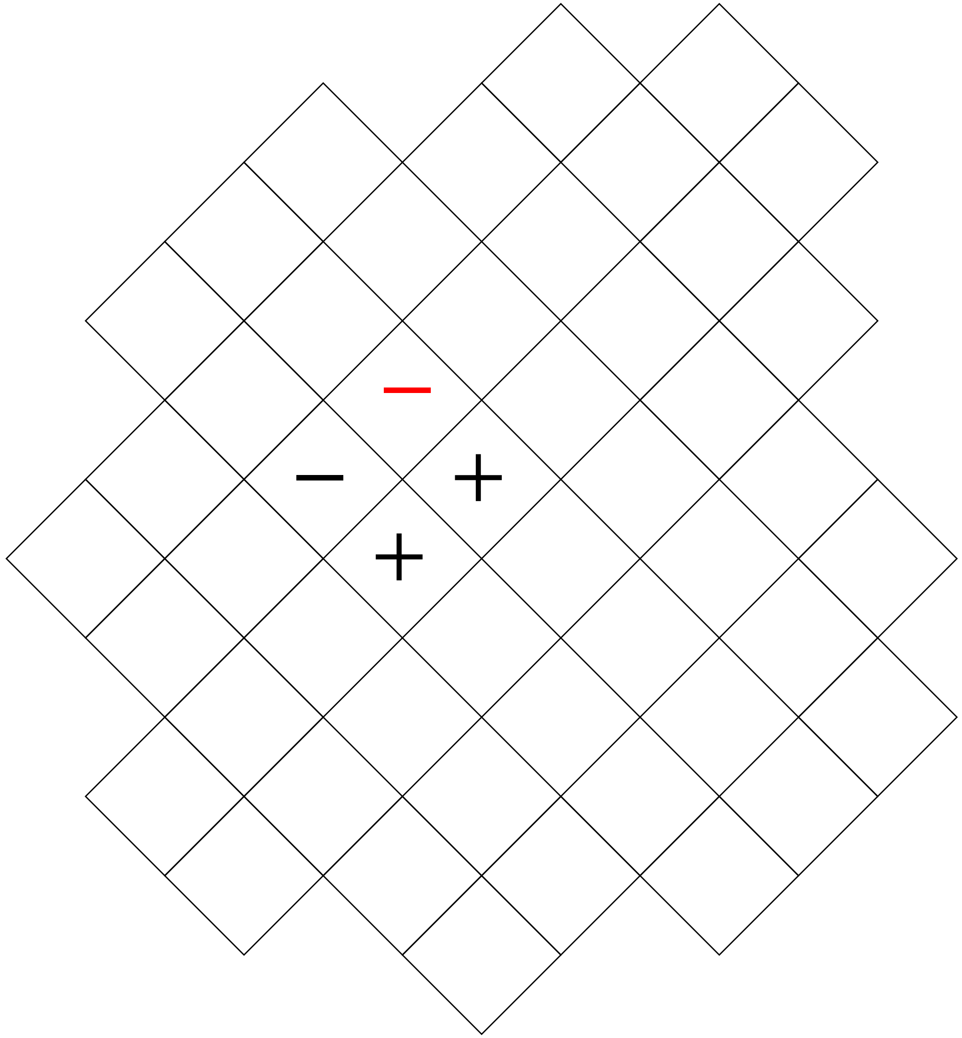

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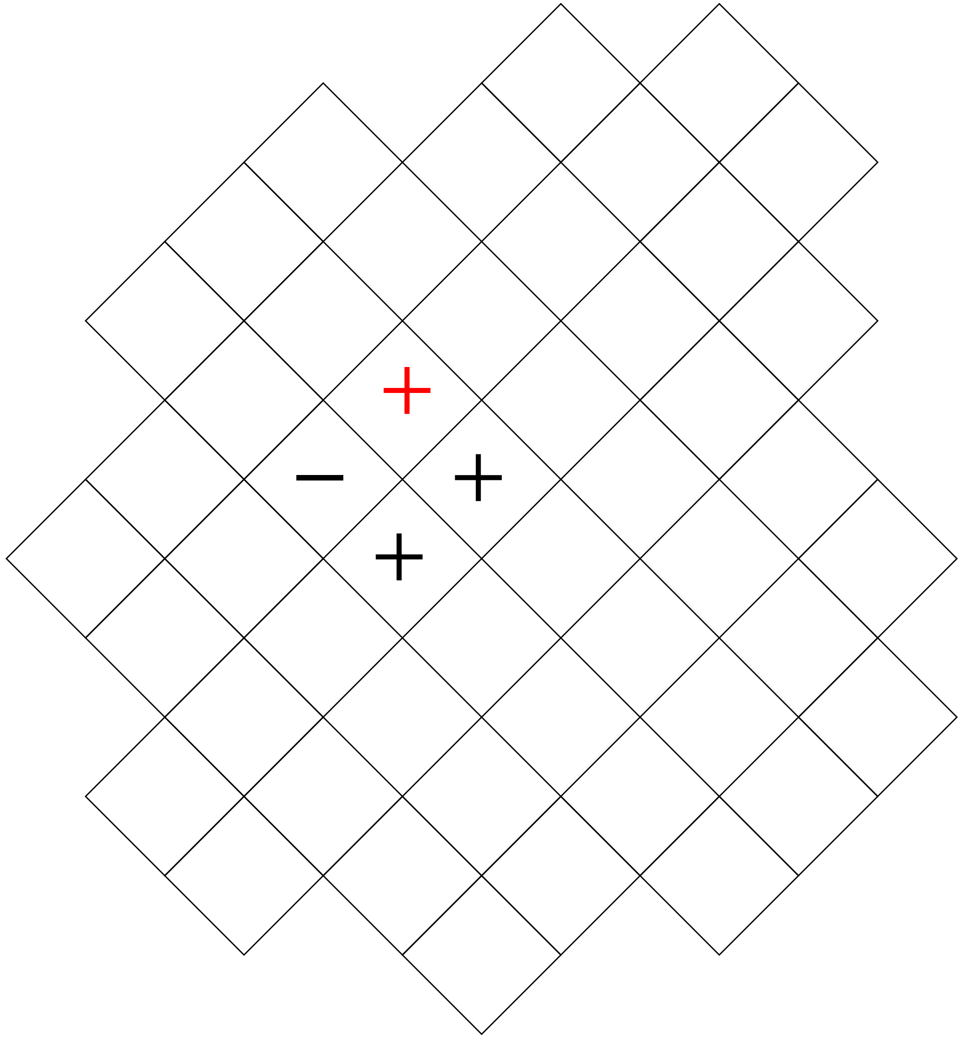
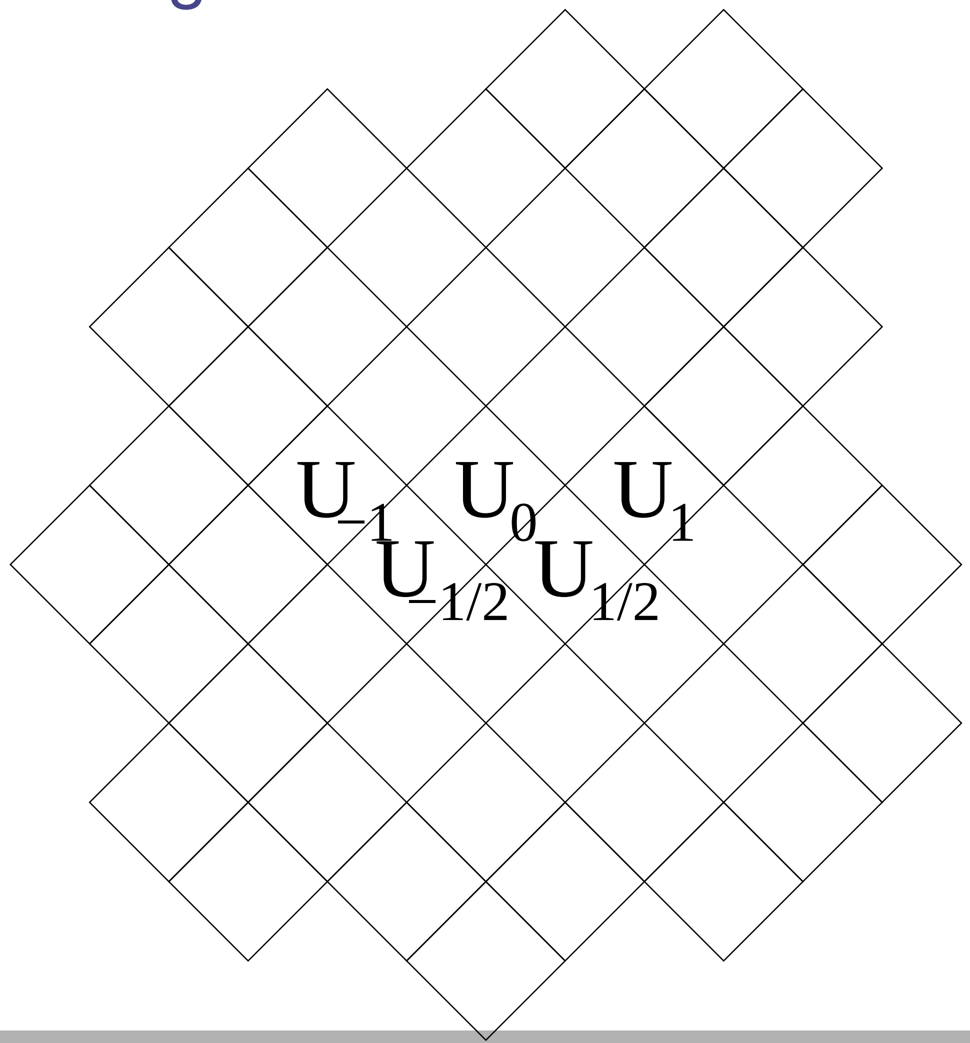
- **Stochastic dynamics:** with probability $1 - p$



I. What is a local physical theory?

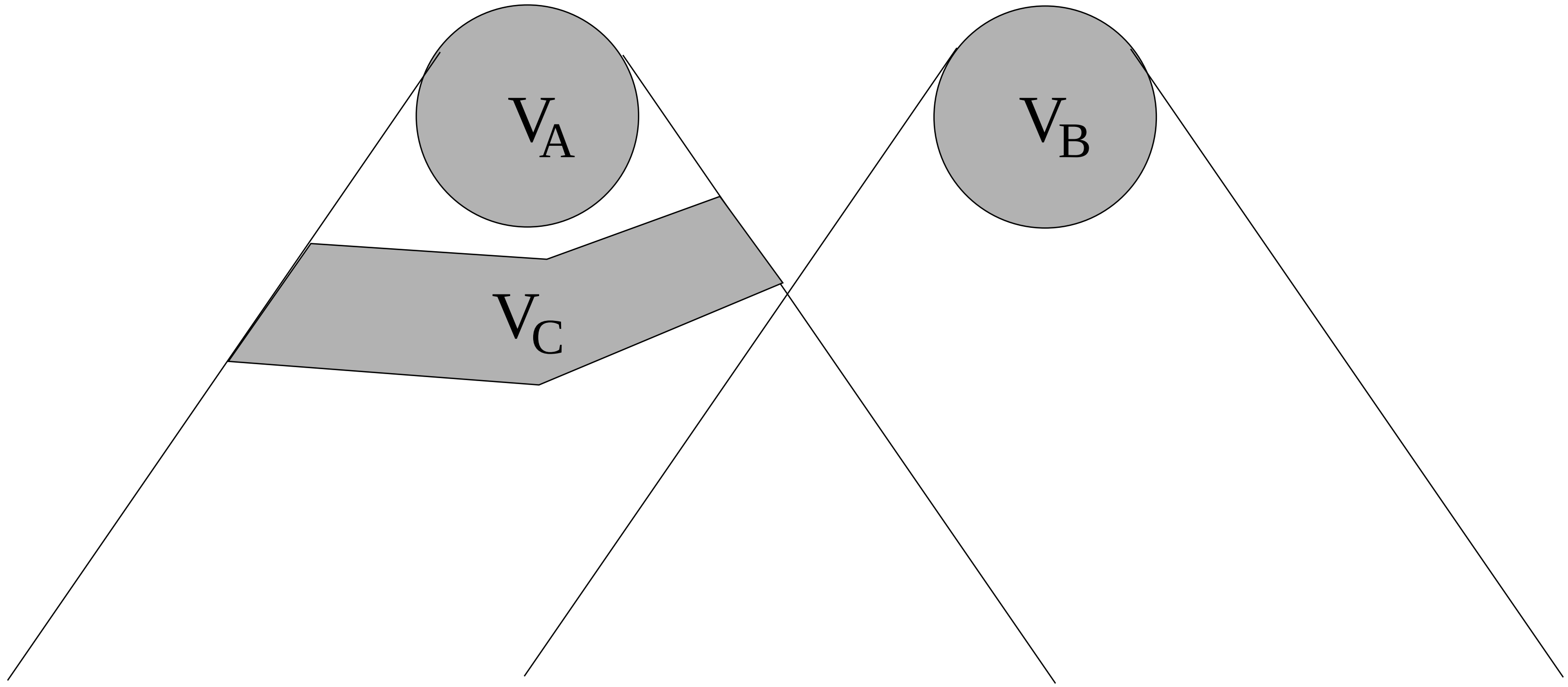
3. **Deterministic LQT** by imposing anticommutation relation between neighbours



II. Bell’s local causality in a LPT

II. Bell's local causality in a LPT

- "A theory will be said to be locally causal if the probabilities attached to values of local beables in a space-time region V_A are unaltered by specification of values of local beables in a space-like separated region V_B, when what happens in the backward light cone of V_A is already sufficiently specified, for example by a full specification of local beables in a space-time region V_C." (Bell, 1990/2004, p. 239-240)



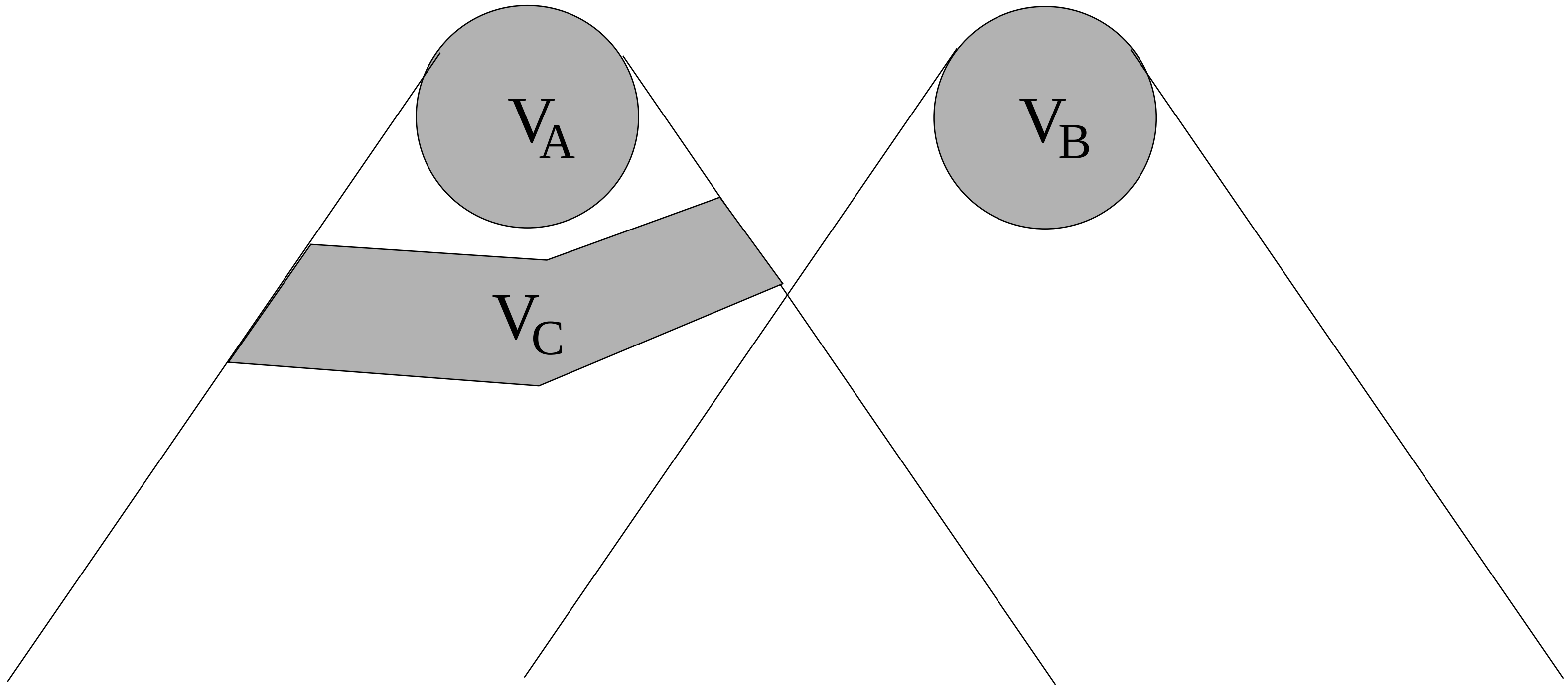
II. Bell's local causality in a LPT

Remarks:

1. "The ***beables*** of the theory are those entities in it which are, at least tentatively, to be taken seriously, as corresponding to something real."
2. "there *are* things which **do go faster than light**. British sovereignty is the classical example. When the Queen dies in London (long may it be delayed) the Prince of Wales, lecturing on modern architecture in Australia, becomes instantaneously King."
3. "***Local* beables** are those which are definitely associated with particular space-time regions. The electric and magnetic fields of classical electro-magnetism, $\mathbf{E}(t,x)$ and $\mathbf{B}(t,x)$ are again examples."

Remarks:

4. “It is important that region V_C **completely shields off** from V_A the overlap of the backward light cones of V_A and V_B.”



II. Bell's local causality in a LPT

Remarks:

5. "And it is important that events in V_C be **specified completely**. Otherwise the traces in region V_B of causes of events in V_A could well supplement whatever else was being used for calculating probabilities about V_A."

II. Bell's local causality in a LPT

Translation:

- "local beable" $\longrightarrow$ element of a local von Neumann algebra
- "complete specification" $\longrightarrow$ an atomic element of a local von Neumann algebra
- "completely shielder-off region" $\longrightarrow$

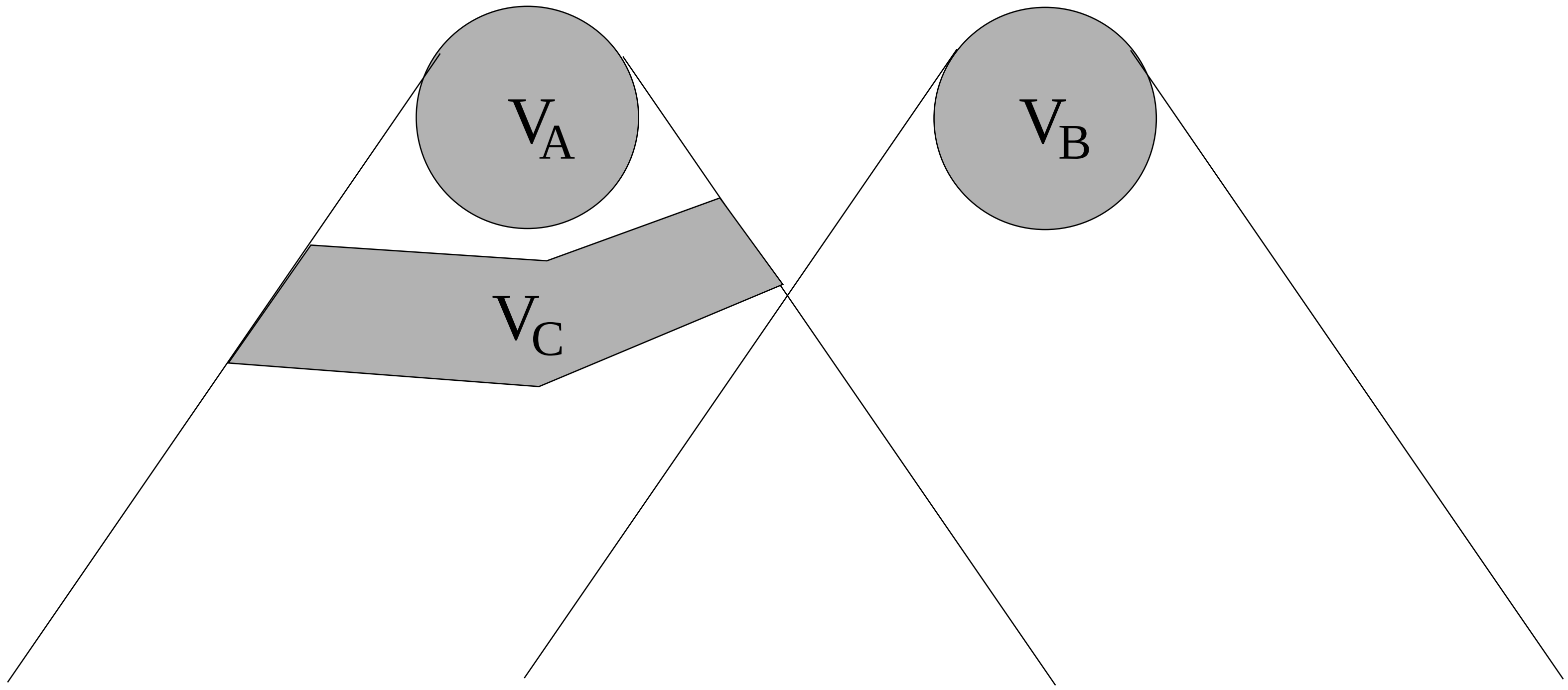
II. Bell's local causality in a LPT

- **"completely shielder-off region":**

  (i) $V_C \subset J_-(V_A)$

  (ii) $V_A \subset V_C''$

  (iii) $V_C \subset V_B'$



II. Bell's local causality in a LPT

- **Definition.** A LPT is called **(Bell) locally causal**, if
  - for any *pair of projections* $A \in \mathcal{N}(V_A)$ and $B \in \mathcal{N}(V_B)$ supported in spacelike separated regions $V_A, V_B \in \mathcal{K}$, and
  - for every locally normal and faithful *state* ϕ establishing a correlation between A and B, $\phi(AB) \neq \phi(A)\phi(B)$, and
  - for any *spacetime region* V_C satisfying Requirements (i)-(iii), and
  - for any *atomic event* C_k in $\mathcal{N}(V_C)$:

$$\frac{\phi(C_kABC_k)}{\phi(C_k)} = \frac{\phi(C_kAC_k)}{\phi(C_k)} \frac{\phi(C_kBC_k)}{\phi(C_k)}$$

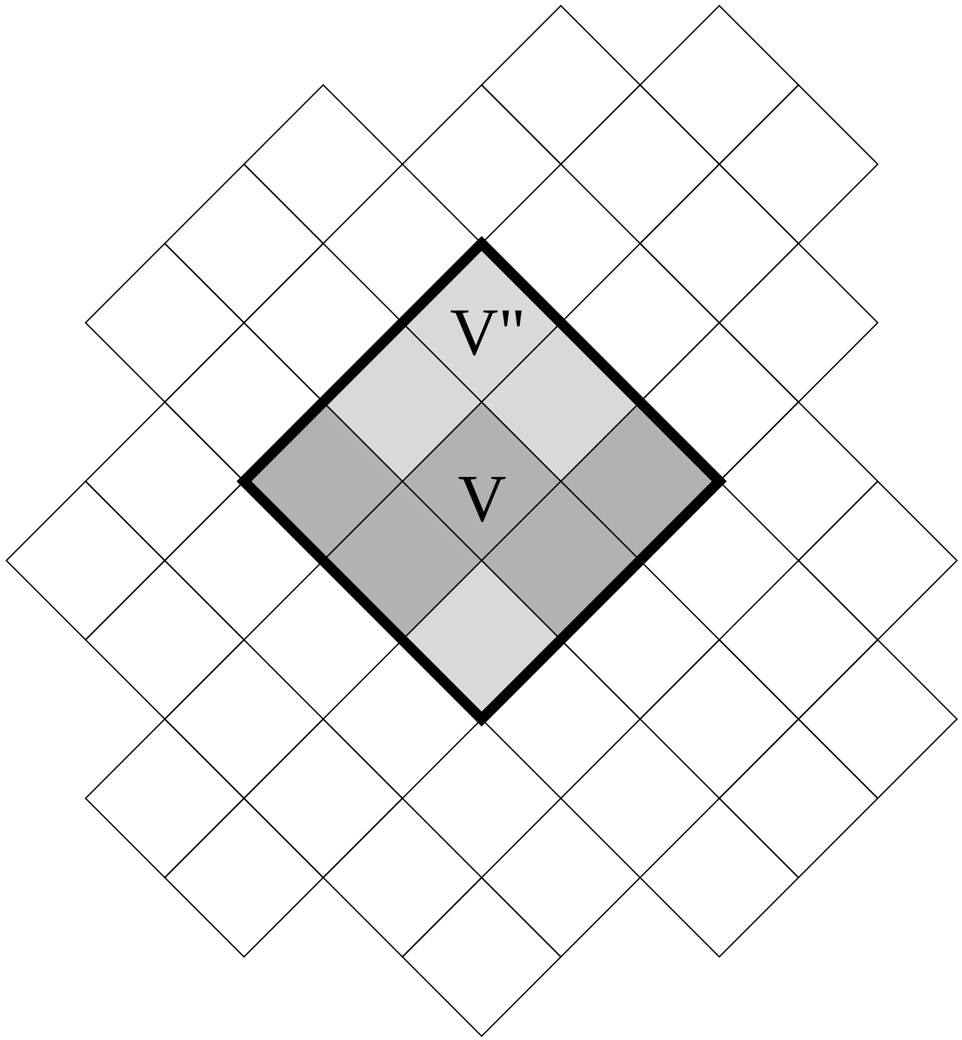
II. Bell's local causality in a LPT

Question:

- When is a LPT locally causal?

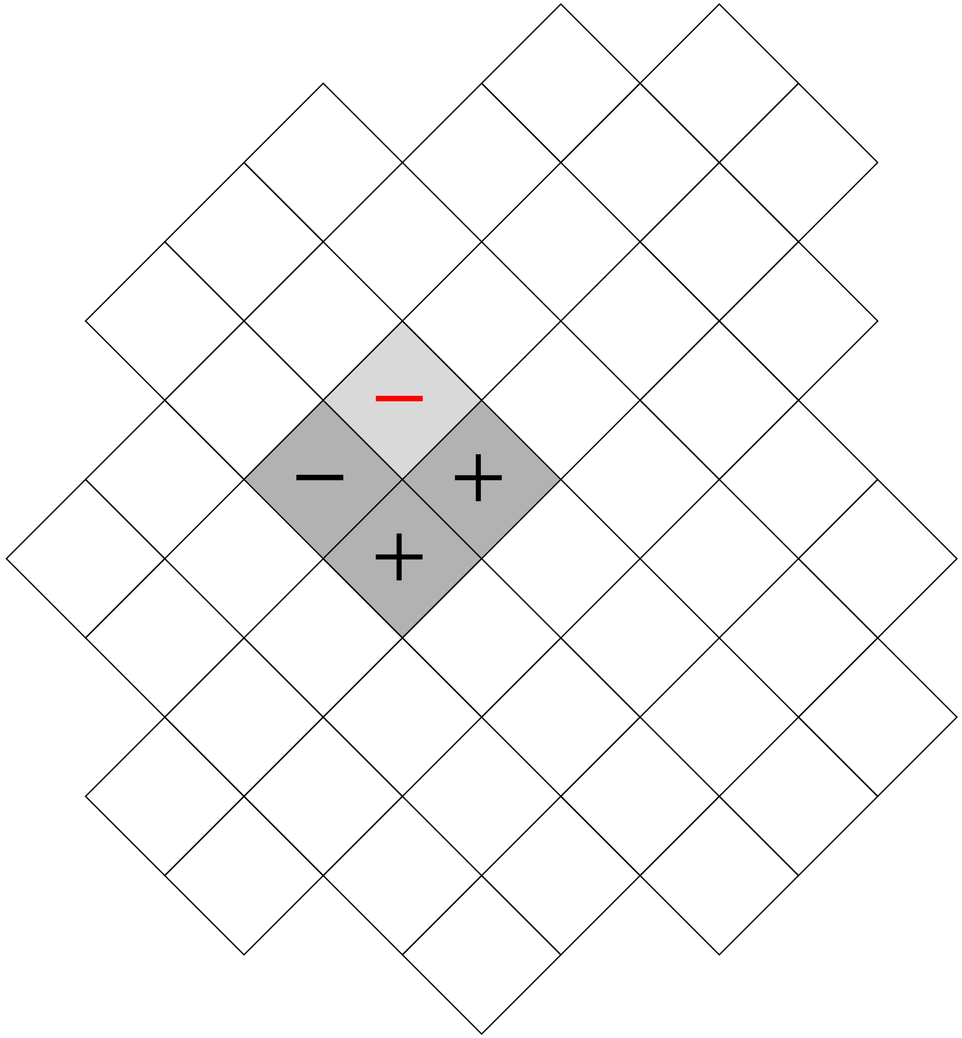
II. Bell's local causality in a LPT

- **Local primitive causality:** $\mathcal{N}(V) = \mathcal{N}(V'')$ for any $V \in \mathcal{K}$



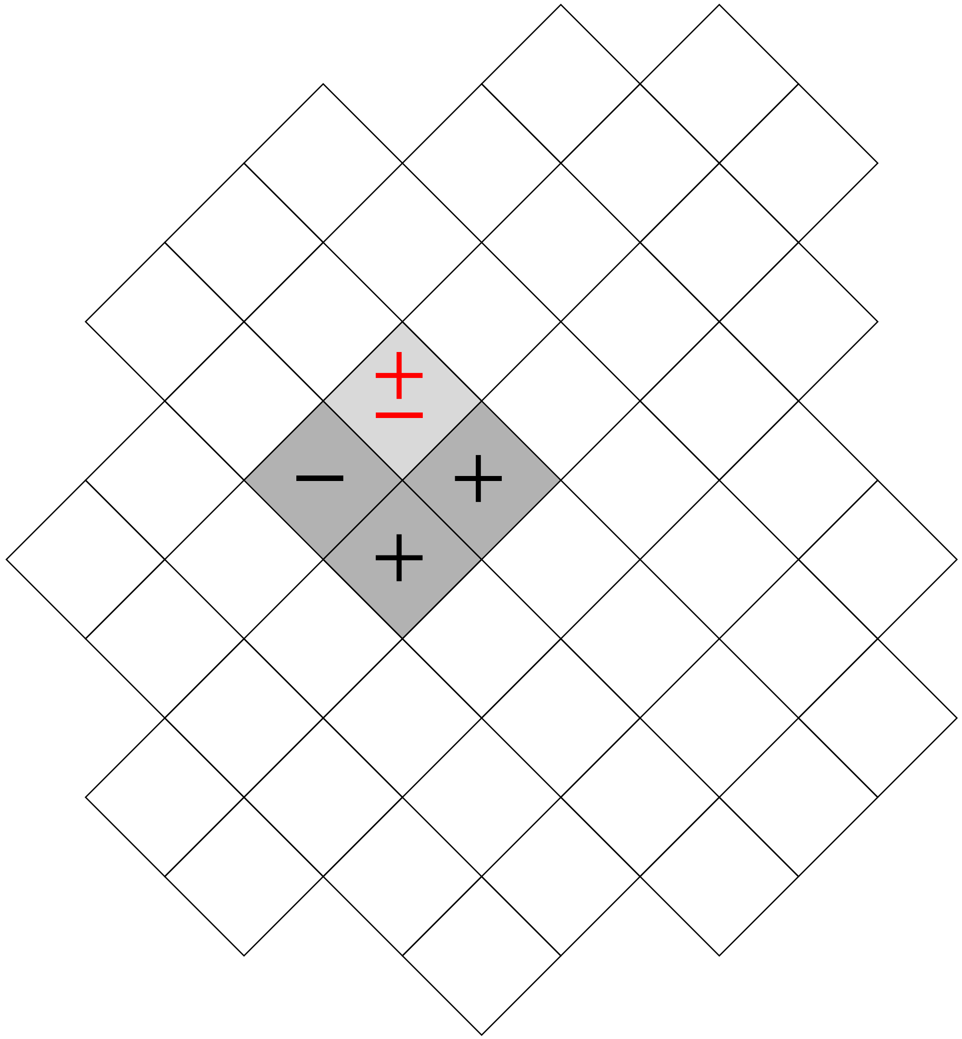
II. Bell's local causality in a LPT

- **Local primitive causality:** holds in deterministic LCTs



II. Bell's local causality in a LPT

- **Local primitive causality:** does not hold in stochastic LCTs



II. Bell's local causality in a LPT

Proposition:

- Any **atomic** LPT satisfying **local primitive causality** is locally causal.

II. Bell's local causality in a LPT

But. . .

- how can a **LQT** be locally causal if local causality implies the Bell inequalities which are violated for certain quantum correlations?

III. Common Cause Principle

III. Common Cause Principle

- **Reichenbach's Common Cause Principle (CCP)**: If there is a correlation between two events and there is no direct causal (or logical) connection between them, then there always exists a common cause of the correlation.

III. Common Cause Principle

- **Correlation:** $\phi(AB) \neq \phi(A)\phi(B)$
- **Common cause:** partition $\{C_k\}_{k\in K}$ of the unit

$$\frac{\phi(C_kABC_k)}{\phi(C_k)} = \frac{\phi(C_kAC_k)}{\phi(C_k)}\frac{\phi(C_kBC_k)}{\phi(C_k)}$$

III. Common Cause Principle

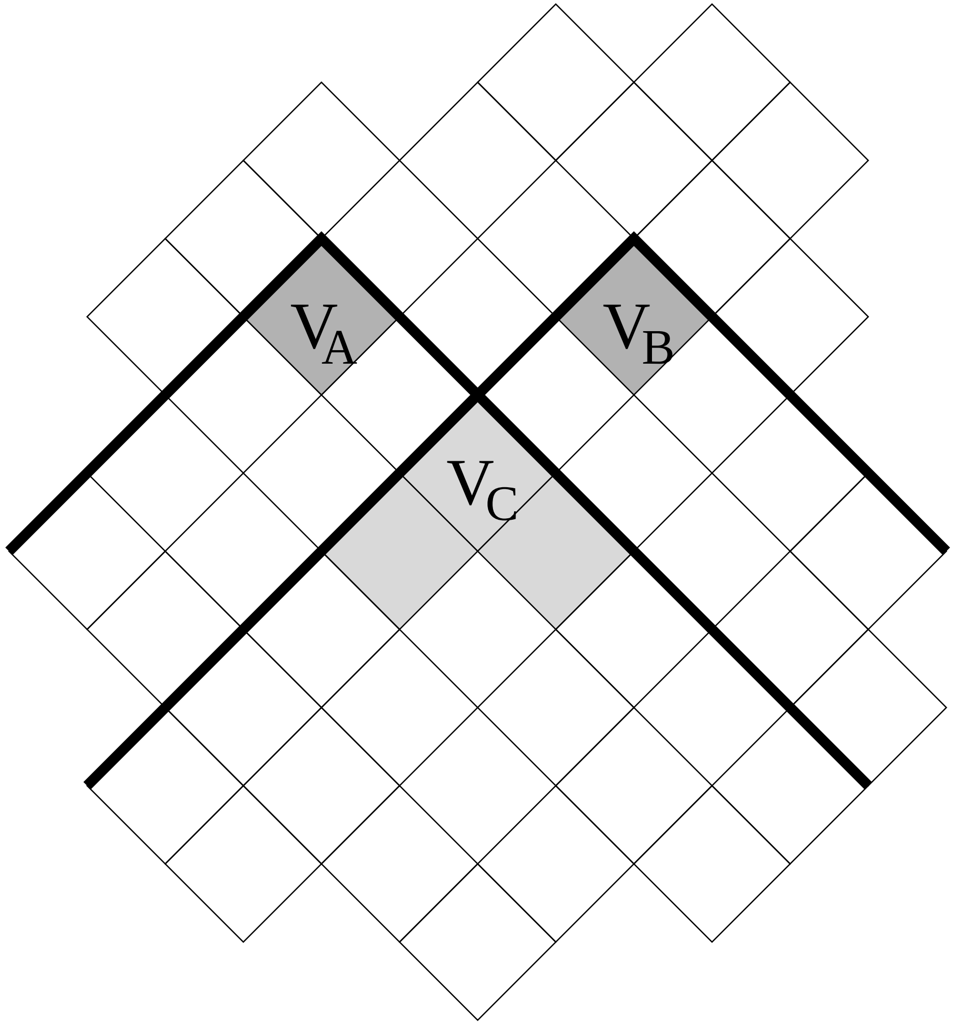
- **Correlation:** $\phi(AB) \neq \phi(A)\phi(B)$
- **Common cause:** partition $\{C_k\}_{k\in K}$ of the unit

$$\frac{\phi(C_k A B C_k)}{\phi(C_k)} = \frac{\phi(C_k A C_k)}{\phi(C_k)} \frac{\phi(C_k B C_k)}{\phi(C_k)}$$

  - **Commuting / Noncommuting common cause:** $\{C_k\}_{k\in K}$ is commuting / not commuting with A and B
  - **Nontrivial common cause**: $C_k \nleq A, A^{\perp}, B$ or $B^{\perp}$ for some $k \in K$

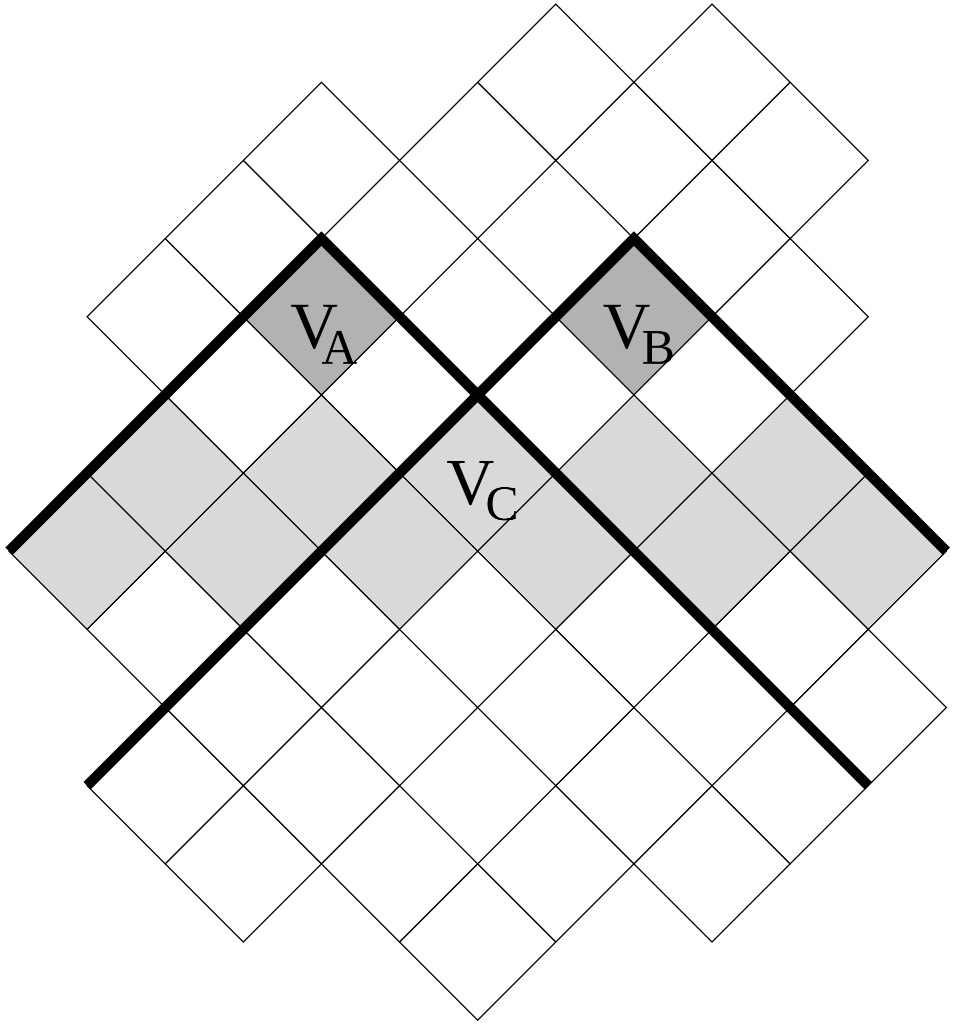
III. Common Cause Principle

- **Common Cause Principle:**

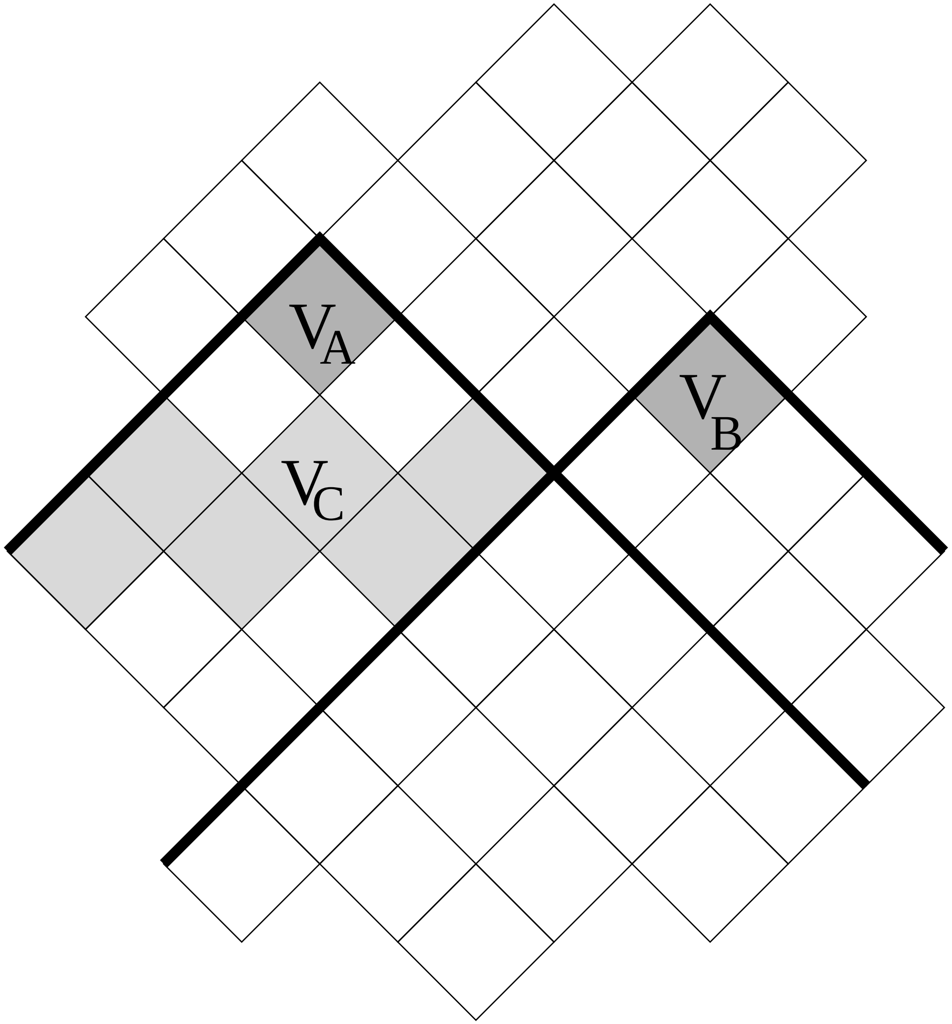


III. Common Cause Principle

- **Weak Common Cause Principle:**



- **Local causality:**



III. Common Cause Principle

Similarities:

1. Both local causality and the CCPs are **properties of a LPT** represented by a net $\{\mathcal{N}(V), V \in \mathcal{K}\}$.
2. The core mathematical requirement of both principles is the **screening-off condition**:

$$\frac{\phi(C_k ABC_k)}{\phi(C_k)} = \frac{\phi(C_k AC_k)}{\phi(C_k)} \frac{\phi(C_k BC_k)}{\phi(C_k)}$$

3. The **Bell inequalities** can be derived from both principles. (But see below.)

III. Common Cause Principle

Differences:

1. For local causality the screening-off condition is required for **every** atomic event. For the CCPs it is required only for events of **one** partition.
2. For local causality the screening-off condition is required only for **atomic** events. For the CCPs one is looking for **nontrivial** common causes.
3. For local causality screener-offs are localized **'asymmetrically'** in the past of V_A (or V_B). For the CCP they are localized **'symmetrically'** in the joint / common past of V_A and V_B.

IV. Bell inequalities

IV. Bell inequalities

- A nice **parallelism**:

$$\text{Local causality} \quad \Longrightarrow \quad \text{Bell inequalities}$$
$$\text{Common Cause Principle} \quad \Longrightarrow \quad \text{Bell inequalities}$$

IV. Bell inequalities

- **Set of correlations:** $\phi(A_m B_n) \neq \phi(A_m)\phi(B_n)$
- **Joint common cause:** partition $\{C_k\}_{k \in K}$ of the unit

$$\frac{\phi(C_k A_m B_n C_k)}{\phi(C_k)} = \frac{\phi(C_k A_m C_k)}{\phi(C_k)} \frac{\phi(C_k B_n C_k)}{\phi(C_k)}$$

IV. Bell inequalities

- **Reduced state:** $\phi_{\{C_k\}}(X) := \sum_k \phi(C_k X C_k)$

IV. Bell inequalities

Proposition:

- Joint common cause $\Longrightarrow$ Bell inequalities for the **reduced state** $\phi_{\{C_k\}}$

$$-1 \leqslant \phi_{\{C_k\}}(A_1B_1 + A_1B_2 + A_2B_1 - A_2B_2 - A_1 - B_1) \leqslant 0$$

- Joint common cause + **commutativity** $\Longrightarrow$ Bell inequalities for the **original state** ϕ

$$-1 \leqslant \phi(A_1B_1 + A_1B_2 + A_2B_1 - A_2B_2 - A_1 - B_1) \leqslant 0$$

IV. Bell inequalities

Proposition:

- Local causality $\Longrightarrow$ Bell inequalities for the **reduced state** $\phi_{\{C_k\}}$

$$-1 \leqslant \phi_{\{C_k\}}(A_1B_1 + A_1B_2 + A_2B_1 - A_2B_2 - A_1 - B_1) \leqslant 0$$

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V. Causal Markov Condition

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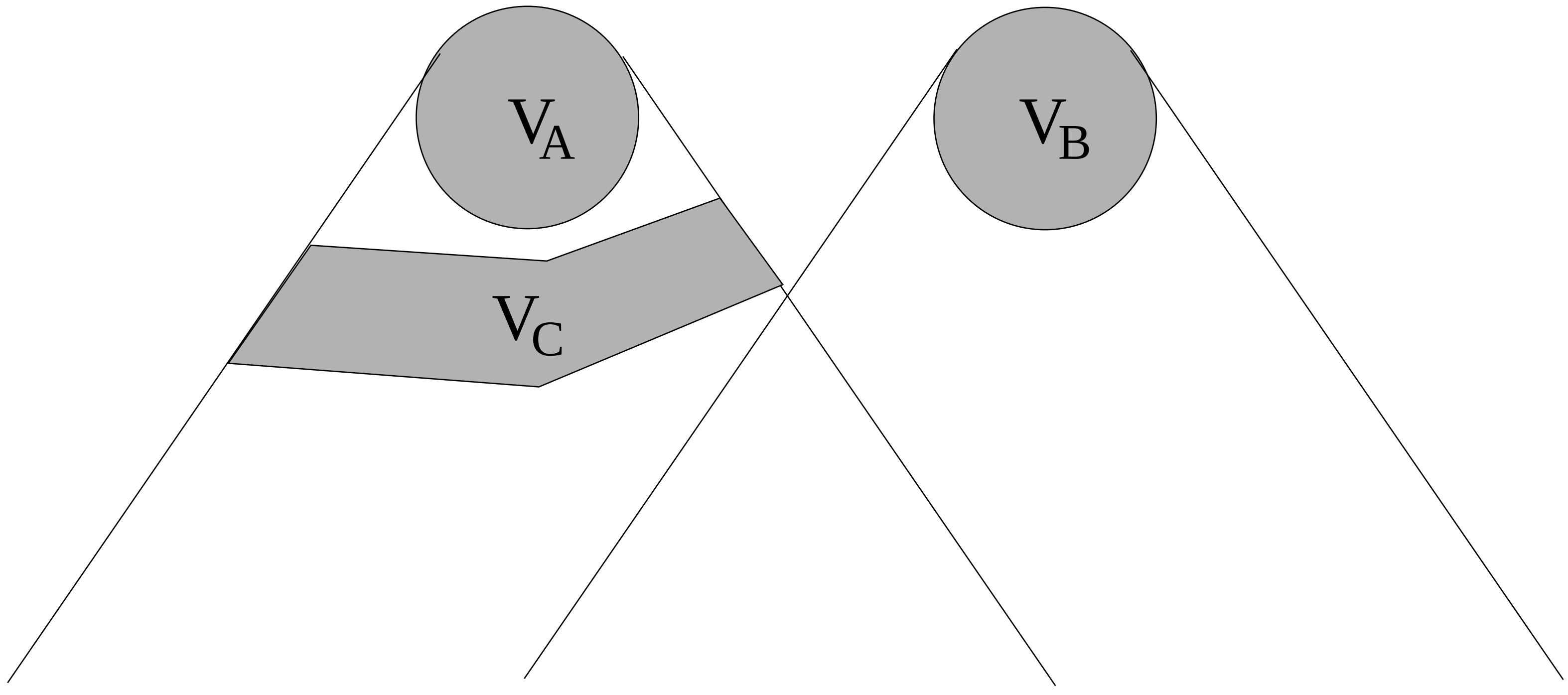
- **Localization:**
  - (i) $V_C \subset J_-(V_A)$
  - (ii) $V_A \subset V''_C$
  - (iii) $V_C \subset V'_B$



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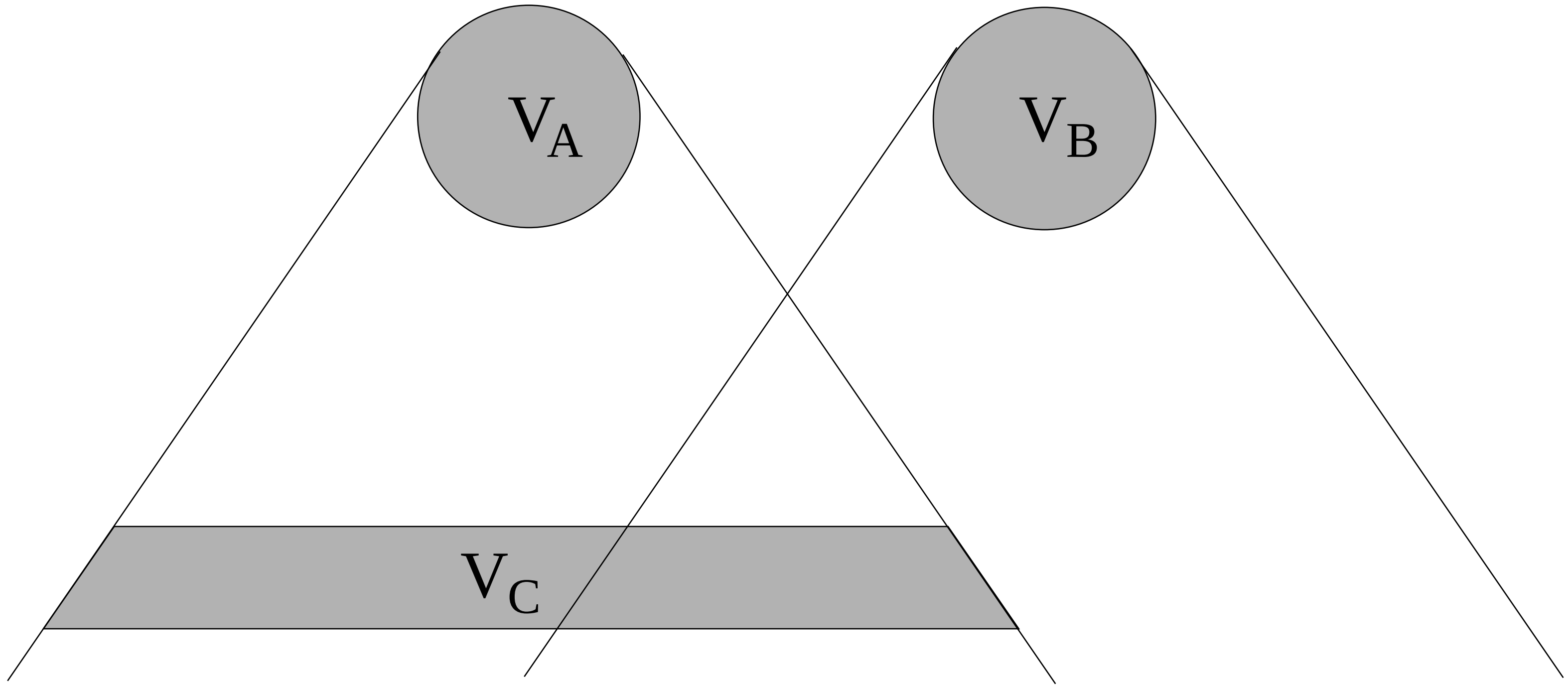

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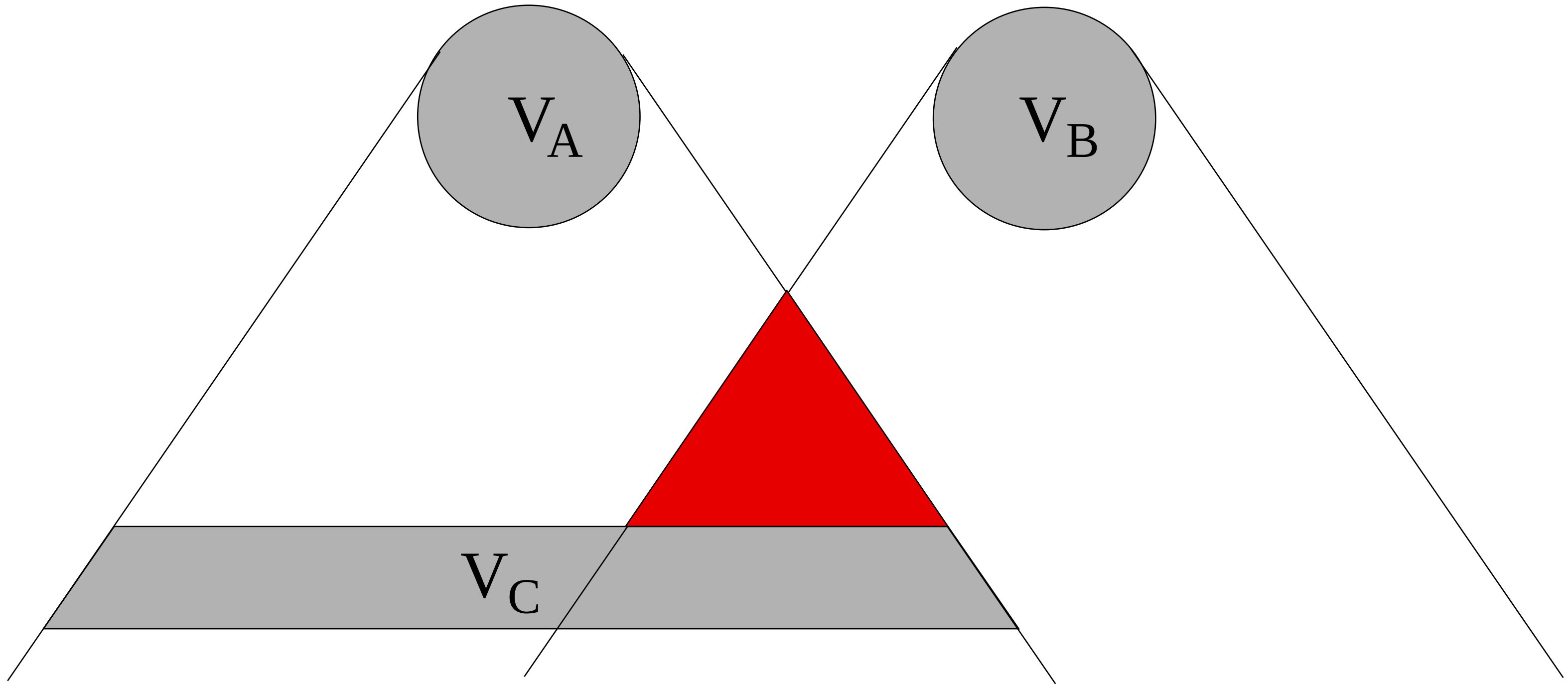
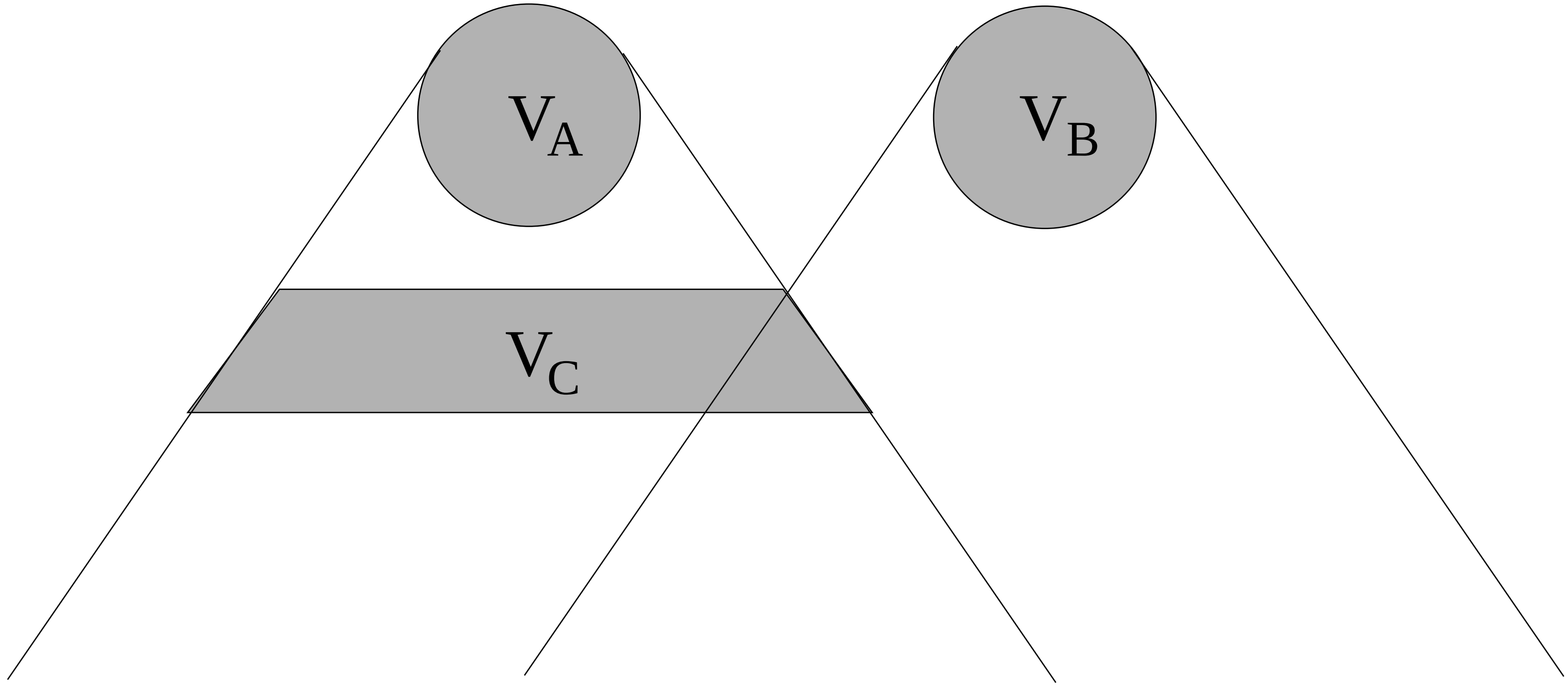


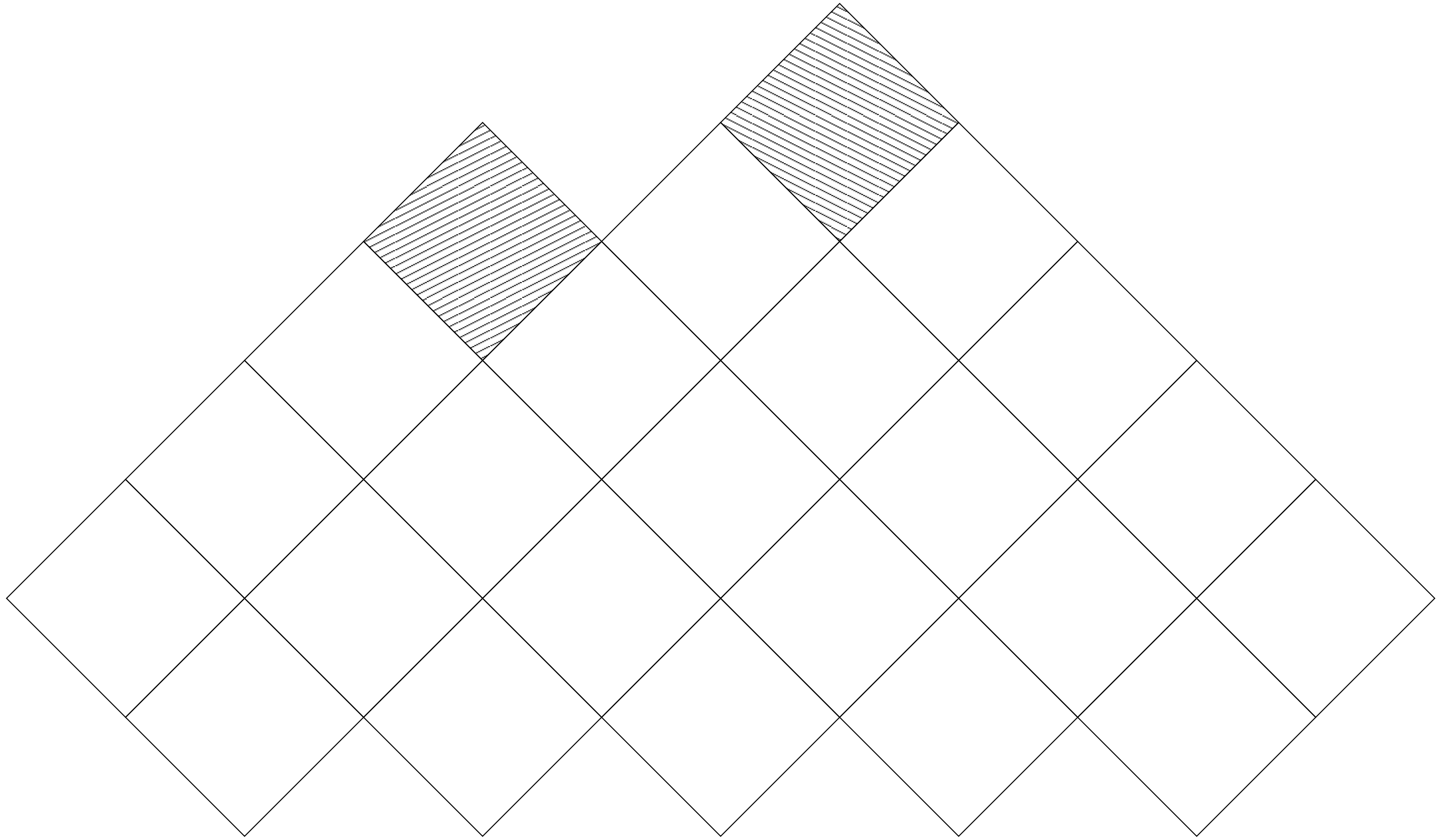
V. Causal Markov Condition

- **Localization:**
  (i) $V_C \subset J_-(V_A)$
  (ii) $V_A \subset V_C''$
  (iii) $J_-(V_C) \supset (J_-(V_A) \cap J_-(V_B))$



V. Causal Markov Condition

- **Shielder-off region:**



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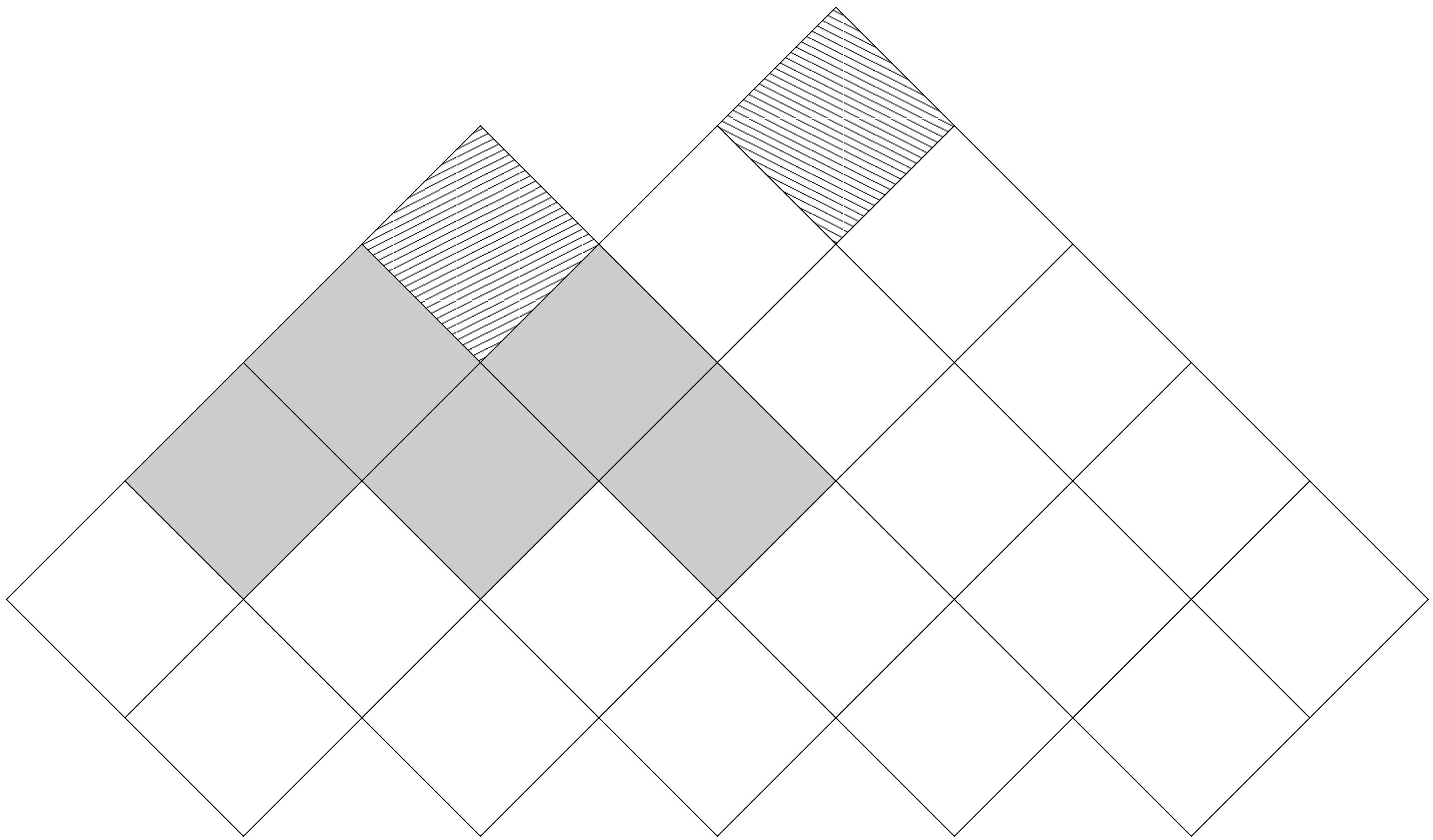

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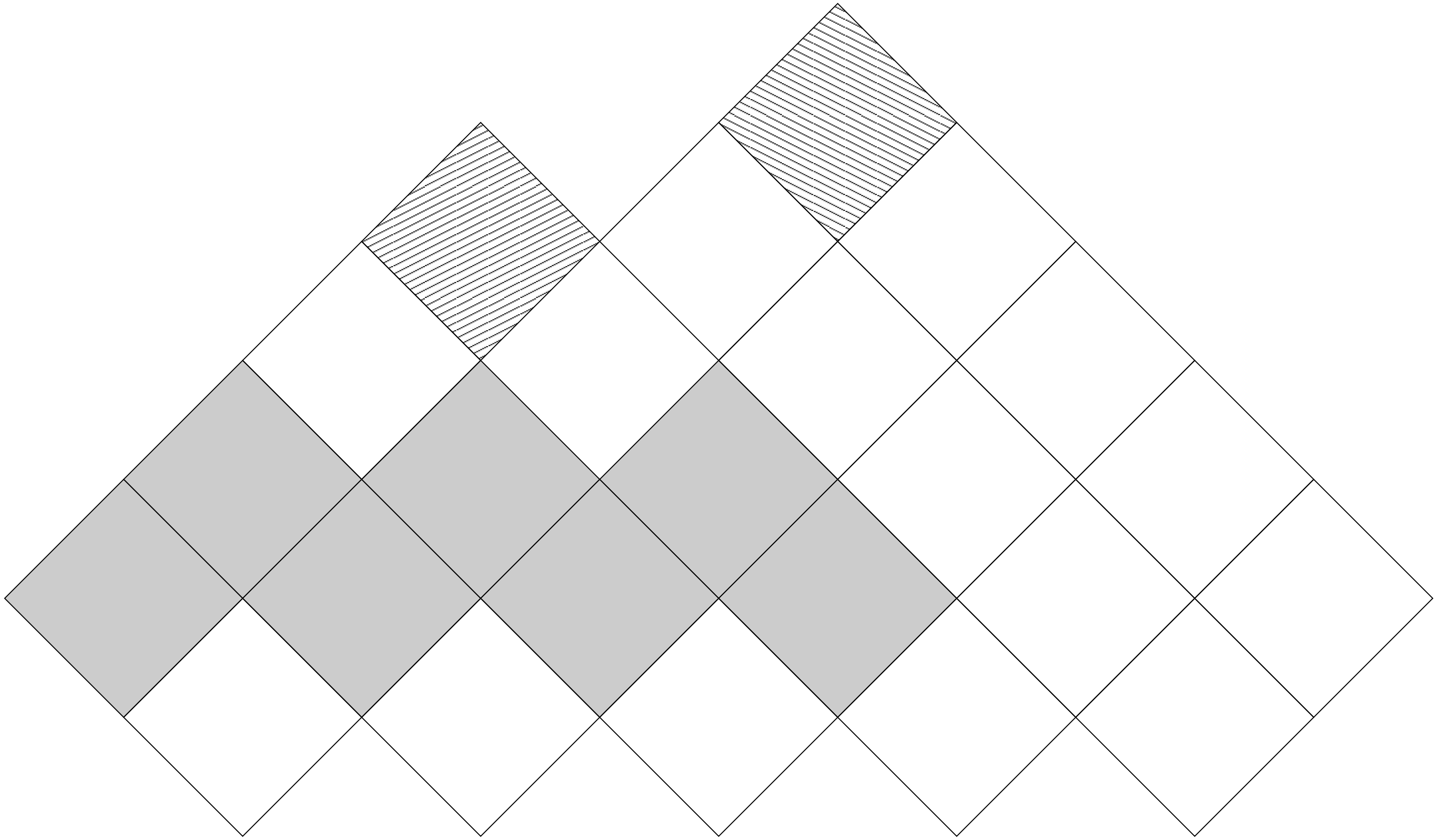

V. Causal Markov Condition

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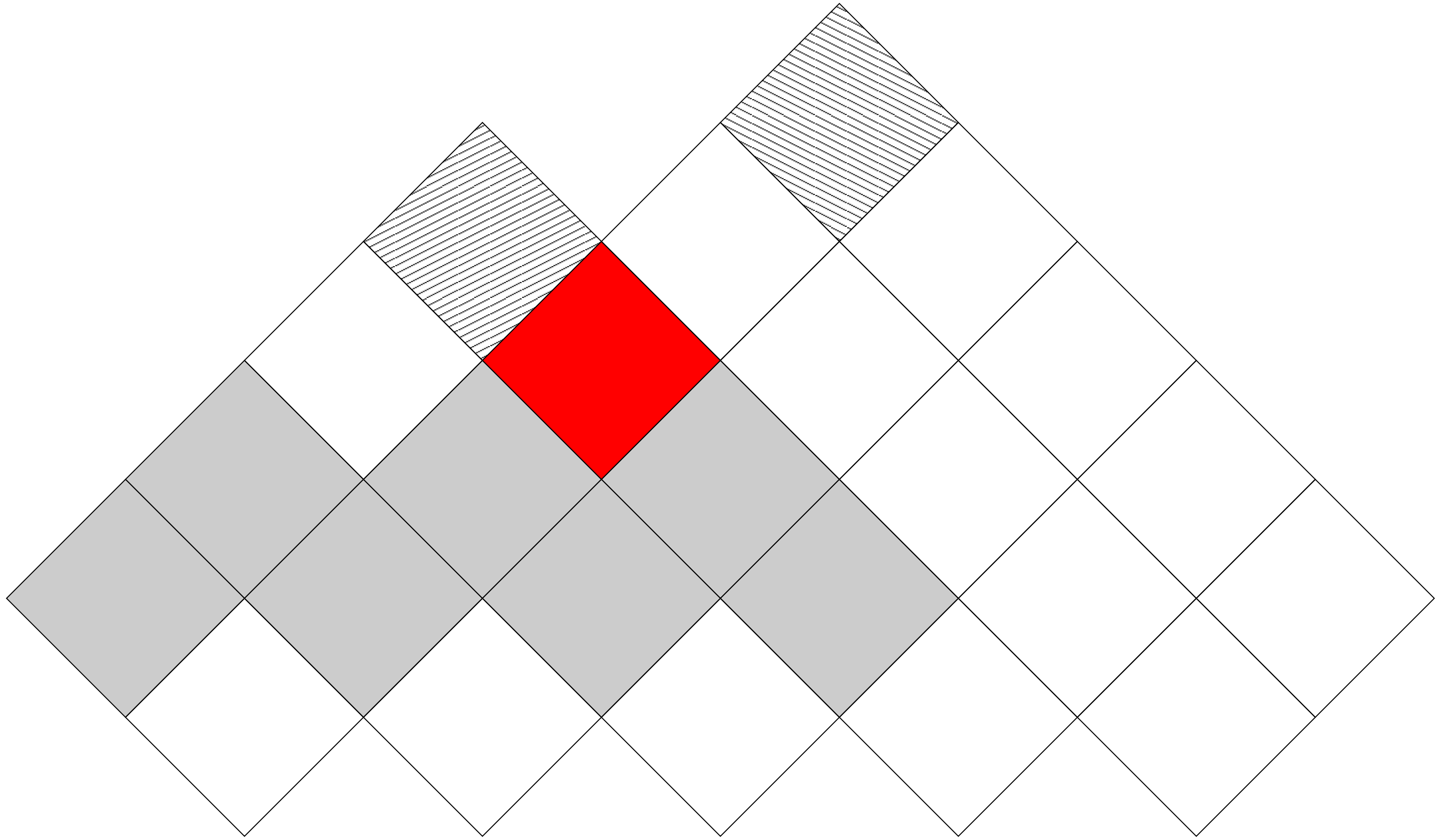
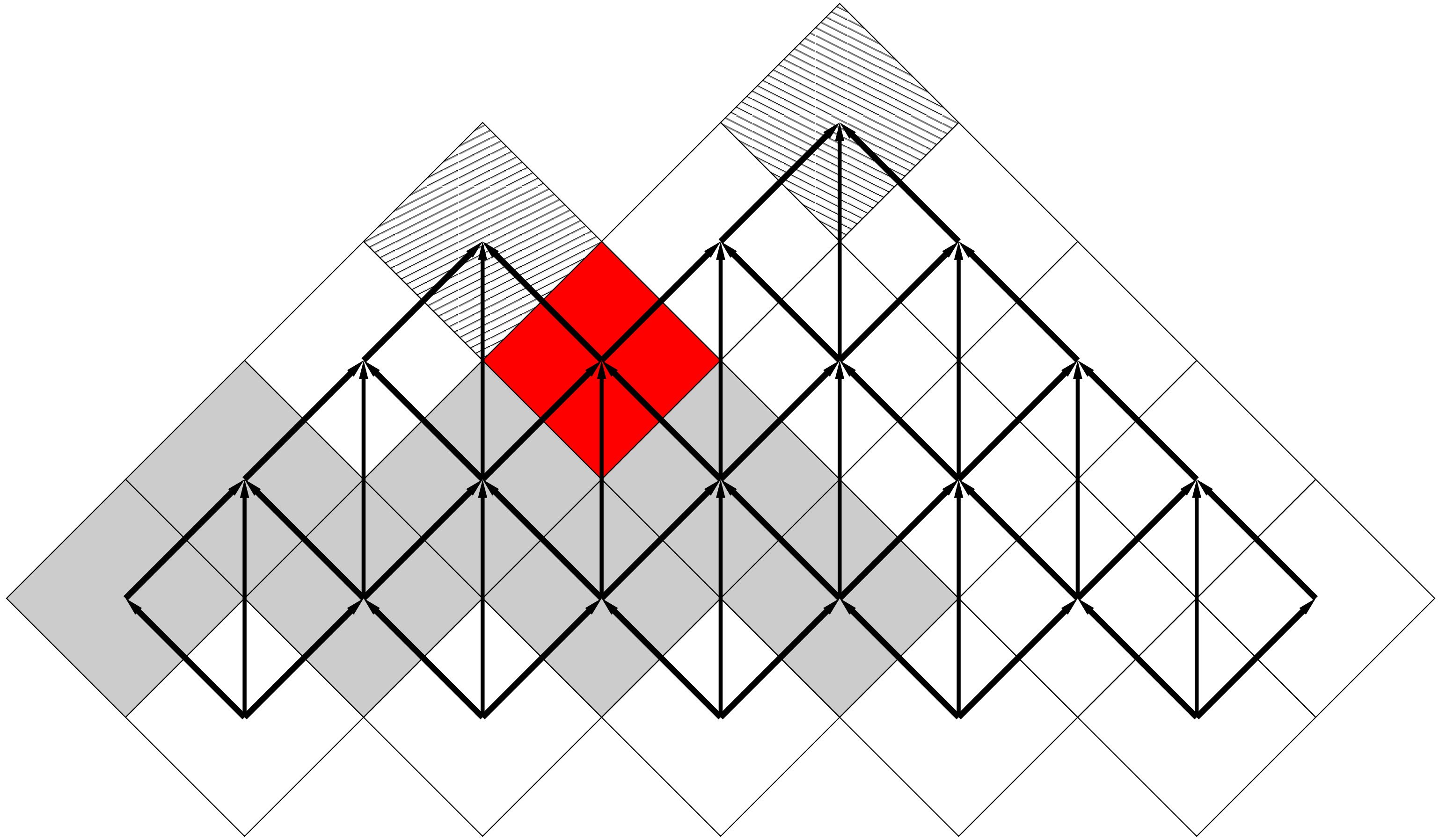


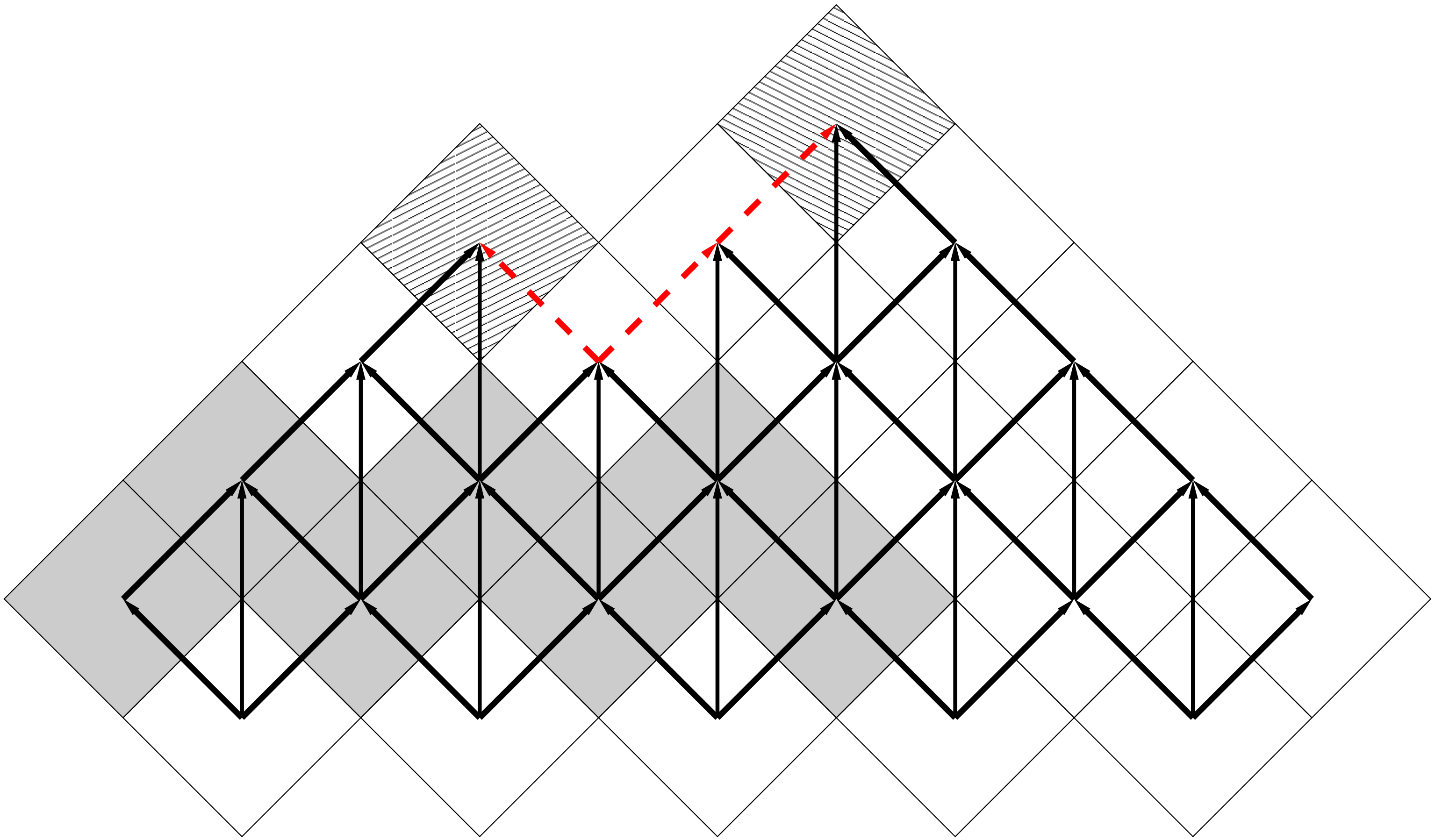
V. Causal Markov Condition

- **Causal graph:**



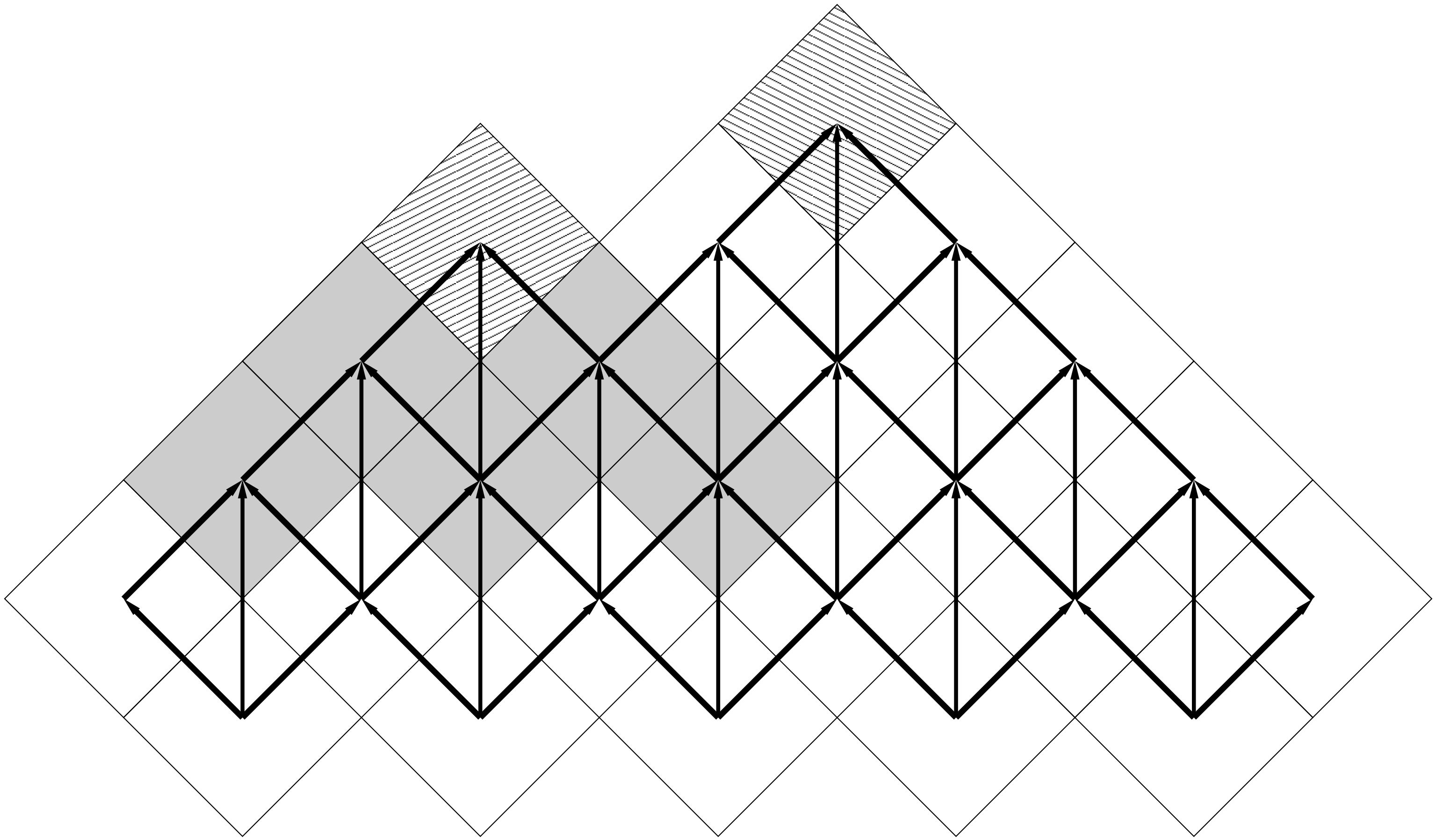
V. Causal Markov Condition

- **d-connected**



V. Causal Markov Condition

- **d-separated**



V. Causal Markov Condition

- **An open problem.** Let $\{\mathcal{N}(V), V \in \mathcal{K}\}$ be a discrete LPT. Construct the Bayesian network $\big(\mathcal{G}(V), \mathcal{V}(V)\big)$ associated to a region V in $\mathcal{K}$. Prove (or falsify) that $\{\mathcal{N}(V), V \in \mathcal{K}\}$ is locally causal in Bell's sense *iff* $\big(\mathcal{G}(V), \mathcal{V}(V)\big)$ fulfils the Causal Markov Condition for every $V \in \mathcal{K}$.

Conclusions

- Bell's notion of local causality presupposes a clear-cut framework integrating probabilistic and spatiotemporal entities. This goal can be met by introducing the notion of a LPT.
- In this general framework one can define Bell's notion of local causality and show sufficient conditions on which a LPT will be locally causal.
- There is a nice parallelism between local causality and the CCPs: Bell inequalities cannot be derived from neither unless the LPT is classical or the common cause is commuting.
- Is Bell's local causality a Causal Markov Condition?

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V. Causal Markov Condition

Theory of Bayesian networks	**Stochastic local classical theory**
Bayesian network $(\mathcal{G}(V), \mathcal{V}(V))$	Associated to every $V \in \mathcal{K}^m$
Causal graph $\mathcal{G}(V)$	Local von Neumann algebra $\mathcal{N}(V)$ with $V \in \mathcal{K}^m$
Vertices	Center of minimal double cones in V
Arrows	Pointing to future timelike related adjacent minimal double cones
Random variables $\mathcal{V}(V)$	Projections localized in the minimal double cones contained in V
Parents	Projections in past timelike related adjacent minimal double cones
Descendants	Projections in future timelike related minimal double cones
Causal Markov Condition	Bell's local causality