QUANTUM FIELD THEORY AND CAUSALITY

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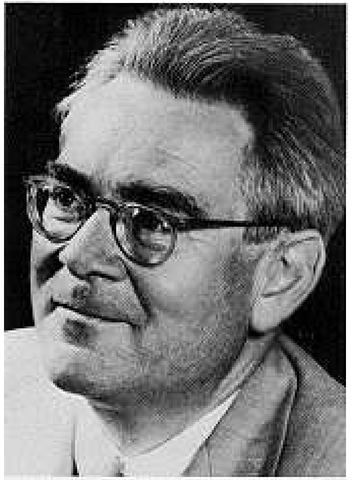
Project

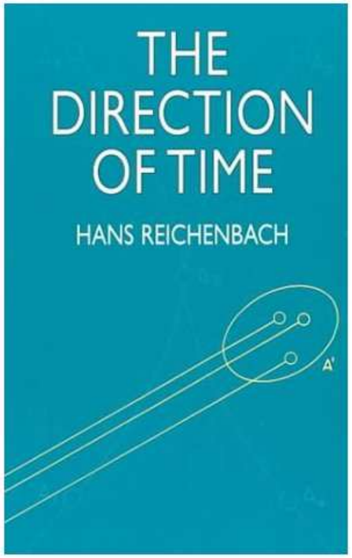
I. Introduction to Reichenbach's Common Cause Principle

II. Introduction to Algebraic Quantum Field Theory

III. Common causal explanation of correlations in Algebraic Quantum Field Theory

I. Reichenbach’s Common Cause Principle





Reichenbach's Common Cause Principle

Common Cause Principle: If there is a correlation between two events A and B and there is no direct causal (or logical) connection between the correlating events then there always exists a common cause C of the correlation.

- What is a common cause?

Reichenbachian common cause

- **Classical probability measure space:** (Ω, Σ, p)

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$$p(AB) > p(A)p(B)$$

- **Reichenbachian common cause:** $C \in \Sigma$

$$\begin{aligned} p(AB|C) &= p(A|C)p(B|C) \\ p(AB|C^{\perp}) &= p(A|C^{\perp})p(B|C^{\perp}) \\ p(A|C) &> p(A|C^{\perp}) \\ p(B|C) &> p(B|C^{\perp}) \end{aligned}$$

Common cause system

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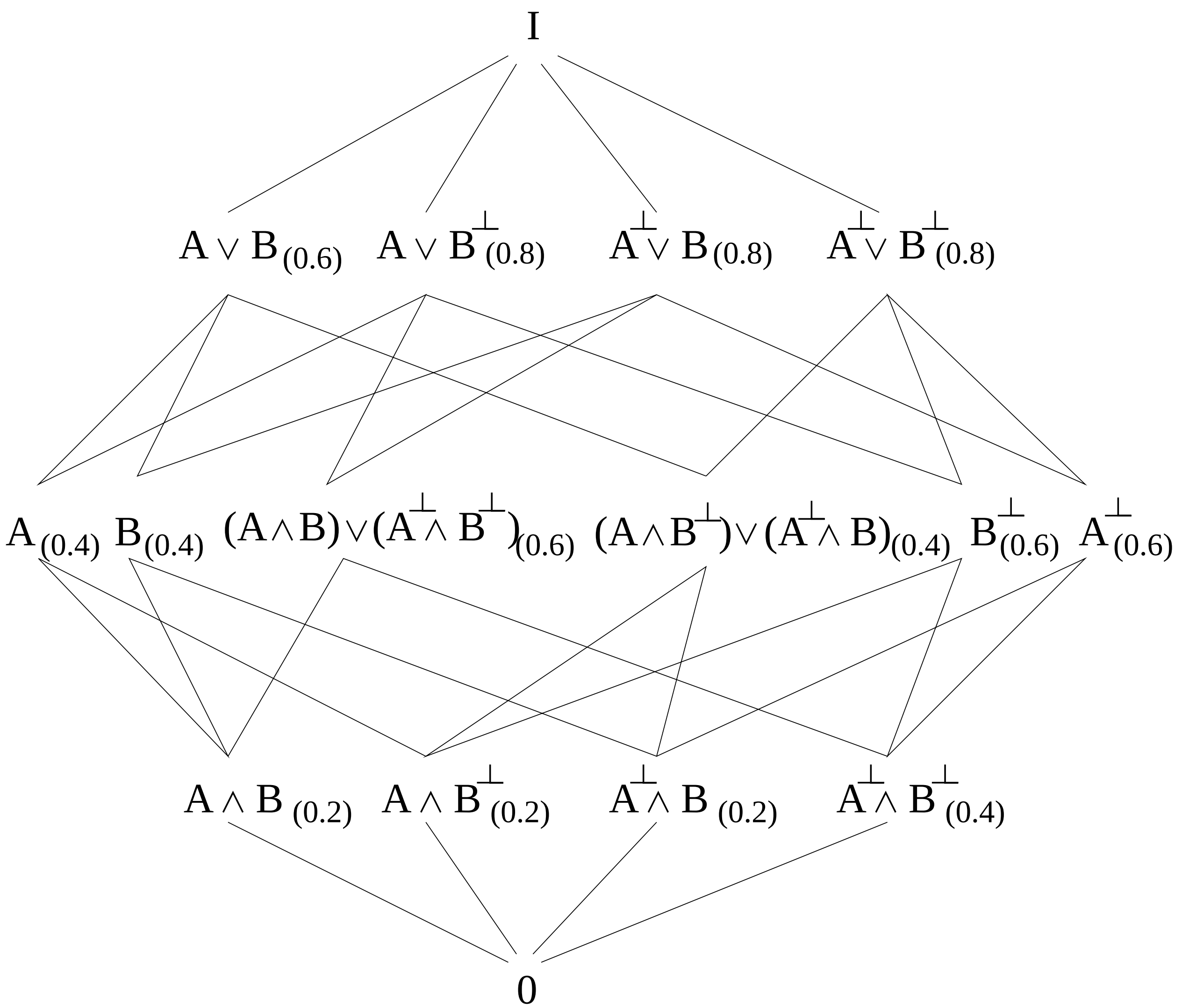
- **Common cause:** common cause system of size 2.
- **Local causality:** ”in the particular case that Λ contains already the *complete* specification of beables in the overlap of the two light cones, supplementary information from region 2 could reasonably be expected to be redundant.” (Bell, 1975)

$$p(A|\Lambda B) \quad = \quad p(A|\Lambda)$$

Trivial common cause system

- **Trivial common cause system:** $\{C_k\}_{k\in K}$ such that $C_k \leq X$ where $X \in \{A, A^{\perp}, B, B^{\perp}\}$
- **Trivial common cause:** $\{C, C^{\perp}\} = \{A, A^{\perp}\}$ or $\{B, B^{\perp}\}$
  - Reichenbach's definition incorporates also direct causes.

Challenging the Common Cause Principle



”Saving” the Common Cause Principle

- **Strategy:** Maybe our description of the physical phenomenon in question is too ”coarse” to provide a common cause for every correlation. However, a finer description would reveal the hidden common causes.
  - **Proposition:** Let (Ω, Σ, p) be a classical probability measure space and let (A_i, B_i) a finite set of pairs of correlating events ($i = 1, 2, \ldots n$). Then there is a (Ω', Σ', p') extension of (Ω, Σ, p) such that for every correlating pair (A_i, B_i) there exists a common cause C_i in (Ω', Σ', p'). (Hofer-Szabó, Rédei, Szabó, 1999, 2000a)
  - The same holds for nonclassical probability measure spaces!

The *common* common cause

- **Question:** If every probability measure space can be common cause extended then what is the problem with the EPR scenario?
- **Answer:**
  - Common causes $\neq$ *common* common causes!
  - Other conditions (locality, no-conspiracy) are also present!
- $$\begin{array}{rcl} \textit{Common}\text{ common cause system} & & \\ \text{Locality} & \Longrightarrow & \text{Bell inequality} \\ \text{No-conspiracy} & & \end{array}$$

Nonclassical common cause system

- **Nonclassical probability measure space:** $(\mathcal{N}, \mathcal{P}(\mathcal{N}), \phi)$

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- **Correlation:** A, $B \in \mathcal{P}(\mathcal{N})$

$$\phi(AB) \neq \phi(A)\phi(B)$$

- **Common cause system:** partition $\{C_k\}_{k \in K}$ in $\mathcal{P}(\mathcal{N})$

  (i) C_k commutes with both A and B

  (ii) if $\phi(C_k) \neq 0$ then:

$$\frac{\phi(ABC_k)}{\phi(C_k)} = \frac{\phi(AC_k)}{\phi(C_k)} \frac{\phi(BC_k)}{\phi(C_k)}$$

Noncommutative common cause system

- **Conditional expectation:**

$$E : \mathcal{N} \to \mathcal{C}, \quad A \mapsto \sum_{k \in K} C_k A C_k$$

a unit preserving positive surjection onto the unital C^*-subalgebra $\mathcal{C} \subseteq \mathcal{N}$ obeying the property $E(B_1 A B_2) = B_1 E(A) B_2; A \in \mathcal{N}, B_1, B_2 \in \mathcal{C}$

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- **Noncommutative common cause system:** partition $\{C_k\}_{k \in K}$ in $\mathcal{P}(\mathcal{N})$

$$\frac{(\phi \circ E)(ABC_k)}{\phi(C_k)} = \frac{(\phi \circ E)(AC_k)}{\phi(C_k)} \frac{(\phi \circ E)(BC_k)}{\phi(C_k)}$$

if $\phi(C_k) \neq 0$

II. Algebraic quantum field theory

$\mathcal{P}_{\mathcal{K}}$**-covariant local quantum theory:** $\{\mathcal{A}(\mathcal{O}), \mathcal{O} \in \mathcal{K}\}$ in a spacetime $\mathcal{S}$ with group $\mathcal{P}$:

(i) **Net** (under inclusion $\subseteq$): a directed poset $\mathcal{K}$ of causally complete, bounded regions of $\mathcal{S}$;

(ii) **Isoton map:** $\mathcal{K} \ni \mathcal{O} \mapsto \mathcal{A}(\mathcal{O})$ satisfying algebraic Haag duality:

$$\mathcal{A}(\mathcal{O}')' \cap \mathcal{A} = \mathcal{A}(\mathcal{O}), \mathcal{O} \in \mathcal{K};$$

Quasilocal observable algebra: inductive limit C^*-algebra of the net;

(iii) **Group homomorphism:** $\alpha \colon \mathcal{P}_{\mathcal{K}} \to \mathrm{Aut}\,\mathcal{A}$ such that

$$\alpha_g(\mathcal{A}(\mathcal{O})) = \mathcal{A}(g \cdot \mathcal{O}), \mathcal{O} \in \mathcal{K}.$$

States

- **States:** normalized positive linear functionals on $\mathcal{A}$
- **GNS:** states (ϕ) $\longrightarrow$ representations ($\pi : \mathcal{A} \to \mathcal{B}(\mathcal{H})$)
- **von Neumann algebra:** weak clousure:
  $\mathcal{N}(\mathcal{O}) := \pi(\mathcal{A}(\mathcal{O}))'', \mathcal{O} \in \mathcal{K}$.

Classification of von Neumann algebras

- **von Neumann lattice:** $\mathcal{P}(\mathcal{N})$, the orthomodular lattice of the projections of $\mathcal{N}$
  - $\mathcal{P}(\mathcal{N})$ generates $\mathcal{N}$: $\mathcal{P}(\mathcal{N})'' = \mathcal{N}$
- **Factor:** $\mathcal{N}$ is a factor von Neumann algebra iff $\mathcal{N} \cap \mathcal{N}' = \{\lambda 1\}$
- **Dimension function:** $d : \mathcal{P}(\mathcal{N}) \to \mathbb{R}^+ \cup \infty$ such that

$$d(A) + d(B) = d(A \wedge B) + d(A \vee B)$$

Classification of von Neumann algebras

Classification of factors: Murray, von Neumann, 1936

Range of d	Type of $\mathcal{N}$	The lattice $\mathcal{P}(\mathcal{N})$
$\{0, 1, 2 \ldots n\}$	I_n	modular, atomic
$\{0, 1, 2 \ldots \infty\}$	I_∞	nonmodular, atomic
$[0, 1]$	II_1	modular, nonatomic
$[0, \infty]$	II_∞	nonmodular, nonatomic
$\{0, \infty\}$	III	nonmodular, nonatomic

- Distributivity: $A \vee (B \wedge C) = (A \vee B) \wedge (A \vee C)$
- Modularity: $A \leq C \implies A \vee (B \wedge C) = (A \vee B) \wedge (A \vee C) = (A \vee B) \wedge C$
- Orthomodularity: $A \leq C \implies A \vee (A^\perp \wedge C) = (A \vee A^\perp) \wedge (A \vee C) = 1 \wedge C = C$

III. Common causal explanation in AQFT

- **Question:** Is the Common Cause Principle valid in algebraic quantum field theory?
- **Answer:** It depends ...

Correlations in quantum field theory

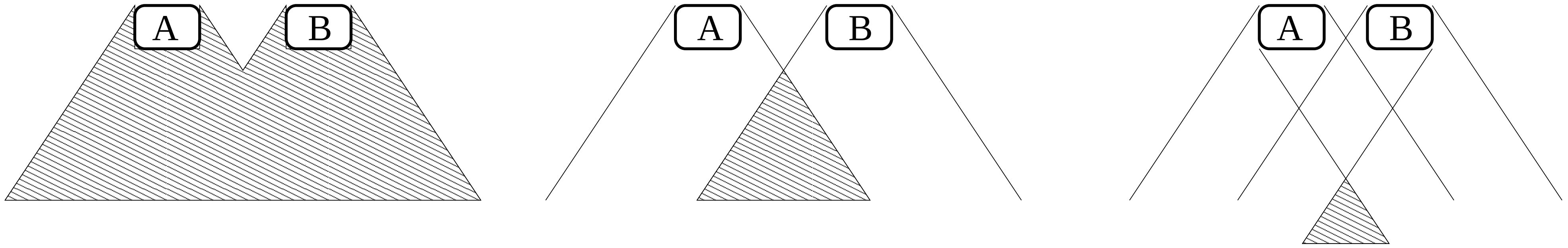
- **Local system:** $(\mathcal{A}(V_1), \mathcal{A}(V_2), \phi)$
  - V_1 and V_2: nonempty convex subsets in $\mathcal{M}$ such that V_1'' and V_2'' are spacelike separated double cones
  - ϕ: locally normal and locally faithful state
- ϕ "typically" generates **correlation** between the projections $A \in \mathcal{A}(V_1)$ and $B \in \mathcal{A}(V_2)$
- $\mathcal{A}(V_1)$ and $\mathcal{A}(V_2)$ are **logically independent**

Question: Does the Common Cause Principle hold for the correlations of the local system?

Where to locate the common cause?

Weak, common and strong past:

$$
\begin{aligned}
wpast(V_1, V_2) &:= I_-(V_1) \cup I_-(V_2) \\
cpast(V_1, V_2) &:= I_-(V_1) \cap I_-(V_2) \\
spast(V_1, V_2) &:= \cap_{x \in V_1 \cup V_2} I_-(x),
\end{aligned}
$$



Common Cause Principles

$(\mathcal{A}(V_1), \mathcal{A}(V_2), \phi)$ satisfies the **Common Cause Principle**: for any pair $A \in \mathcal{A}(V_1)$, $B \in \mathcal{A}(V_2)$ there exists a common cause system in $\mathcal{A}(V)$ such that $V \subset cpast(V_1, V_2)$.

- **Weak and Strong Common Cause Principle** similarly for $wpast(V_1, V_2)$ and $spast(V_1, V_2)$, respectively.
- **Noncommutative Common Cause Principles** similarly for noncommutative common cause system.

The Weak Common Cause Principle holds

- **Proposition:** The Weak Common Cause Principle holds in Poincaré covariant algebraic quantum field theory (Rédei, Summers, 2002)
- **Conditions:**
  - (i) isotony,
  - (ii) Einstein causality,
  - (iii) relativistic covariance,
  - (iv) irreducible vacuum representation,
  - (v) weak additivity,
  - (vi) von Neumann algebras of type III,
  - (vii) local primitive causality.

The Weak Common Cause Principle holds

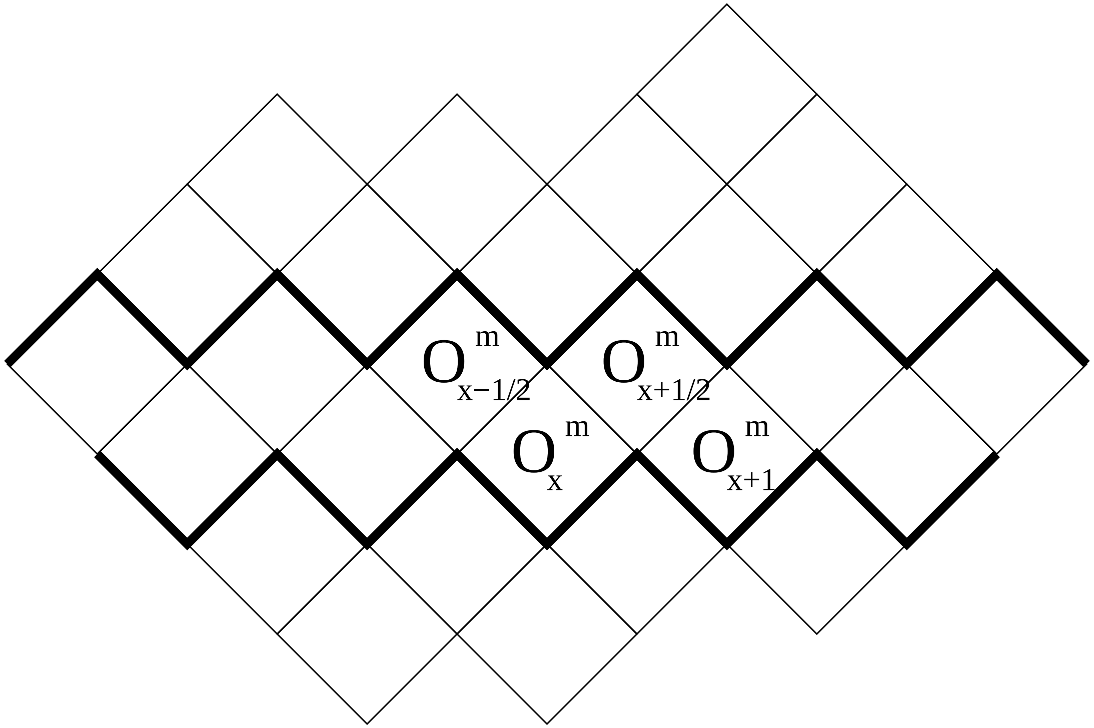
Proof:

- Suppose $C < AB$. Then $\{C, C^{\perp}\}$ will be a nontrivial solution:
  - C: the common cause condition trivially fulfils,
  - $C^{\perp}$: fixes $\phi(C)$ such that $0 < \phi(C) < \phi(AB)$
- Type III von Neumann algebras $\longrightarrow$ for every projection $P \in \mathcal{P}(\mathcal{N})$ and every positive real number $r < \phi(P)$ there exists a projection $C \in \mathcal{P}(\mathcal{N})$ such that $C < P$ and $\phi(C) = r$
- Isotony, local primitive causality $\longrightarrow A, B, C \in \mathcal{A}(V)$ such that $V \subset wpast(V_1, V_2)$ $\blacksquare$

Common Cause Principle in AQFT

- **Question:** Does the Weak Common Cause Principle hold in *every* local quantum theory?
- **Answer:** There is a tight connection between the fate of the (commutative) Common Cause Principle and the type of the local algebras.

Ising model



- A thickened Cauchy surface in the two dimensional Minkowski space $\mathcal{M}^2$

Ising model

- **Intervals:** $(i,j) := \{i, i+\frac{1}{2}, \dots, j-\frac{1}{2}, j\} \subset \frac{1}{2}\mathbf{Z}$
  the space coordinates of the center of minimal double cones on a thickened Cauchy surface
- **Minimal double cone:** $\mathcal{O}_i^m$
- **Double cone:** $\mathcal{O}_{i,j}$, smallest double cone containing $\mathcal{O}_i^m$ and $\mathcal{O}_j^m$
- **Cauchy surface net:** $\mathcal{K}_{CS}^m$, poset of double cones based on the Cauchy surface
- **Net:** $\mathcal{K}^m$, by integer time translation

Ising model

- **Group:** $G = \mathbf{Z}_2 := \{e, g | g^2 = e\}$
- **One-point algebra:**

$$\mathcal{A}(i,i) \cong \begin{cases} \mathbf{C}\mathbf{Z}_2, & i \in \mathbf{Z}, \\ \mathbf{C}(\mathbf{Z}_2), & i \in \mathbf{Z} + \frac{1}{2}, \end{cases}$$

- **Algebraic generators:**

$$U_i := \begin{cases} A_i(g), & i \in \mathbf{Z}, \\ A_i(\chi_e - \chi_g), & i \in \mathbf{Z} + \frac{1}{2}, \end{cases}$$

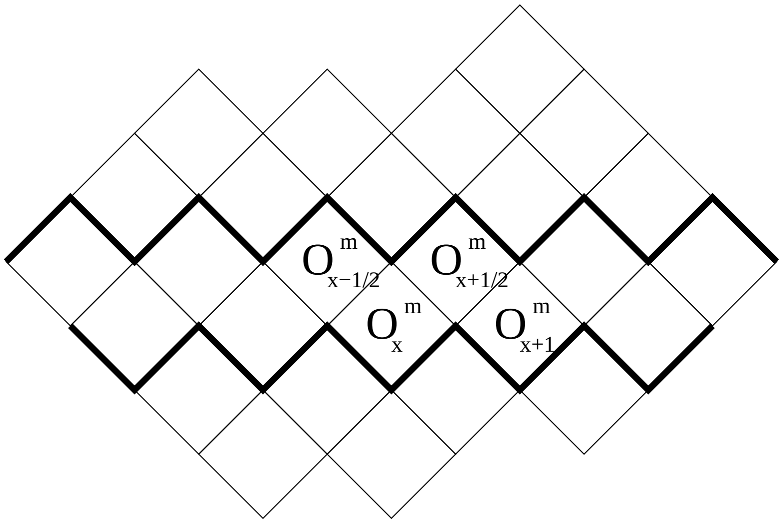
where $\chi_e, \chi_g \in \mathbf{C}(\mathbf{Z}_2)$ are characteristic functions.

Ising model

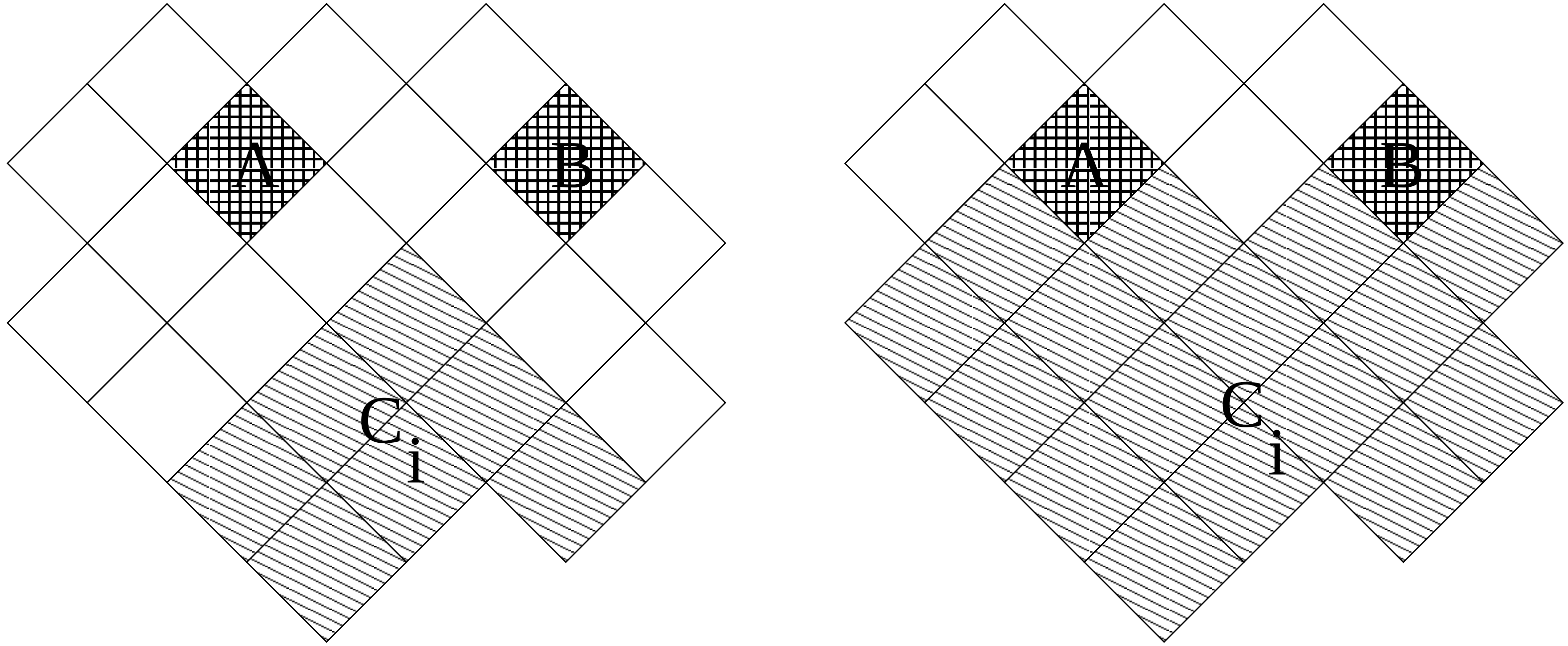
- **Commutation relations:**

$$U_i U_j = \begin{cases} -U_j U_i, & \text{if } |i-j| = \frac{1}{2} \\ U_j U_i, & \text{otherwise} \end{cases}$$

- **Dynamics:** $\beta = \beta(\theta_1, \theta_2, \eta_1, \eta_2)$ automorphisms of $\mathcal{A}$
  - Due to the dynamics local primitive causality holds.



Common Cause Principles



- The Common and the Weak Common Cause Principle

The Weak Common Cause Principle fails

- **Proposition:** Fixing a causal time evolution let us choose two nonzero projections $A \in \mathcal{A}(\mathcal{O}_a)$ and $B \in \mathcal{A}(\mathcal{O}_b)$ localized in two spacelike separated double cones $\mathcal{O}_a, \mathcal{O}_b \in \mathcal{K}^m$. One can construct faithful states on $\mathcal{A}$ such that the Weak Common Cause Principle fails. (Hofer-Szabó, Vecsernyés, to be published)

The Weak Common Cause Principle fails

Proof: Since $\mathcal{A}$ is a UHF algebra there is a unique (non-degenerate) normalized trace $\mathrm{Tr}: \mathcal{A} \to \mathbf{C}$ on it, which coincides with the unique normalized trace on any unital full matrix subalgebras of $\mathcal{A}$. One can find double cones $\tilde{\mathcal{O}}_x \supseteq \mathcal{O}_x, x = a, b$ in $\mathcal{K}^m$ that are also spacelike separated and are (integer) time translates of cones $\mathcal{O}_{i(x),i(x)-\frac{1}{2}+n(x)} \in \mathcal{K}^m_{CS}$ with $i(x) \in \frac{1}{2}\mathbf{Z}$ and $n(x) \in \mathbf{N}$ for $x = a, b$. Then $\mathcal{A}(\tilde{\mathcal{O}}_x)$ is isomorphic to the full matrix algebra $M_{2^{n(x)}}(\mathbf{C})$. Let $\tilde{\mathcal{O}} \in \mathcal{K}^m$ be a double cone that contains both $\tilde{\mathcal{O}}_a$ and $\tilde{\mathcal{O}}_b$ and that is a time translate of a cone $\mathcal{O}_{i,i-\frac{1}{2}+n} \in \mathcal{K}^m_{CS}$ with $i \in \frac{1}{2}\mathbf{Z}$ and $n \in \mathbf{N}$. Therefore $\mathcal{A}(\tilde{\mathcal{O}})$ is isomorphic to the full matrix algebra $M_{2^n}(\mathbf{C})$. Hence, $A, A^\perp$ and $B, B^\perp$ are projections in two commuting full matrix algebras in a full matrix algebra, that is the (mut ually orthogonal) projections $P = AB, A^\perp B^\perp, AB^\perp, A^\perp B$ have nonzero rational traces $m_P/2^n$ with $m_P \in \mathbf{N}$ and $\sum_P m_P = 2^n$. Then

$$X \mapsto \phi(X)_\lambda := \mathrm{Tr}(\sum_P \lambda_P \frac{2^n}{m_P} PX), \quad 0 < \lambda_P, \ \sum_P \lambda_P = 1 \tag{1}$$

defines a faithful state ϕ_λ on $\mathcal{A}$ due to the faithfulness of the trace.

The Weak Common Cause Principle fails

Proof: The requirement of positive correlation $\phi_\lambda(AB) > \phi_\lambda(A)\phi_\lambda(B)$ and the common cause equation read as

$$\begin{aligned} \lambda_{AB}\lambda_{A^\perp B^\perp} &> \lambda_{AB^\perp}\lambda_{A^\perp B}, && (2)\\ \frac{\lambda_{AB}\lambda_{A^\perp B^\perp}}{m_{AB}m_{A^\perp B^\perp}}\mathrm{Tr}(ABC_k)\mathrm{Tr}(A^\perp B^\perp C_k) &= \frac{\lambda_{AB^\perp}\lambda_{A^\perp B}}{m_{AB^\perp}m_{A^\perp B}}\mathrm{Tr}(AB^\perp C_k)\mathrm{Tr}(A^\perp BC_k). && (3)\end{aligned}$$

Let us choose the λ parameters in a way to satisfy (2), moreover let the products $\lambda_{AB}\lambda_{A^\perp B^\perp}$ and $\lambda_{AB^\perp}\lambda_{A^\perp B}$ be rational and irrational, respectively. Such numbers trivially exist; e.g. $\lambda_{AB} = \lambda_{A^\perp B^\perp} = \frac{1}{4}$, $\lambda_{AB^\perp} = \frac{1}{4} + \frac{\pi}{20}$ and $\lambda_{A^\perp B} = \frac{1}{4} - \frac{\pi}{20}$. However, if the projections $C_k, k \in K$ are elements of a local (hence, finite dimensional) algebra $\mathcal{A}(\mathcal{O}_c)$ then the traces of the products of commuting projections in (3) have rational values. Thus (3) fulfills only if both sides are zero, that is only if $C_k \leq X$, with $X = A, A^\perp, B, B^\perp$ for $k \in K$. Therefore all of the solutions are trivial common cause systems which are excluded by definition in the Weak Common Cause Principle. ■

Remarks

- The proof is purely an algebraic-probabilistic one; no mention of the localization of C_k.
- It falsifies the Common and the Strong Common Cause Principle as well.
- Nontrivial quasilocal common causes may exist in $\mathcal{N}$.
- It can be trivally extended to Hopf spin models with causal dynamics.
- ϕ is faithful but neither space nor time translation invariant.

What if we abandon commutativity?

- **Question:** Do the *noncommutative* Common Cause Principles hold in every local physical theory?
- **Answer:** They might.

Illustration

- Fix some parameters of the causal dynamics.
- Choose two projections:

$$\begin{aligned} A &:= \frac{1}{2}\beta(1+U_0) = \frac{1}{2}(1+U_{-\frac{1}{2}}U_0U_{\frac{1}{2}}) \in \mathcal{A}(\mathcal{O}^m(1,0)), \\ B &:= \frac{1}{2}\beta(1+U_1) = \frac{1}{2}(1+U_{\frac{1}{2}}U_1U_{\frac{3}{2}}) \in \mathcal{A}(\mathcal{O}^m(1,1)). \end{aligned}$$

- Choose the falsifying state ϕ above.
- Let C be defined as:

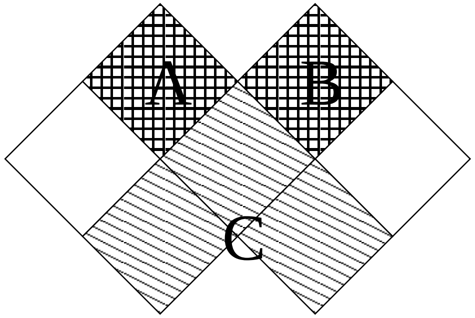
$$C = \frac{1}{2}(1+a_1U_{\frac{1}{2}}+a_2U_1+ia_3U_0U_{\frac{1}{2}}); \quad a_1, a_2, a_3 \in \mathbf{R}, \sum_{i=1}^{3} a_i^2 = 1.$$

Illustration

- $\{C, C^{\perp}\}$ will be a common cause of the correlation:

$$\begin{aligned} \phi(CABC)\,\phi(CA^{\perp}B^{\perp}C) &= \phi(CAB^{\perp}C)\,\phi(CA^{\perp}BC) \\ \phi(C^{\perp}ABC^{\perp})\,\phi(C^{\perp}A^{\perp}B^{\perp}C^{\perp}) &= \phi(C^{\perp}AB^{\perp}C^{\perp})\,\phi(C^{\perp}A^{\perp}BC^{\perp}) \end{aligned}$$

- $\{C, C^{\perp}\} \subset \mathcal{A}(\mathcal{O}_{0,1})$ that is in the $cpast(\mathcal{O}_a, \mathcal{O}_b)$



Remarks

- This does not prove the validity of the noncommutative Common Cause Principles in the Ising model!
- Why to require commutativity for the common cause?

Conclusions

Two reactions to the failure of the Common Cause Principles:

1. A discrete model is only an approximation; it does not contain all the observables therefore the common cause might remain burried beyond the coarse description.
2. A discrete model is a self-contained physical model; the commuting Common Cause Principle is not universally valid. Abandon commutativity!

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