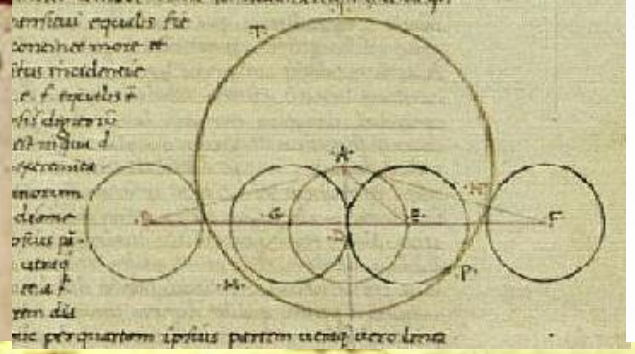
Ptolemaiosz és Kopernikusz összehasonlítása

a szövegek tükrében

PTOLEMAEI MATHEMATICÆ constructionis Liber primus græce & latine editus.

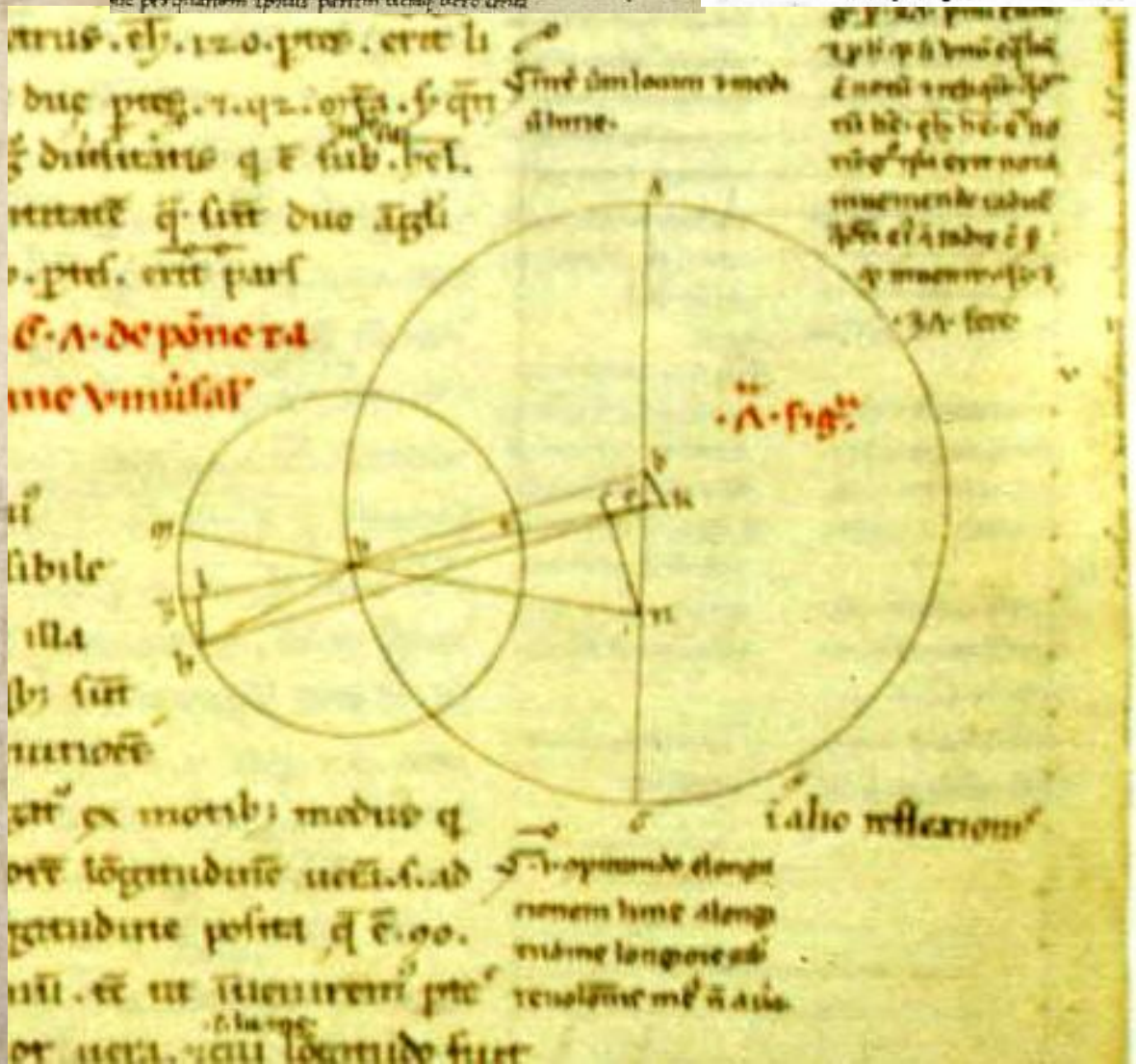
ADDITÆ EXPLICATIONES ALIquot locorum ab ERASMO RHEINHOLT Saluelденsi.

VVITTEBERGAE
Ex Officina Iohannis Lufft.
ANNO
1549.



Almagestũ CL. Ptolemei
Pheludiensis Alexandrini Astronomorũ principis: Opus ingens ac nobile omnes Celestũ motus continens. Felicibus Astris eat in lucem: Ductu Petri Liechtenstein Coloniensis Germani. Anno Virginei Partus. 1515. Die. 10. Ia. Venetijs ex officina eiusdem litteraria.

Cum priuilegio.



NICOLAI COPERNICI TORINENSIS DE REVOLVTIONIbus orbium cœleſtium, Libri VI.

IN QVIBVS STELLARVM ET FIXARVM ET ERRATICARVM MOTVS, EX VETEribus atq; recentibus obſeruationibus, reſtituit hic autor. Præterea tabulas expeditas luculentasq; addidit, ex quibus eoſdem motus ad quoduis tempus Mathematum ſtudioſus facillime calculare poterit.

ITEM, DE LIBRIS REVOLVTIONVM NICOLAI Copernici Narratio prima, per M. Georgium Ioachimum Rheticum ad D. Ioan. Schonerum ſcripta.

Cum Gratia & Priuilegio Cæſ. Maieſt.

BASILEAE, EX OFFICINA HENRICPETRINA.

NICOLAI COPERNICI TORINENSIS DE REVOLVTIONIBVS ORBIum cœleſtium, Libri VI.

Habes in hoc opere iam recens nato, & ædito, ſtudioſe lector, Motus ſtellarum, tam fixarum, quàm erraticarum, cum ex ueteribus, tum etiam ex recentibus obſeruationibus reſtitutos: & nouis inſuper ac admirabilibus hypotheſibus ornatos. Habes etiam Tabulas expeditiſsimas, ex quibus eoſdem ad quoduis tempus quàm facillime calculare poteris. Igitur eme, lege, fruere.

Ἀγαθοῦ τύχης ἕνεκα ἔστω.

Norimbergæ apud Ioh. Petreium,
Anno M. D. XLIII.

Ptolemaiosz: *Almagest*
1. sz. közepe

Könyvei:

I. Ált. bevezetés, a világ szerkezete + matematikai alapvetés
II. Égi alapjelenségek, naphossz vált.
III. Nap mozgása, mozgástáblázatok
IV. és V. Hold mozgása
VI. Fogyatkozások
VII. és VIII. Csillagszféra, csillagkatalógus
IX. Merkúr, bolygómozgás alapfogalmai
X. Vénusz, Mars
XI. Jupiter, Szaturnusz
XII. Retrográd mozgások magyarázata
XIII. Pályahajlások, láthatóságok

Kopernikusz: *De Revolutionibus…*
1543

Könyvei:

I. Ált. bevezetés, a világ szerkezete + matematikai alapvetés
II. U.ez folyt., de a Föld forog
III. Napéjegyenlőség, napforduló, precesszió + Nap
IV. Hold
V. Bolygók: táblázatok, összevetés a régi elméletekkel
VI. Pályahajlások

Következő 5 fólia:

- 1-3: Ptolemaiosz I.10: Húrok közötti összefüggések (a.k.a. trigonometrikus átalakítások) (angol ford.)
- 4-5: Kopernikusz I.12: u.a. (angol ford.)

Latin fejezetcím:

De magnitudine rectarum in circulo linearum. Cap. XII.

$$\left.\begin{array}{r}\therefore \text{ Crd } 90^\circ \approx 84;51,10^p \\ \text{and Crd } 120^\circ \approx 103;55,23^p\end{array}\right\} \text{ where the diameter is } 120^p.$$

We can, then, consider the above chords as established individually by the above straightforward procedures. It will immediately[58] be obvious that if any chord be given, the chord of the supplementary arc is given in a simple fashion,
H36 since the sum of their squares equals the square on the diameter. For instance, since the chord of 36° was shown to be $37;4,55^p$, and the square of this is $1375;4,15^p$, and the square on the diameter is 14400^p, the square on the chord of the supplementary arc (which is 144°) will be the difference, namely 13024;55,45, and so

$$\text{Crd } 144^\circ \approx 114;7;37^p.$$

Similarly for the other chords [of the supplements].

We shall next show how the remaining individual chords can be derived from the above [chords], first of all setting out a theorem which is extremely useful for the matter at hand.

[See Fig. 1.2.] Let there be a circle with an arbitrary quadrilateral ABGD inscribed in it. Join AG and BD.

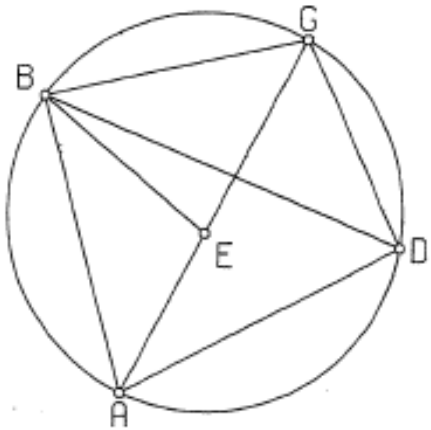


Fig. 1.2

We must prove that

$$AG.BD = AB.DG + AD.BG.^{59}$$

[Proof:] Make ∠ ABE = ∠ DBG.

Then, if we add ∠ EBD common,

$$\angle ABD = \angle EBG.$$

[58] Reading αὐτόθεν (with D) for ἐντεῦθεν at H35,18.

[59] This proposition, commonly known as 'Ptolemy's Theorem', is not in fact attested before him. It remains uncertain whether any of the earlier chord tables (e.g. Menelaus') used any geometrical basis beyond the half-angle theorem (see n. 60 and Toomer[2] 18–19).

But ∠ BDA = ∠ BGE also, since they subtend the same segment. H37

∴ triangle ABD ||| triangle BGE.

∴ BG:GE = BD:DA.

∴ BG.AD = BD.GE.

Again, since ∠ ABE = ∠ DBG,

and ∠ BAE = ∠ BDG,

triangle ABE ||| triangle BGD.

∴ BA:AE = BD:DG.

∴ BA.DG = BD.AE.

But it was shown that

BG.AD = BD.GE.

Therefore, by addition, AG.BD = AB.DG + AD.BG.

Q.E.D.

Having established this preliminary theorem, we draw [Fig. 1.3] semi-circle ABGD on diameter AD, and draw from A two chords, AB, AG, each given in H38 size in terms of a diameter of 120^p. Join BG.

I say that BG too is given.

[Proof:] Join BD,GD.

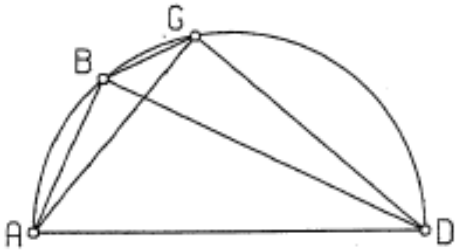


Fig. 1.3

Then, clearly, BD and GD too will be given, since they are chords of [arcs] supplementary [to the arcs of the given chords AB and AG].

Now since ABGD is a cyclic quadrilateral,

AB.GD + AD.BG = AG.BD.

But AG.BD and AB.GD are given.

∴ AD.BG is given by subtraction.

And AD is a diameter.

Therefore chord BG is given.

And we have shown that, if two arcs and the corresponding chords are given, the chord of the difference between the two arcs will also be given.

It is obvious that by means of this theorem we shall be able to enter [in the table] quite a few chords derived from the difference between the individually calculated chords, and notably the chord of 12°, since we have those of 60° and H39 72°.

Let us now consider the problem of finding the chord of the arc which is half that of some given chord.[60]

Let [Fig. 1.4] ABG be a semi-circle on diameter AG. Let GB be a given chord. Bisect arc GB at D, join AB, AD, BD, DG, and drop perpendicular DZ from D on to AG.

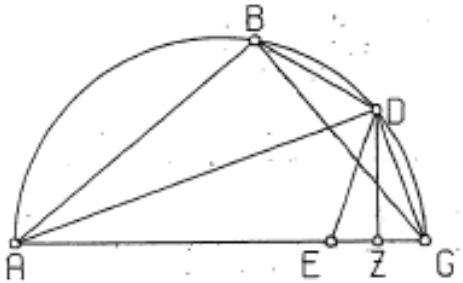


Fig. 1.4

I say that

$$ZG = \tfrac{1}{2}(AG - AB).$$

[Proof:] Let AE = AB, and join DE.

Then since [in the triangles ABD, ADE]

AB = AE, and AD is common,

the two pairs of sides AB, AD, and AE, AD are equal.

Furthermore ∠ BAD = ∠ EAD.

∴ base BD = base DE.

But BD = DG [by construction]

∴ DG = DE.

So, since, in the isosceles triangle DEG, perpendicular DZ has been drawn from apex to base

EZ = ZG.

But EG = [AG − AE =] AG − AB.

∴ $ZG = \tfrac{1}{2}(AG - AB)$.

Now, if the chord of arc BG is given, the supplementary chord AB is immediately given.

Therefore ZG, which is $\tfrac{1}{2}(AG - AB)$, is also given.

But, since, in the right-angled triangle AGD, the perpendicular DZ has been drawn,

triangle ADG ||| triangle DGZ (both right-angled).[61]

∴ AG:GD = GD:GZ.

∴ $AG.GZ = GD^2$.

[60] Although Ptolemy's formula for the chord of the half-angle can easily be derived from his general theorem (see Toomer[2] 16–17), he introduces instead another theorem, which goes back to Archimedes (see *HAMA* 23–4). It is a plausible inference that this is because the latter theorem was the sole basis of earlier chord tables, notably Hipparchus', as I have argued, Toomer[2] 18–19.

[61] Euclid VI 8.

But AG.GZ is given.

Therefore GD^2 is given, and so chord GD, which subtends an arc half of [the arc of the given chord] BG, is also given.

By means of this theorem too a large number of chords will be derived by halving [the arcs of] the previously determined chords, and notably, from the chord of 12°, the chords of 6°, 3°, $1\tfrac{1}{2}$° and $\tfrac{3}{4}$°. By calculation we find the chord of $1\tfrac{1}{2}$° to be approximately $1;34,15^p$ where the diameter is 120^p, and the chord of $\tfrac{3}{4}$° H41
to be approximately $0;47,8^p$ in the same units.

Again, [see Fig. 1.5] let there be a circle ABGD on diameter AD, with centre Z. From A let there be cut off in succession two given arcs, AB, BG. Join the corresponding chords AB, BG; they too will be given.

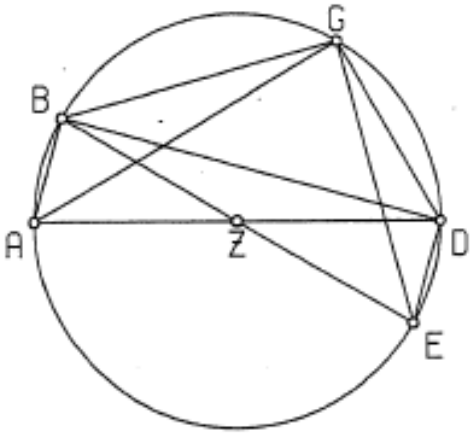


Fig. 1.5

I say, that if we join AG, that [chord] too will be given.

[Proof:] Draw through B diameter BZE, and join BD,DG,GE,DE. It is immediately clear that from BG one can derive GE, and from AB one can derive BD and DE [all as chords of the supplementary arc]. By an argument similar to the preceding [p. 51], since BGDE is a cyclic quadrilateral, in which BD and GE are diagonals, the product of the diagonals will be equal to the sum of the products of the opposite sides [i.e. BD.GE = BG.DE + BE.GD]. Therefore, since (BD.GE) and (BG.DE) are both given, (BE.GD) is also given. But BE also is given, being a diameter: therefore the remaining[62] part, GD, will also H42
be given, and hence GA, the [chord of the] supplement.

Therefore, if two arcs and the corresponding chords are given, the chord corresponding to the sum of these two arcs will be given by means of this theorem.

It is obvious that by combining [in this way] the chord of $1\tfrac{1}{2}$° with all the chords we have already obtained, and then computing successive chords, we will be able to enter [in the table] all chords [of arcs] which when doubled are

[62] Reading ἡ λοιπὴ (with A) at H42,1 for λοιπὴ ('by subtraction').

divisible by three {i.e. multiples of $1\frac{1}{12}°$}. Then the only chords remaining to be determined will be those between the $1\frac{1}{12}°$ intervals, two in each interval, since our table is made at $\frac{1}{2}°$ intervals. If, therefore, we can find the chord of $\frac{1}{2}°$, this will enable us to complete {the table with} all the remaining intermediate chords, by finding the sum or difference {of $\frac{1}{2}°$} from the given chords at either end of the {$1\frac{1}{12}°$} intervals. Now, if a chord, e.g. the chord of $1\frac{1}{2}°$, is given, the chord corresponding to an arc which is one-third of the previous one cannot be found by geometrical methods.[63] {If this were possible, we should immediately
H43 have the chord of $\frac{1}{2}°$}. Therefore we shall first derive the chord of 1° from those of $1\frac{1}{2}°$ and $\frac{3}{4}°$. {We shall do this} by establishing a lemma which, though it cannot in general exactly determine the sizes {of chords}, in the case of such very small quantities can determine them with a negligibly small error.

I say, then, that if two unequal chords be given, the ratio of the greater to the lesser is less than the ratio of the arc on the greater to the arc on the lesser.

[See Fig. 1.6] Let there be a circle ABGD, in which there are drawn two unequal chords, the lesser AB and the greater BG.

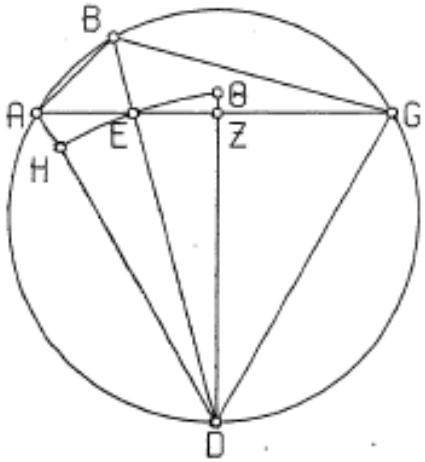


Fig. 1.6

I say that

$$GB:BA < \text{arc } BG : \text{arc } BA.$$

[Proof:] Let ∠ ABG be bisected by [chord] BD. Join AEG, AD and GD. Then, since ∠ ABG is bisected by chord BED,

$$GD = AD$$

H44 $$\text{and } GE > EA.^{64}$$

[63] This is true: the problem of finding Crd n from given Crd $3n$ can be reduced to a cubic equation of the kind which cannot (except for a few particular values of n) be solved by Euclidean geometry (using straight line and circle). See Toomer[3] 138.

[64] Derivable from Euclid VI 3, which states that the bisector of the angle at the apex of a triangle divides the base in the ratio of the two sides enclosing the angle. Here, since BG > BA, GE > EA.

So drop perpendicular DZ from D on to AEG.

Then, since AD > ED and ED > DZ, a circle drawn on centre D with radius DE will cut AD and pass beyond DZ. Let it be drawn as HEΘ, and let DZ be produced to Θ. Now, since sector DEΘ is greater than triangle DEZ, and triangle DEA is greater than sector DEH,

triangle DEZ: triangle DEA < sector DEΘ: sector DEH.

But triangle DEZ: triangle DEA = EZ:EA,[65]

and sector DEΘ: sector DEH = ∠ ZDE:∠ EDA.

$$\therefore ZE:EA < \angle ZDE:\angle EDA.$$

So, *componendo*,

$$ZA:EA < \angle ZDA:\angle ADE.$$

And, doubling the first members [of the ratios],

$$GA:AE < \angle GDA:\angle EDA.$$ H45

Then, *dividendo*,

$$GE:EA < \angle GDE:\angle EDA.$$

But GE:EA = GB:BA,[66]

and ∠ GDB:∠ BDA = arc GB:arc BA.

$$\therefore GB:BA < \text{arc } GB:\text{arc } BA.$$

Having established this, let us draw [Fig. 1.7] circle ABG, and in it two chords, AB and AG. Let us suppose, first, that AB is the chord of $\frac{3}{4}°$ and AG the chord of 1°. Then, since

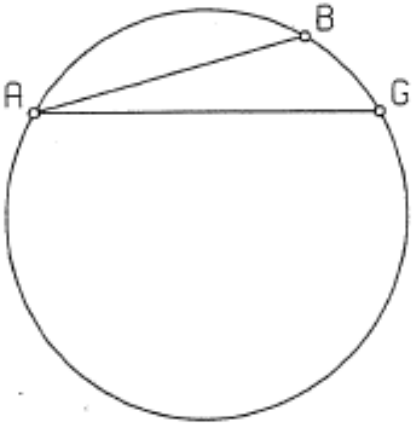


Fig. 1.7

$$AG:BA < \text{arc } AG:\text{arc } AB$$

$$\text{and arc } AG = \frac{4 \text{ arc } AB}{3},$$

$$GA < \frac{4AB}{3}.$$

[65] Euclid VI 1.

[66] Euclid VI 3.

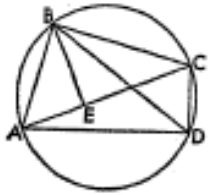
to the sum of the squares on the sides comprehending the right angle; therefore —since the side of the decagon, which subtends 36° of the circumference, has been shown to have 61,803 parts whereof the diameter has 200,000 parts—the chord which subtends the remaining 144° of the semicircle has 190,211 parts.

And in the case of the side of the pentagon, which is equal to 117,557 parts of the diameter and subtends an arc of 72°, a straight line of 161,803 parts is given, and it subtends remaining 108° of the circle.

Second Theorem

If a quadrilateral is inscribed in a circle, the rectangle comprehended by the diagonals is equal to the two rectangles which are comprehended by the two pairs of opposite sides.

For let the quadrilateral $ABCD$ be inscribed in a circle; I say that the rectangle comprehended by the diagonals AC and DB is equal to those comprehended by AB, CD and by AD, BC.



For let us make

angle ABE = angle CBD.

Therefore by addition

angle ABD = angle EBC,

taking angle EBD as common to both. Moreover

angle ACB = angle BDA.

because they stand on the same segment of the circle; and accordingly the two similar triangles BCE and BDA will have their sides proportional. Hence

$$BC : BD = EC : AD.$$

And

rect. EC, BD = rect. BC, AD.

But also the triangles ABE and CBD are similar, because

angle ABE = angle CBD.

And

angle BAC = angle BDC,

because they intercept the same arc of the circle.

So again,

$$AB : BD = AE : CD$$

And

rect. AB, DC = rect. AE, BD.

But it has already been made clear that

rect. AD, BC = rect. BD, EC.

Accordingly, taken as a whole,

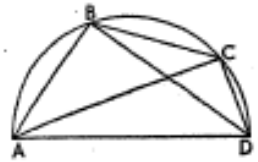
rect. BD, AC = rect. AD, BC + rect. AB, CD,

as it was opportune to have shown.

Third Theorem

Hence if straight lines subtending unequal arcs in a semicircle are given, the chord subtending the arc whereby the greater arc exceeds the smaller is also given.

[13[b]] In the semicircle $ABCD$ with diameter AD, let the straight lines AB and AC subtending unequal arcs be given. To us, who wish to discover the chord subtending BC, there are given by means of the aforesaid the chords BD and CD subtending the remaining arcs of the semicircle, and these chords bound



the quadrilateral $ABCD$ in the semicircle. The diagonals AC and BD have been given together with the three sides AB, AD, and CD. And, as has already been shown,

rect. AC, BD = rect. AB, CD + rect. AD, BC.

Therefore,

rect. AD, BC = rect. AC, BD − rect. AB, CD.

Accordingly, in so far as the division may be carried out,

$$(AC\text{-}BD - AB\text{-}CD) \div AD = BC,$$

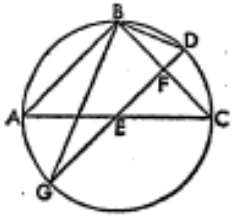
which was sought.

Further when, for example, the sides of the pentagon and hexagon are given from the above, by this computation a line is given subtending 12°—which is the difference between the arcs—and it is equal to 20,905 parts of the diameter.

Fourth Theorem

Given a chord subtending any arc, the chord subtending half of the arc is also given.

Let us describe the circle ABC, whose diameter is AC, and let the arc BC be given together with the chord subtending it, and let the line EF from the centre E cut BC at right angles. Accordingly by Euclid, III, 3, it will bisect chord BC at F, and the arc at D. Let the chords subtending arcs AB and BD be drawn. Since the triangles ABC and EFC are right and also similar—for they have angle ECF in common; therefore, as



$$CF = \tfrac{1}{2}\, BFC,$$

so

$$EF = \tfrac{1}{2}\, AB.$$

But chord AB is given, for it subtends the remaining arc of the semicircle. Therefore EF is given; and so is line DF the remainder of the radius. Let the diameter DEG be completed, and let BG be joined. Therefore in triangle BDG line BF falls from the right angle at B perpendicular to the base. Accordingly,

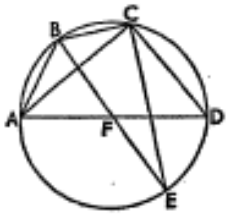
rect. GD, DF = sq. BD.

Therefore BD is given in length, and it subtends half of the arc BDC.

And since a chord subtending 12° has already been given, the chord subtending 6° is given as 10,467 parts; that subtending 3°, as 5235 parts; that subtending 1½°, as 2618 parts; and that subtending 45′, as 1309 parts.

[14[a]] Fifth Theorem

Again, when chords are given subtending two arcs, the chord subtending the whole arc made up of them is also given.



Let there be given in the circle the two chords subtending the arcs AB and BC; I say that the chord subtending the whole arc ABC is also given.

For let the diameters AFD and BFE be drawn, and also the chords BD and CE, which are given by means of the foregoing, on account of chords AB and BC being given; and

chord DE = chord AB.

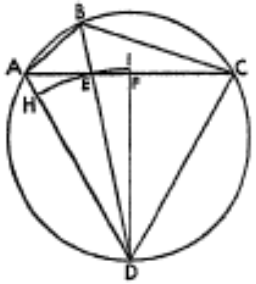
The joining of CD completes the quadrilateral $BCDE$, whose diagonals BD and CE are given together with the three sides BC, DE, and BE; and the remaining side CD will be given by the second theorem; accordingly chord CA which subtends the remaining part of the semicircle will be given, and it subtends the whole arc ABC and is what was sought.

Furthermore, since so far there have been discovered chords which subtend 3°, 1½°, and ¾°; by means of these intervals a table can be constructed with the most exact ratios. Nevertheless if we ascend through the degrees and add one arc to another arc either by halves or by some other mode, there is not unjustified doubt concerning the chords subtending those arcs, as the graphical ratios by which they can be shown are lacking to us. Nothing, however, prevents us from going on with that by some mode which is this side of error perceptible to sense and which is least unconsonant with the assumed number. This was what Ptolemy too sought as regards the chords subtending arcs of 1° or of ½°; and he admonished us in the first place.

Sixth Theorem

The ratio of the arcs is greater than the ratio of the greater to the smaller of the chords.

Let there be in a circle two unequal successive arcs AB and BC, and let BC be the greater.



I say that

arc BC : ard AB > chord BC : chord AB.

These chords comprehend angle B, and let that be bisected by line BD. And let AC be joined, which cuts BD at point E. Similarly let AD and CD be joined; then

$$AD = CD,$$

because they subtend equal arcs.

Accordingly, since in triangle ABC, the line which bisects the angle also cuts AC [14ᵇ] at E, then

EC, segment of base : $AE = BC : AB$ [Euclid, VI, 3)

and since

$$BC > AB,$$

then

$$EC > EA.$$

Let DF be erected perpendicular to AC; it will bisect AC at point F. And F must necessarily be found in the greater segment EC. And since in every triangle the greater angle is subtended by the greater side, in the triangle DEF

side DE > side DF,

and further,

$$AD > DE,$$

wheretofore the circumference described with D as center and DE as radius will cut AD and pass beyond DF. Therefore let it cut AD at H, and let it be extended in the straight line DFI.

Since

sect. EDI > trgl. EDF,

while

trgl. DEA > sect. DEH,

therefore

trgl. DEF : trgl. DEA < sect. DEI : sect. DEH.

But sectors are proportional to their arcs or to the angles at the centre; while triangles under the same vertex are proportional to their bases. Accordingly

angle EDF : angle ADE > base EF : base AE.

Therefore, *componendo*,

angle FDA : angle ADE > base AF : base AE.

And, in the same way,

angle CDA : angle ADE > base AC : base AE.

But, *separando*,

angle CDE : angle EDA > base CE : base EA.

But

angle CDE : angle EDA = arc CB : arc AB.

And

base CE : base AE = chord CB : chord AB.

Therefore

arc CB : arc AB > chord BC : chord AB,

as was to be shown.

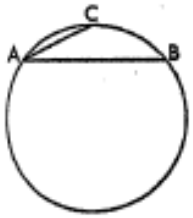
Problem

But since the arc is always greater than the straight line subtending it—as the straight line is the shortest of those lines which have the same termini—nevertheless in going from greater to lesser sections of the circle, the inequality approaches equality, so that finally the circular line and the straight line go out of existence simultaneously at the point of tangency on the circle. Therefore it is necessary that just before that moment they differ from one another by no discernible difference.

For example, let arc AB be 3° and arc AC 1½°. It has been shown that

ch. AB = 5235,

where diameter = 200,000,



and that

ch. AC = 2618.

And though

arc AB = 2 [15ᵃ] arc AC,

Yet

ch. AB < 2 ch. AC

and

ch. AC—2617 = 1.

But if we make

arc AB = 1½°

and

arc AC = ¾°,

then

ch. AB = 2618

and

ch. AC = 1309,

and even though chord AC ought to be greater than half of chord AD, it is seen to be no different from the half. And the ratios of the arcs and the straight

Ptolemaiosz ábrái

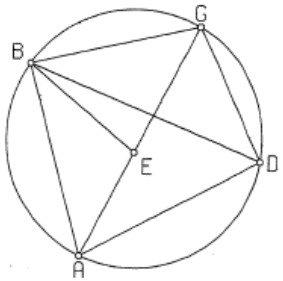


Fig. 1.2

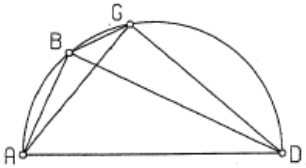


Fig. 1.3

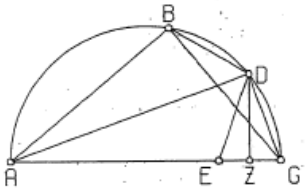


Fig. 1.4

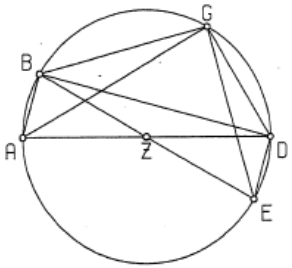


Fig. 1.5

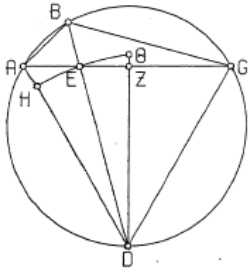


Fig. 1.6

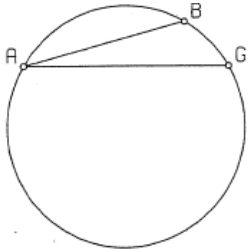
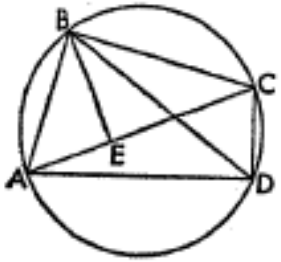
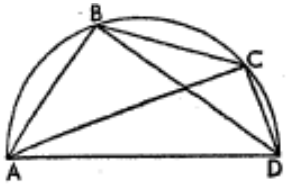
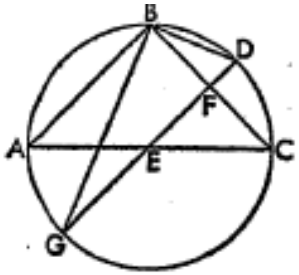
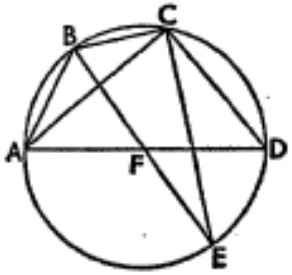


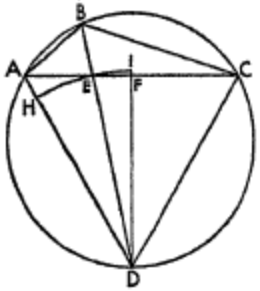
Fig. 1.7

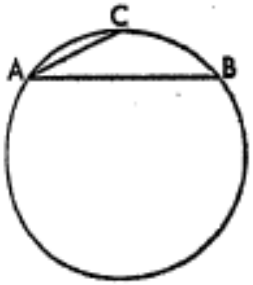












Kopernikusz ábrái

Következő 3 fólia:

- 1: Ptolemaiosz csillagkatalógusának kezdete (angol ford.)
- 2: Kopernikusz csillagkatalógusának kezdete (angol ford.)
- 3: Kopernikusz csillagkatalógusának kezdete (latin)

5 {*Tabular layout of the constellations in the northern hemisphere*}[95]

[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
	[I] Constellation of Ursa Minor				
1	The star on the end of the tail	♊ 0⅙	+66	3	α UMi
2	The one next to it on the tail	♊ 2½	+70	4	δ UMi
3	The one next to that, before the place where the tail joins [the body]	*♊ 10⅙[96]	+74⅓	4	ε UMi
4	The southernmost of the stars in the advance side of the rectangle	♊ 29⅔	+75⅔	4	ζ UMi
5	The northernmost of [those in] the same side	♋ 3⅔	+77⅓	4	η UMi
6	The southern star in the rear side	*♋ 17½[97]	+72⅚	2	β UMi
7	The northern one in the same side	♋ 26⅙	+74⅚	2	γ UMi
	{7 stars, 2 of the second magnitude, 1 of the third, 4 of the fourth}				
	Nearby star outside the constellation:				
8	The star lying on a straight line with the stars in the rear side [of the rectangle] and south of them	♋ 13	+71⅙	4	5 UMi
	{1 star of the fourth magnitude}				

[Number in constellation]	Description	Longitude in degrees	Latitude in degrees	Magnitude	[Modern designation]
	[II] Constellation of Ursa Major				
1	The star on the end of the snout	♊ 25⅓	+39⅚	4	o UMa
2	The more advanced of the stars in the two eyes	♊ 25⅚	+43	5	2(A) UMa
3	The one to the rear	♊ 26⅓	+43	5	π^2 UMa
4	The more advanced of the two stars in the forehead	♊ 26⅙	+47⅙	5	ρ UMa
5	The one to the rear	*♊ 26⅔[98]	+47	5	σ^2 UMa
6	The star on the tip of the advance ear	♊ 28⅙	+50½	5	24(d) UMa
7	The more advanced of the two stars in the neck	♋ 0½	+43⅚	4	τ UMa
8	The one to the rear	♋ 2½	+44⅓	4	23(h) UMa
9	The northernmost of the two stars in the chest	♋ 9	+42	4	υ UMa
10	The southernmost of them	♋ 11	*+44[99]	<4	φ UMa
11	The star on the left knee	♋ 10⅔	+35	3	θ UMa
12	The northernmost of the [two] in the front left paw[100]	♋ 5½	+29⅓	3	ι UMa
13	The southernmost of them	♋ 6⅓	+28⅓	3	κ UMa
14	The star above the right knee	♋ 5⅔	+36	4	18(e) UMa
15	The star below the right knee	♋ 5⅚	+33[101]	4	15(l) UMa
16–19	The stars in the quadrilateral:				
16	the one on the back	♋ 17⅔	+49	2	α UMa
17	the one on the flank	♋ 22⅙	+44½	2	β UMa
18	the one on the place where the tail joins [the body]	♌ 3⅙	+51	3	δ UMa
19	the remaining one, on the left hind thigh	♌ 3	+46½	2	γ UMa
20	The more advanced of the [two stars] in the left hind paw	♋ 22⅔	+29⅓	3	λ UMa
21	The one to the rear of it	♋ 24⅙	+28¼	3	μ UMa

CATALOGUE OF THE SIGNS AND OF THE STARS

And First Those of the Northern Region

Constellations	Longitude Deg.	Min.		Latitude Deg.	Min.	Magnitude
Ursa Minor, or the Little Bear, or Cynosura						
The [star] at the tip of the tail	53	30	N	66	0	3
The [star] to the east in the tail	55	50	N	70	0	4
The [star] at the base of the tail	69	20	N	74	0	4
The more southern [star] on the western side of the quadrilateral	83	0	N	75	20	4
The northern [star] on the same side	87	0	N	77	40	4
The more southern of the stars on the eastern side	100	30	N	72	40	2
The more northern on the same side	109	30	N	74	50	2
7 stars: 2 of second magnitude, 1 of third, 4 of fourth						
The most southern unconstellated star near the Cynosure, in a straight line with the eastern side	103	20	N	71	10	4
Ursa Major, or the Great Bear						
The star in the muzzle	78	40	N	39	50	4
The western star in the two eyes	79	10	N	43	0	5
The star to the east of that	79	40	N	43	0	5
The more western star of the two in the forehead	79	30	N	47	10	5
The star to the east in the forehead	81	0	N	47	0	5
The western star in the right ear	81	30	N	50	30	5
The more western of the two in the neck	85	50	N	43	50	4
The eastern	92	50	N	44	20	4

Northern Signs

The more northern of the two in the breast	94	20	N	44	0	4
The more southern	93	20	N	42	0	4
The star at the knee of the left foreleg	89	0	N	35	0	3
The more northern of the two in the left forefoot	89	50	N	29	0	3
The more southern	88	40	N	28	30	3
At the knee of the right foreleg	89	0	N	36	0	4
The star below the knee	101	10	N	33	30	4
The star on the shoulder	104	0	N	49	0	2
The star on the flanks	105	30	N	44	30	2
The star at the base of the tail	116	30	N	51	0	3
The star in the left hind leg	117	20	N	46	30	2
The more western of the two in the left hind foot	106	0	N	29	38	3
The star to the east of that	107	30	N	28	15	3
[47a] The star in the hollow of the left leg	115	0	N	35	15	4
The more northern of the two which are in the right hind foot	123	10	N	25	50	3

Northern Signs

Constellations	Longitude Deg.	Min.		Latitude Deg.	Min.	Magnitude
The more southern	123	40	N	25	0	3
The first of the three in the tail after the base	125	30	N	53	30	2
The middle star	131	20	N	55	40	2
The star which is last and at the tip of the tail	143	10	N	54	0	2
27 stars: 6 of second magnitude, 8 of third, 8 of fourth, and 5 of fifth						
Unconstellated Stars near the Great Bear						
The star to the south of the tail	141	10	N	39	45	3
The more obscure star to the west	133	30	N	41	20	5
The star between the forefeet of the Bear and the head of the Lion	98	20	N	17	15	4
The star more to the north than that one	96	40	N	19	10	4
The last of the three obscure stars	99	30	N	20	0	obscure
The one to the west of that	95	30	N	22	45	obscure
The one more to the west	94	30	N	23	15	obscure
The star between the forefeet and the Twins	100	20	N	22	15	obscure
8 unconstellated stars: 1 of third magnitude, 2 of fourth, 1 of fifth, 4 obscure						
Draco, or the Dragon						
The star in the tongue	200	0	N	76	30	4
On the jaws	215	10	N	78	30	4 greater
Above the eye	216	30	N	75	40	3
In the cheek	229	40	N	75	20	4
Above the head	233	30	N	75	30	3
The most northern star in the first curve of the neck	258	40	N	82	20	4
The most southern	295	50	N	78	15	4
The star in between	262	10	N	80	20	4
The star to the east of them at the second curve	282	50	N	81	10	4
The more southern star on the western side of the quadrilateral	331	20	N	81	40	4
The more northern star on the same side	343	50	N	83	0	4
The more northern star on the eastern side	1	0	N	78	50	4
The more southern on the same side	346	10	N	77	50	4
The more southern star in the triangle at the third curve	4	0	N	80	30	4
The more western of the other two in the triangle	15	0	N	81	40	5
The star to the east	19	30	N	80	15	5
<The star to the east> in the triangle to the west	66	20	N	83	30	4
The more southern of the remaining two in the same triangle	43	40	N	83	30	4

SIGNORVM STELLARVMQVE DESCRIPTIO CANONICA, ET PRIMO quæ sunt Septentrionalis plagæ.

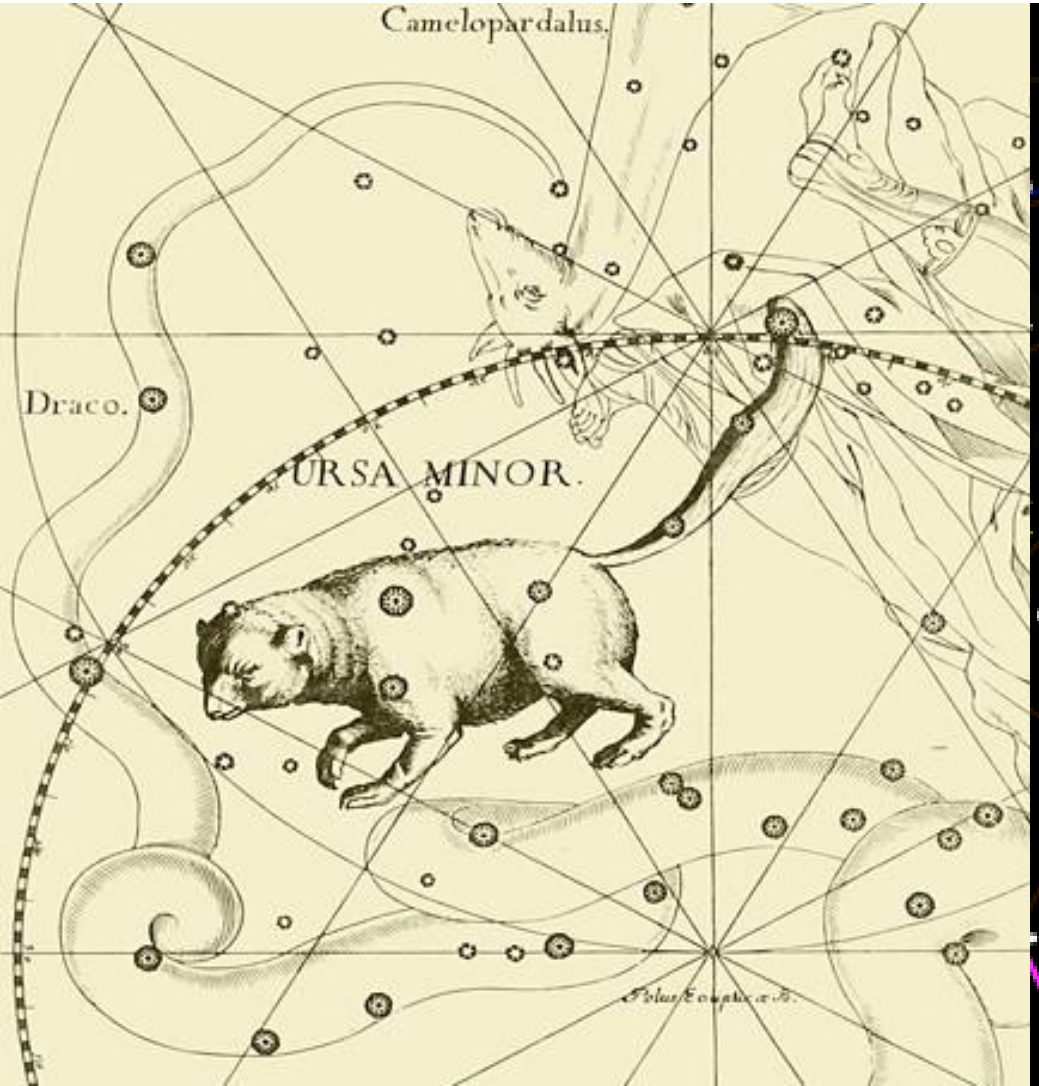
Formæ stellarum	Lõgitudinis partes.	Latitudinis partes	magnitudo
VRSAE MINORIS SIVE CYNOSVRAE.			
In extremo caudæ.	53 1/2	66 0	3
Sequens in cauda.	55 1/2 1/3	70 0	4
In eductione caudæ.	69 1/3	74 0	4
In latere q̃drãguli p̃cedẽte australior	83 0	75 1/3	4
Eiusdem lateris Borea.	87 0	77 1/2 1/6	4
Earũ quæ in latere sequẽte australior	100 1/2	72 1/2 1/6	2
Eiusdem lateris Borea.	109 1/2	74 1/2 1/3	2
Stellæ 7. quarum secudæ magnitudinis 2. tertiæ 1. quartæ 4.			
Et q̃ circa Cynosurã informis in latere sequẽte ad rectã lineã maxĩe aust.	103 1/3	71 1/6	4
VRSÆ MAIORIS QVAM ELICEN VOCANT.			
Quæ in rostro.	78 1/2 1/6	39 1/2 1/3	4
In binis oculis præcedens.	79 1/6	43 0	5
Sequens hanc.	79 1/2 1/6	43 0	5
In fronte duarum præcedens.	79 1/2	47 1/6	5
Sequens in fronte.	81 0	47 0	5
Quæ in dextra auricula præcedente.	81 1/2	50 1/2	5
Duarum in collo antecedens.	85 1/2 1/3	43 1/2 1/3	4
Sequens.	92 1/2 1/3	44 1/3	4
In pectore duarum Borea.	94 1/3	44 0	4
Australior.	93 1/3	42 0	4
In genu sinistro anteriori.	89 0	35 0	3
Duarũ in pede sinistro priori borea.	89 1/2 1/3	29 0	3
Quæ magis ad Austrum.	88 1/2 1/6	28 1/2	3
In genu dextro priori.	89 0	36 0	4
Quæ sub ipso genu.	101 1/6	33 1/2	4
Quæ in humero.	104 0	49 0	2
Quæ in ilibus.	105 1/2	44 1/2	2
Quæ in eductione caudæ.	116 1/2	51 0	3
In sinistro crure posteriore.	117 1/3	46 1/2	2
Duarũ p̃cedẽs in pede sinistro poster.	106 0	29 1/2	3
Sequens hanc.	107 1/2	28 1/6	3

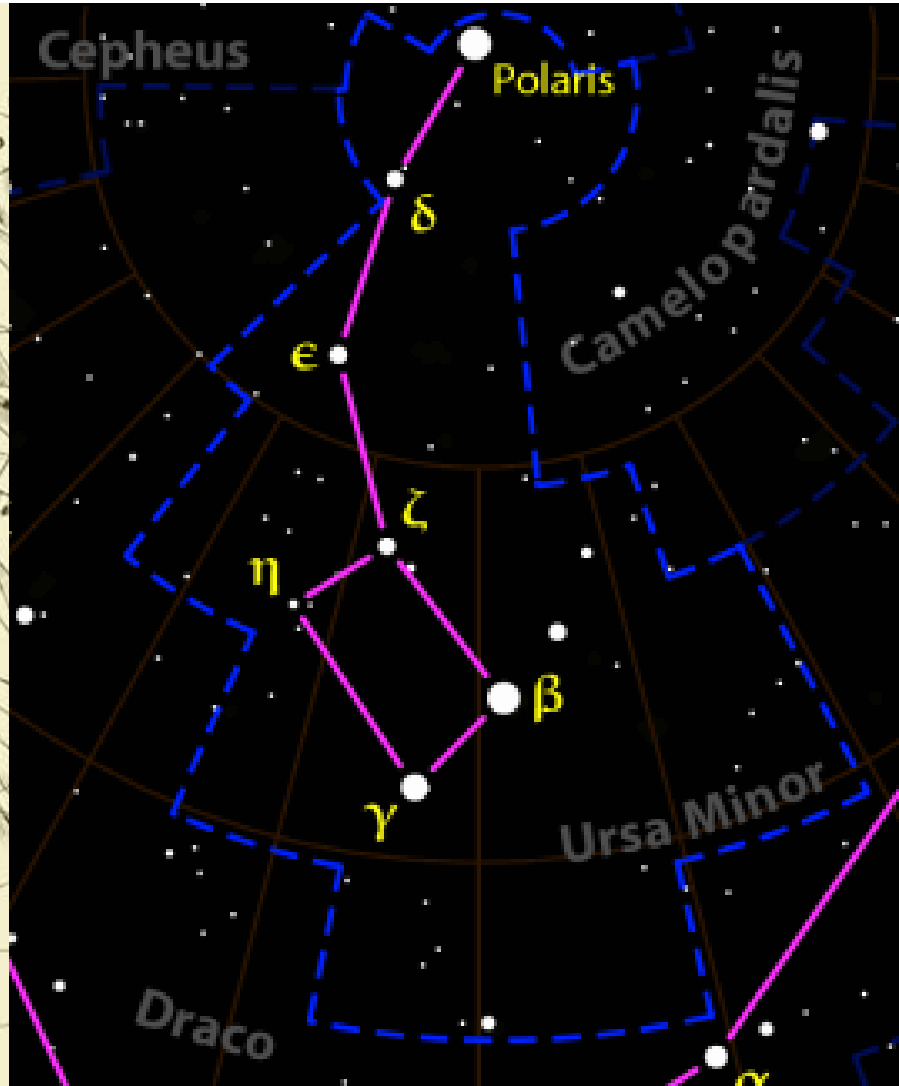
Quæ

BOREAE PLAGAE. Formæ stellarum	Lõgit. partes.	Latit. partes	magnitu.	
VRSAE MAIORIS &c.				
Quæ in sinistra cauitate.	115 0	35 1/2 1/4	4	
Duarũ q̃ in pede dextro posteriore	123 1/6	25 1/2 1/3	3	
Quæ magis ad Austrũ. (Borea.	123 1/2 1/6	25 0	3	
Prima triũ in cauda post eductionẽ.	125 1/2	53 1/2 1/2	2	
Media earum.	131 1/3	55 1/2 1/6	2	
Vltima & in extrema cauda.	143 1/6	54 0	2	
Stellæ 27. quarũ secundæ magnitud. 6. tertiæ 8. quartæ 8. quintæ. 5.				
QVAE CIRCA ELICEN INFORMES.				
Quæ à cauda in Austrum.	141 1/6	39 1/2 1/4	3	
Antecedens hanc obscurior.	133 1/2	41 1/3	5	
Inter ursæ pedes priores, & caput Le	98 1/3	17 1/4	4	
Quæ magis ab hac in boreã. (onis.	96 1/2 1/6	19 1/6	4	
Vltima trium obscurarum.	99 1/2	20 0		obscura
Antecedens hanc.	95 1/3	22 1/2 1/4		obscura
Quæ magis antecedit.	94 1/2	23 1/4		obscura
Quæ intra priores pedes & geminos.	100 1/3	22 1/4		obscura
Informiũ 8. quarũ magnitud. tertiæ 1. quartæ 2. quintæ 1. obscuræ 4				
DRACONIS.				
Quæ in lingua.	200 0	76 1/3	4	
In ore.	215 1/6	78 1/2	4	maior
Supra oculum.	216 1/3	75 1/2 1/6	3	
In gena.	229 1/2 1/6	75 1/2	4	
Supra caput.	233 1/2	75 1/2	3	
In prima colli inflexione Borea.	258 1/2 1/6	82 1/2	4	
Australis ipsarum.	295 1/2 1/3	78 1/4	4	
Media earundem.	262 1/6	80 1/3	4	
Quæ seqũ has ab ortu ĩ cõuersiõe se:	282 1/2 1/3	81 1/6	4	
Austrina lateris p̃cedẽtis q̃drilateri.	331 1/3	81 1/2 1/6	4	
Borea eiusdem lateris.	343 1/2 1/3	83 0	4	
Borea lateris sequentis.	1 0	78 1/2 1/3	4	
Australis eiusdem lateris.	346 1/6	77 1/2 1/3	4	
In inflexiõe tertia australis trianguli	4 0	80 1/2	4	
Reliquarum trianguli præcedens.	15 0	81 1/2 1/6	5	
Quæ sequitur.	19 1/2	80 1/4	5	
In triangulo antecedente trium.	66 1/3	84 1/2	4	
Reliquarũ eiusdẽ trianguli australis.	43 1/2 1/6	83 1/2	4	

m iij Quæ

A szóban forgó Kis Medve csillagkép:





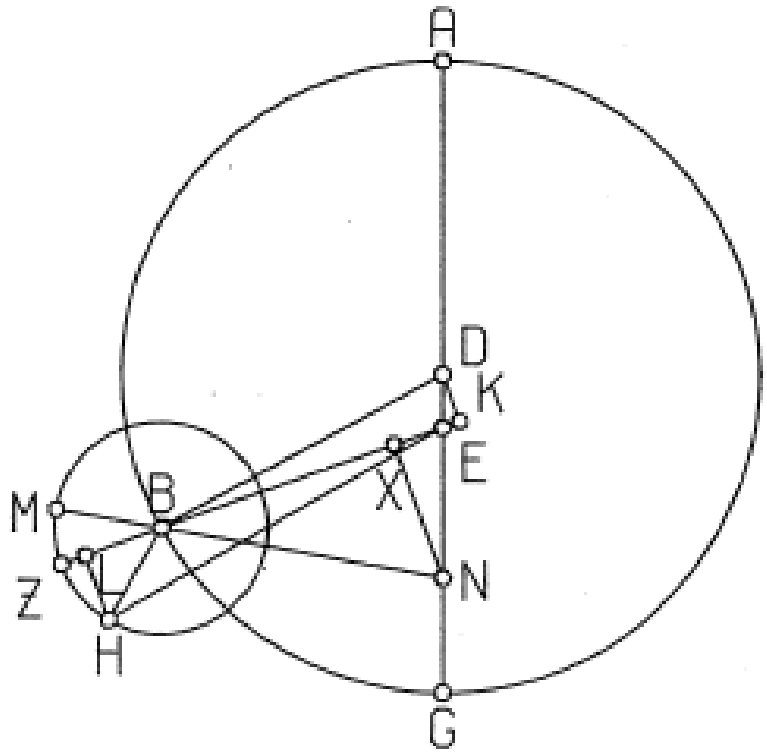
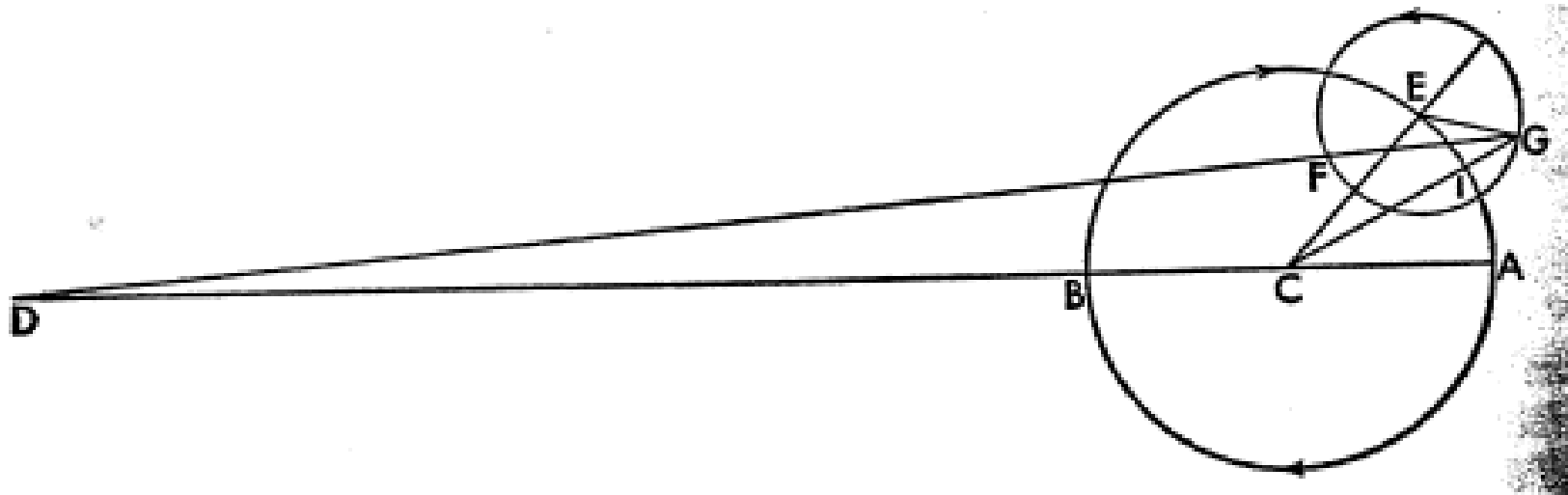


Fig. 5.6

Ptolemaiosz ábrája a Hold pozíció-számításához

Kopernikusz ábrája a Hold pozíció-számításához



Ptolemaiosz: *Almagest*
I. könyv eleje

1. Előszó (filozófiai előkészítés)
2. A felépítés vázlata („tartalomjegyzék”)
3. Az egek gömbként mozognak
4. A Föld mint egész is érezhetően gömb alakú
5. A Föld az egek közepén van
6. A Föld pontként aránylik az egekhez
7. A Föld nem is mozog helyről helyre

…

Kopernikusz: *De Revolutionibus…*
I. könyv eleje

Előszók, ajánlás

Tartalomjegyzék

1. A világ gömb alakú
2. A föld is gömb alakú
3. Hogyan alkot a szárazföld és a víz egy közös gömböt
4. Az égitestek mozgása szabályos körmozgás, vagy azok összetétele
5. Van-e a Földnek körmozgása? A Föld helye
6. Arról, hogy az egek óriásiak
7. Miért hitték régen, hogy a Föld a világ közepén nyugszik
8. Válasz a fenti elégtelen érvekre

…

Ptolemaiosz: *Almagest* – I. könyv
3. Az egek gömbként mozognak

Valószínű, hogy a régiek a következőfajta megfigyelések alapján szereztek először tudomást ezekről a dolgokról. Látták, hogy a Nap, a Hold és más csillagok olyan körökön mozognak keletről nyugatra, amelyek mindig párhuzamosak egymással, és úgy tűnt, mintha a föld alól előbukkanva fokozatosan felemelkednének, majd hasonlóan körbemennének és aztán ereszkednének, mígnem úgymond a földre zuhannak és tejesen eltűnnek, hogy aztán egy ideig tartó láthatatlanság után megújulva ismét felkeljenek. És látták, hogy mind a mozgások periódusa, mind a kelések és nyugvások helye, egészében véve rögzített és ugyanaz…

(…)

Összefoglalva, ha egy gömb forgásán kívül bármilyen más mozgást feltételezünk az égitestek számára, akkor szükségszerűen következik, hogy a Földtől mért távolságuknak változnia kell, akárhol és akárhogyan helyezkedjék is el a Föld. Így a megfigyelő számára úgy kell, hogy tűnjék, hogy a csillagok mérete és kölcsönös távolsága ugyancsak változik a körülfordulás ideje alatt, mivel egyszer közelebb kell lenniük hozzá, máskor pedig távolabb. Ám semmiféle ilyen változást nem látunk…

(…)

Továbbá, bizonyos fizikai megfontolások is ugyanerre a nézetre vezetnek. Például: minden test közül az éter az, amelynek összetevő részei a legfinomabbak, és a leginkább hasonlítanak egymásra. Nos, a hasonló részekkel rendelkező testeket hasonló részekkel rendelkező felszínek veszik körül, ám az összes felszín közül hasonló részekkel csak a kör (a síkban) és a gömb (a térben) rendelkezik. Mivel az éter nem síkban van, hanem térben, ezért következik, hogy gömb alakúnak kell lennie…

Kopernikusz: *De Revolutionibus…* – I. könyv
3. A világ gömb alakú

Kezdetben észre kell vennünk, hogy a világ gömb alakú. Ez vagy azért van, mert ez az alak a legtökéletesebb mind között, hiszen teljes egész és nincsenek benne törések, vagy mert ennek az alaknak a legnagyobb a térfogata, és így különösen alkalmas arra, hogy magába foglaljon minden dolgot, vagy mert az elkülönült részei, vagyis a Nap, a Hold és a csillagok szintén ilyen alakúnak látszanak, vagy mert a világon minden ezt az alakot igyekszik felvenni, amikor magára van hagyva, mint az a vízcseppek és egyéb folyadékok esetén látható. Így senki sem habozna kijelenteni, hogy az égboltnak is ilyen az alakja.

Ptolemaiosz: *Almagest* – I. könyv
4. A Föld egésze is érezhetően gömb alakú

… Látjuk, hogy a Nap, a Hold és más csillagok nem mindenki számára ugyanakkor kelnek fel, hanem a keletiek számára korábban, a nyugatiak számára pedig később. Ugyanis úgy találjuk, hogy a fogyatkozási jelenségeket, és különösen a holdfogyatkozást, ami mindenki számára ugyanakkor történik, a különböző megfigyelések különböző időpontokra (vagyis délhez képest nem egyenlő távolságra) teszik. Ehelyett a keletiek által rögzített időpont mindig későbbi, mint a nyugatiak által megfigyelt. Úgy találjuk, hogy az időpontkülönbségek arányosak a megfigyelési helyek közti távolsággal. Így ésszerű arra következtetni, hogy a Föld gömbölyű, hiszen egy egyenletesen görbülő felület (amikor egészében tekintjük) szabályosan takar ki rendre a megfigyelők számára.

(…)

Henger alakú sem lehet, kelet-nyugat irányú görbülettel, és a világ pólusai felé mutató lapos oldalakkal, mint ahogy néhányan hihetőnek találnák. Ez világos a következőkből: a görbült felületen élők számára egyetlen csillag sem lenne mindig látható, hanem mindig felkelnének és lenyugodnának, míg más csillagok, egyenlő távolságra a pólusoktól, sosem lennének láthatók. Valójában viszont minél északabbra utazunk, annál inkább eltűnnek a déli csillagok, és annál jobban feltűnnek az északiak. Így aztán világos, hogy a Föld görbülete szabályosan takar ki a megfigyelők számára észak-déli irányban, és ez bizonyítja a Föld gömb alakját minden irányban.

Van egy további megfontolás is, mely szerint ha hegyek vagy magas helyek felé hajózunk, bárhonnan és bárhová, akkor úgy látjuk, hogy fokozatosan növelik méretüket, mintha csak kibukkannának a tengerből, melynek mélyén addig tartózkodtak: ezt pedig a víz felszínének görbülete okozza.

Ptolemaiosz: *Almagest* – I. könyv
6. A Föld úgy aránylik az egekhez, mint egy pont

… Ennek erős bizonyítéka, hogy a csillagok mérete és távolsága minden időben egyenlőnek és ugyanakkorának tűnik a Föld bármely pontjáról…

(…)

Egy másik tiszta bizonyíték az, hogy a Föld bármely pontján a megfigyelő látóvonalán átmenő sík, melyet horizontnak nevezünk, mindig félbevágja az égbolt egészét. Ez nem történhetne meg, ha a Föld mérete észlelhető nagyságú lenne az égitestek távolságához képest, mert ekkor csupán a Föld középpontján átmenő sík vághatná félbe az égboltot, míg a felszín bármely pontján átmenő sík alul egy nagyobb és felül egy kisebb részt vágna ki az égből.

Ptolemaiosz: *Almagest* – I. könyv
7. A Föld nem mozog helyről helyre sem

Megmutatható az eddigi érvekkel, hogy a Föld nem mozoghat egyik említett irányba sem, sőt nem mozdulhat középpont-béli helyzetéből. Ugyanis ha bárhol máshol lenne, mint középen, a már említett jelenségek jelentkeznének…

(…)

…A Földnek, amely tehát gömb alakú és a világegyetem közepén van, akármelyik részén igaz, hogy a súllyal rendelkező testek mozgásának – vagyis valódi, természetes mozgásának – iránya mindig és mindenütt merőleges arra a síkra, amely a becsapódás pontjában húzható…

(…)

Ám némelyek úgy gondolják, hogy meggyőzőbb nézettel állnak elő, amikor a fentiekkel egyetértenek, hiszen semmit sem hozhatnak fel ellene, de szerintük nincs bizonyíték azon feltételezésük ellen sem, hogy az égbolt mozdulatlan, és a Föld nyugatról keletre forog ugyanazon tengely körül, és naponta körülbelül egyszer megfordul… De nem veszik észre, hogy bár talán semmi nem mond ennek ellent az égi jelenségek közül, legalábbis ami az egyszerűbb megfontolásokat illeti, abból, ami itt a Földön és a levegőben történik, látható, hogy egy ilyen elképzelés nevetséges…

Kopernikusz: *De Revolutionibus…* – I. könyv

4. Az égitestek mozgása szabályos, örök körmozgás, vagy ilyen mozgások összetétele

(…)

Be kell tehát vallanunk, hogy ezek a mozgások körkörösek, vagy körmozgásokból tevődnek össze, amennyiben a mondott szabálytalanságokat egy állandó törvény szerint periodikus visszatérésekkel tapasztaljuk, ez azonban nem lehetne így, ha nem lennének körmozgások. Ugyanis csak a kör képes arra, hogy visszahozza azt, ami már elmúlt, és így aztán a Nap körkörös mozgásokból összetett mozgásával például visszahozza nekünk a nappalok egyenlőtlenségét vagy a négy évszakot…

Kopernikusz: *De Revolutionibus…* – I. könyv
5. Rendelkezik-e a Föld körkörös mozgással? És mi a helye?

… Ugyanis minden látható helyváltozás előfordulása vagy a látott dolog, vagy a megfigyelő mozgására vezethető vissza, vagy pedig kettőjük szükségszerűen egyenlőtlen mozgására…

(…)

… És mivel az égbolt tartalmaz és ölel körbe mindeneket a világon, nem világos, hogy miért ne kellene a mozgást inkább a tartalmazottnak tulajdonítanunk, nem pedig a tartalmazónak, más szóval inkább a hellyel rendelkezőnek, és nem a helyet szolgáltatónak…

(…)

Mert ha valaki tagadja, hogy a Föld a világ közepén vagy centrumában van, és ugyanakkor nem ismeri el, hogy kettejük távolsága elég nagy ahhoz, hogy mérhető legyen az állócsillagok távolságához, mégis úgy gondolja, hogy e távolság láthatóan nagy a Nap és a bolygók pályaköreinek viszonylatában, és ha ezek után azt gondolja, hogy a testek mozgása azért tűnik szabálytalannak, mert más középpont körül vannak elrendezve, mint a Föld középpontja, akkor talán képessé válik arra, hogy előhozakodjon egy tökéletesen elfogadható magyarázattal a szabálytalannak látszó mozgásra. Ugyanis az a tény, hogy a bolygók néha közelebbinek tűnnek a Fölhöz lenni, máskor pedig távolabbinak, szükségszerűen azt sugallja, hogy a Föld középpontja nem a mozgásuk középpontja. De még nem világos, hogy vajon a Föld közeledik hozzájuk és távolodik tőlük, vagy pedig ők közelednek és távolodnak.