Does special relativity theory tell us anything new about space and time?

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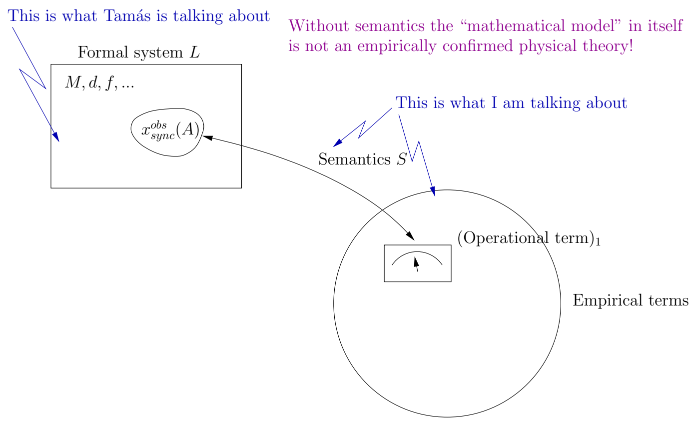
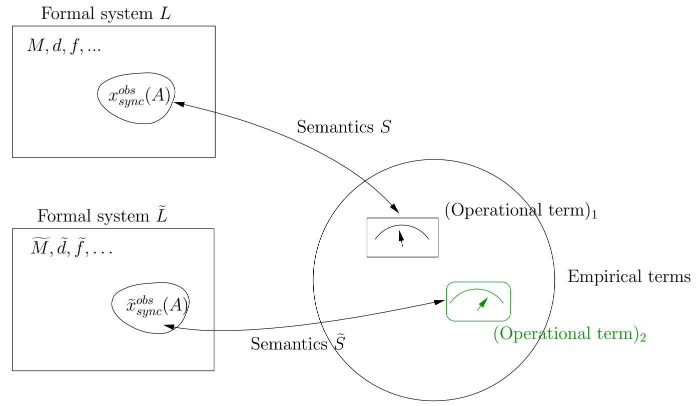
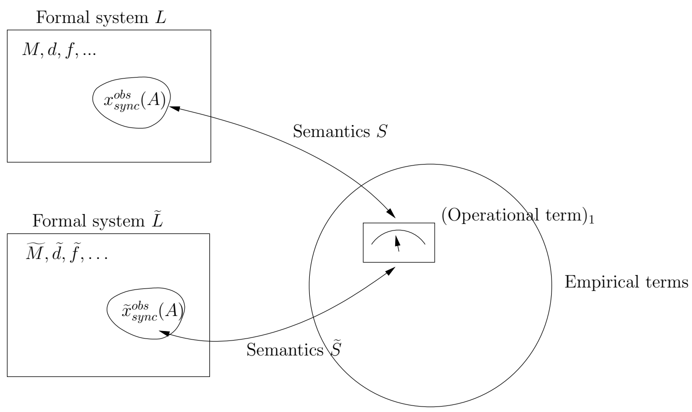
Methodological Preamble

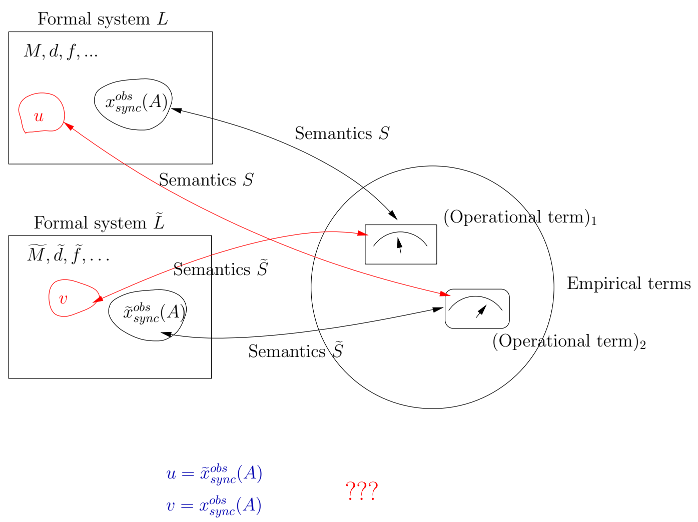


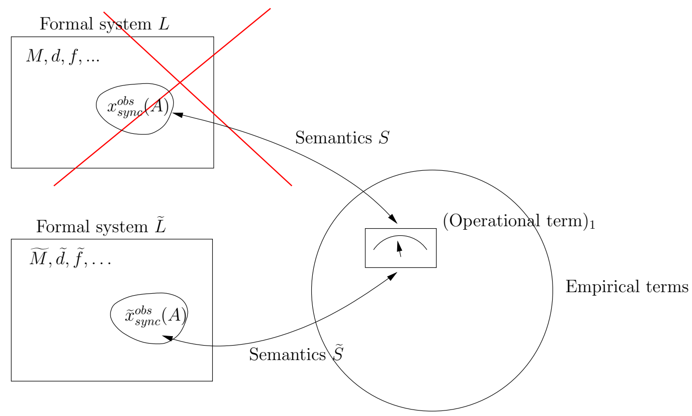


The situation is, however, completely different if $\tilde{x}^{obs}_{sync}(A)$ and $x^{obs}_{sync}(A)$ have different empirical meanings!



Assume that both (L, S) and $(\tilde{L}, \tilde{S})$ are intended to be full and faithful descriptions of physical reality.





If $\tilde{x}^{obs}_{sync}(A) \neq x^{obs}_{sync}(A)$ then one of the models (L, S) $(\tilde{L}, \tilde{S})$ is not faithful

Empirical definition of space and time tags

Step 1

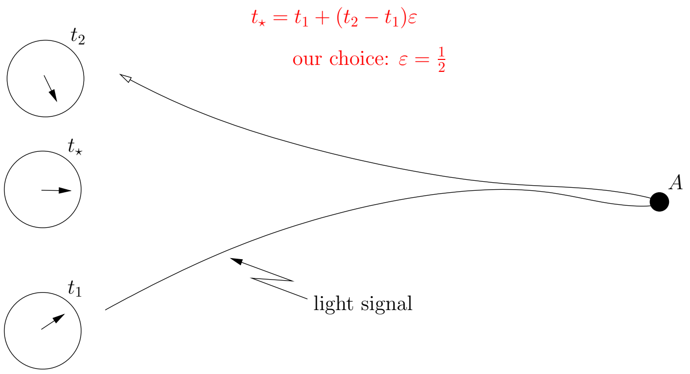
We chose one single "clock" in the universe.

time := the reading of this clock

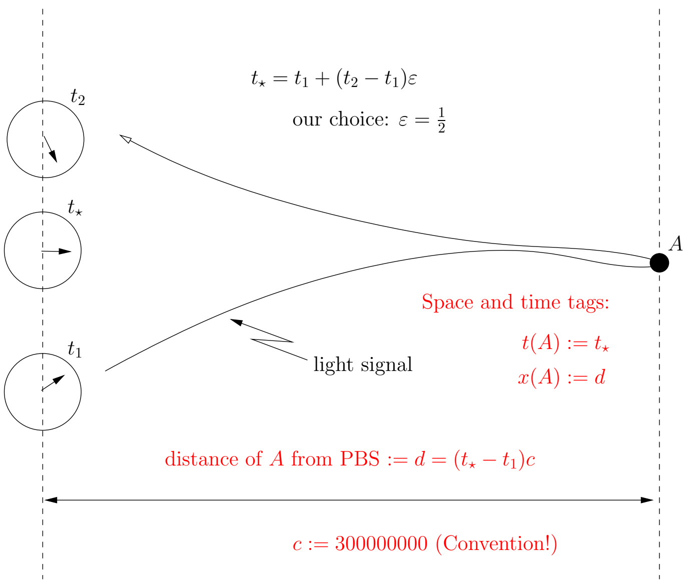
(Say, our choice is the clock floating in the universe together with the Paris Bureau of Standards (PBS).)

Step 2

We define synchronization in the following way (convention!):



Step 3

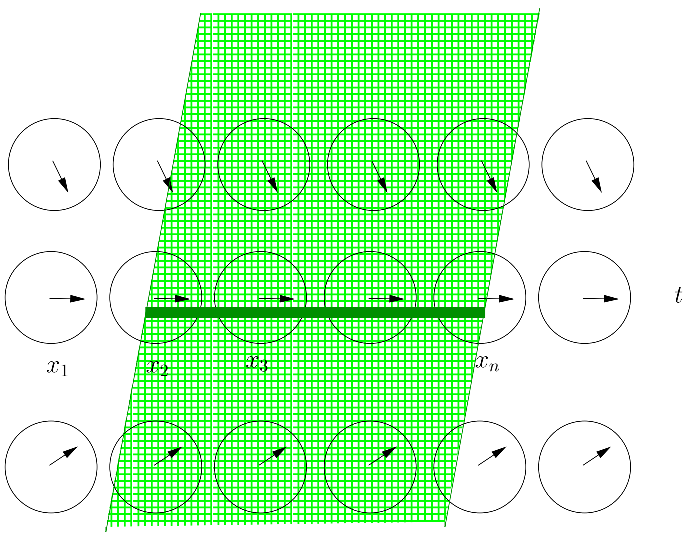


Step 4

We can place everywhere a copy of the standard clock synchronized with the one at PBS, which are at rest relative to PBS.

Step 5

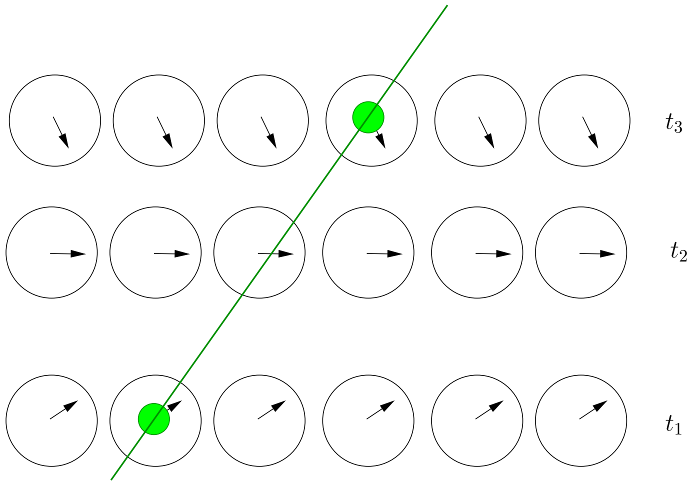
The length of a rod:



When the local clocks show time t the two assistants at the ends of the rod read off the space tags of the local events. This method is independent of whether the rod is moving or is at rest relative to PBS!

Step 6

Similarly, we can measure the time interval between two strokes of a clock-like process:



And this method also is independent of whether the object in question is moving or is at rest relative to PBS!

Step 7

If I do something with the rod (say I am cooling it down), then

$$\text{length}_{earlier} > \text{length}_{later}$$

By definition, I say that the rod has been contracted! This contraction is an objective deformation of the rod.

Step 8

Questions:

1. Does the length of a rigid rod change if we change its velocity?
2. Doest the phase of a clock change if we change its velocity?

These are intelligent questions, and one can answer them by performing experiments!

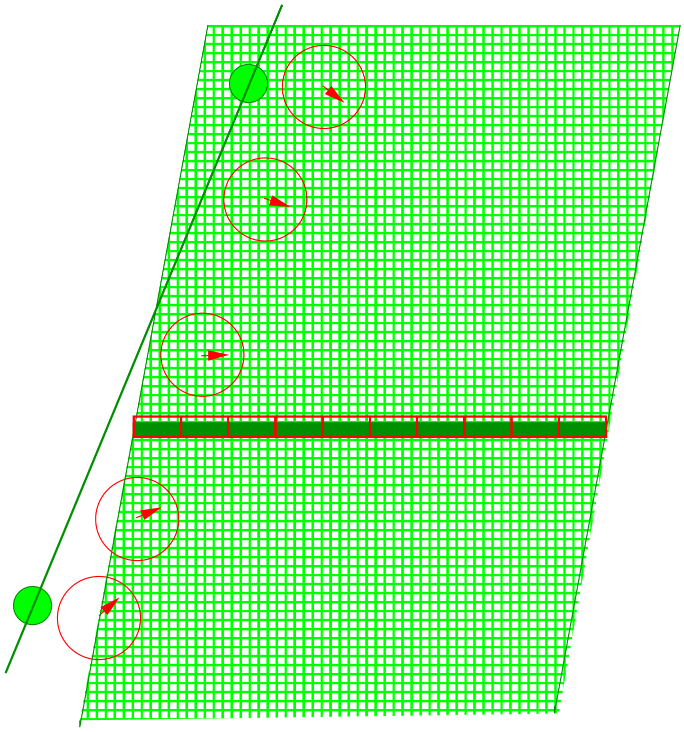
Step 9

Before the end of 19th century all experimental results supported the hypothesis that

1. The length of a rigid rod does not change if we change its velocity?
2. The phase of a clock does not change if we change its velocity?

Step 10

Consequently, we can measure the time interval and the distance in an alternative way:



Step 11

On this basis, it is meaningful to introduce the concept of space and time tags assigned by an observer K' moving at velocity v relative to the observer K at rest in PBS, using co-moving clocks and co-moving measuring rods. It follows from the non-deformation of clocks and rods that

$$\begin{aligned} t^{K'}(A) &= t^K(A) \\ x^{K'}(A) &= x^K(A) - vt^K(A) \end{aligned}$$

Step 12

Later experimental findings:

1. The phase of a clock changes when we change its velocity. If the period of the clock is T_0^K when it is at rest relative to the PBS, then its period is $T^K = \frac{T_0^K}{\sqrt{1-\frac{v^2}{c^2}}}$ when it is moving at velocity v relative to the PBS.
2. The length of a rigid rod changes if we change its velocity. If the length of the rod is l_0^K when it is at rest relative to PBS, then its length is $l^K = l_0^K\sqrt{1-\frac{v^2}{c^2}}$ when it is moving at velocity v relative to the PBS.

Remarks

- Does the *etalon* clock in PBS play a privileged role?
  –Yes!
- Does it mean that we are the center of the universe?
  – No! We are not the center of the universe but we are the center of our language!
- Do the meanings of the terms "time", "distance", "simultaneous", "contraction", "dilatation", "moving", "rest", etc. depend on the choice of the *etalon* clock?
  –Yes!
- Do the meanings of the terms "time", "distance", "simultaneous", "contraction", "dilatation", "moving", "rest", etc. depend on the choice of the value of ε and other details of the synchronization?
  –Yes!
- It it possible that "dilatation" in the language of one (etalon, synchronization) means "contraction" in the language of other (etalon, synchronization)?
  –Yes!
- Does it mean that the contraction of the rod is not an objective physical deformation?
  –No! A nod in Bulgarian means "No"! In English, "Yes". Is your Bulgarian girlfriend's / boyfriend's nod an objective "No"? –Yes!

Step 13

Taking into account these deformations, there are two possible philosophies:

The classical (Lorentz, FitzGerald) views When we are measuring with clocks and rods moving relative to the PBS, we obtain improper results. Therefore we have to make corrections taking into account the deformations of the measuring equipments. The final result can be expressed in terms of t^K, x^K:

$$\begin{aligned} t^{K'}(A) &= t^K(A) \\ x^{K'}(A) &= x^K(A) - vt^K(A) \end{aligned}$$

The "relativistic" views We do not take into account the deformations of the measuring equipments, and define the space and time tags to be equal to the readings of the moving (therefore deformed) measuring equipments, without corrections.

$$\widetilde{t}^{K'}(A) = \frac{t^K(A) - \frac{v\,x^K(A)}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (1)$$

$$\widetilde{x}^{K'}(A) = \frac{x^K(A) - v\,t^K(A)}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (2)$$

Thus, we have two different definitions of "space and time" tags in the system of the moving observer K':

$$x^K(A) \equiv \widetilde{x}^K(A) \qquad (3)$$
$$t^K(A) \equiv \widetilde{t}^K(A) \qquad (4)$$
$$x^{K'}(A) \not\equiv \widetilde{x}^{K'}(A) \qquad (5)$$
$$t^{K'}(A) \not\equiv \widetilde{t}^{K'}(A) \qquad (6)$$

where $\equiv$ denotes the identical operational definition. In spite of the different operational definitions, it could be a *contingent* fact of nature that $x^{K'}(A) = \widetilde{x}^{K'}(A)$ and/or $t^{K'}(A) = \widetilde{t}^{K'}(A)$ for every event A. But a little reflection reveals that this is not the case. It follows from special relativity that $\widetilde{x}^K(A), \widetilde{t}^K(A)$ are related with $\widetilde{x}^{K'}(A), \widetilde{t}^{K'}(A)$ through the Lorentz transformation, while $x^K(A), t^K(A)$ are related with $x^{K'}(A), t^{K'}(A)$ through the corresponding Galilean transformation, therefore, taking into account identities (3)–(4), $x^{K'}(A) \neq \widetilde{x}^{K'}(A)$ and $t^{K'}(A) \neq \widetilde{t}^{K'}(A)$, if $v \neq 0$.

Thus, our first partial conclusion is that *different physical quantities are called "space" tag, and similarly, different physical quantities are called "time" tag in special relativity and in classical physics.*[a] In order to avoid further confusion, from now on "space" and "time" tags will mean the physical quantities defined according to classical physics—, and "space" and "time" in the sense of the relativistic definitions will be called $\widetilde{\text{space}}$ and $\widetilde{\text{time}}$.

[a]This was first recognized by Bridgeman (1927, p. 12), although he did not investigate the further consequences of this fact.

Special relativity does not tell us anything new about space and time

Classical physics and relativity theory would be different theories of space and time if they accounted for physical quantities x and t differently. If there were any event A and any inertial frame of reference $K^\star$ in which the space or time tag assigned to the event by special relativity, $\left[x^{K^\star}(A)\right]_{relativity}$, $\left[t^{K^\star}(A)\right]_{relativity}$, were different from the similar tags assigned by classical physics, $\left[x^{K^\star}(A)\right]_{classical}$, $\left[t^{K^\star}(A)\right]_{classical}$. If, for example, there were any two events simultaneous in relativity theory which were not simultaneous according to classical physics, or vice versa—to touch on a sore point.

But a little reflection shows that this is not the case. All assertions of special relativity about the space and time tags $x^{K'}(A)$ and $t^{K'}(A)$ are the same as the corresponding assertions of classical physics. To see this we can utilize the operational identities (3)–(4) and express everything through, say, x^K and t^K.

According to the empirical definition of t and x,

$$\begin{aligned}\left[x^{K'}(A)\right]_{relativity} &= \widetilde{x}^K(A) - \widetilde{v}^K(K')\widetilde{t}^K(A)\\ &= x^K(A) - v^K(K')t^K(A) = \left[x^{K'}(A)\right]_{classical}\end{aligned}$$

Similarly,

$$\left[t^{K'}(A)\right]_{relativity} = \widetilde{t}^K(A) = t^K(A) = \left[t^{K'}(A)\right]_{classical}$$

Lorentz theory and special relativity are completely identical theories

Since Lorentz theory adopts the classical theory of spacetime, it does not differ from special relativity in its assertions about space and time. However, beyond what special relativity claims about space and time, it also has another claim about $\widetilde{\text{space}}$ and $\widetilde{\text{time}}$, that is, about "the physical behavior of measuring-rods and clocks"—in Einstein's words. Now we show that the two theories have identical assertions about $\widetilde{x}$ and $\widetilde{t}$, that is,

$$\begin{aligned}\left[\widetilde{x}^{K'}(A)\right]_{relativity} &= \left[\widetilde{x}^{K'}(A)\right]_{LT}\\ \left[\widetilde{t}^{K'}(A)\right]_{relativity} &= \left[\widetilde{t}^{K'}(A)\right]_{LT}\end{aligned}$$

According to relativity theory, the $\widetilde{\text{space}}$ and $\widetilde{\text{time}}$ tags in K' and in K are related through the Lorentz transformations. From (3)–(4) one can deduce:

$$\left[\widetilde{t}^{K'}(A)\right]_{relativity} = \frac{t^K(A) - \frac{v\,x^K(A)}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \tag{7}$$

$$\left[\widetilde{x}^{K'}(A)\right]_{relativity} = \frac{x^K(A) - v\,t^K(A)}{\sqrt{1-\frac{v^2}{c^2}}} \tag{8}$$

Taking the assumptions of Lorentz theory that the standard clock slows down by factor $\sqrt{1-\frac{v^2}{c^2}}$ and that a rigid rod suffers a contraction by factor $\sqrt{1-\frac{v^2}{c^2}}$ when they are gently accelerated from K to K', one can *directly* calculate the $\widetilde{\text{space}}$ tag $\widetilde{x}^{K'}(A)$ and the $\widetilde{\text{time}}$ tag $\widetilde{t}^{K'}(A)$. First, let us calculate the reading of the clock slowly transported in K' from the origin to the locus of an event A. Again, we will take into account the identities (3)–(4). The clock is moving with a varying velocity[a]

$$v_C^K(t^K) = v + w^K(t^K)$$

where $w^K(t^K)$ is the velocity of the clock relative to K', that is, $w^K(0) = 0$ when it starts at $x_C^K(0) = 0$ (as we assumed, $t^K = 0$ and the transported clock shows 0 when the origins of K and K' coincide) and $w^K(t_1^K) = 0$ when the clock arrives at the place of A. The reading of the clock at the time t_1^K will be

$$T = \int_0^{t_1^K} \sqrt{1 - \frac{\left(v + w^K(t)\right)^2}{c^2}}\, dt \tag{9}$$

Since w^K is small we may develop in powers of w^K, and we find from (9) when neglecting terms of second and higher order

$$T = \frac{t_1^K - \frac{\left(t_1^K v + \int_0^{t_1^K} w^K(t)\,dt\right)v}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{t^K(A) - \frac{x^K(A)v}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \tag{10}$$

(where, without loss of generality, we take $t_1^K = t^K(A)$). Thus, according to the definition of $\widetilde{t}$, we have

$$\left[\widetilde{t}^{K'}(A)\right]_{LT} = \frac{t^K(A) - \frac{v\,x^K(A)}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

which is equal to $\left[\widetilde{t}^{K'}(A)\right]_{relativity}$ in (7).

Now, taking into account that the length of the co-moving meter stick is only $\sqrt{1-\frac{v^2}{c^2}}$, the distance of event A from the origin of K is the following:

$$x^K(A) = t^K(A)v + \widetilde{x}^{K'}(A)\sqrt{1-\frac{v^2}{c^2}}$$

and thus

$$\left[\widetilde{x}^{K'}(A)\right]_{LT} = \frac{x^K(A) - v\,t^K(A)}{\sqrt{1-\frac{v^2}{c^2}}} = \left[\widetilde{x}^{K'}(A)\right]_{relativity}$$

This completes the proof. The two theories are completely identical.

[a]For the sake of simplicity we continue to restrict our calculation to one space dimension. For the general calculation of the phase shift suffered by moving clocks, see Jánossy 1971, pp. 142–147.

Consequently, there is full agreement between the Lorentz theory and special relativity theory in the following statements:

- $\widetilde{\text{Velocity}}$—which is called "velocity" by relativity theory—is not an additive quantity,

$$\widetilde{v}^{K'}(K''') = \frac{\widetilde{v}^{K'}(K'') + \widetilde{v}^{K''}(K''')}{1 + \frac{\widetilde{v}^{K'}(K'')\widetilde{v}^{K''}(K''')}{c^2}}$$

  while velocity—that is, what we traditionally call "velocity"—is an additive quantity,

$$v^{K'}(K''') = v^{K'}(K'') + v^{K''}(K''')$$

  where K', K'', K''' are arbitrary three frames. For example,

$$v^{K'}(light\,signal) = v^{K'}(K'') + v^{K''}(light\,signal)$$

- The $\left(\widetilde{x}_1, \widetilde{x}_2, \widetilde{x}_3, \widetilde{t}\right)$-map of the world can be conveniently described through a Minkowski geometry, such that the "$\widetilde{t}$-simultaneity" can be described through the orthogonality with respect to the 4-metric of the Minkowski space, etc.

- The (x_1, x_2, x_3, t)-map of the world, can be conveniently described through a traditional spacetime geometry like $E^3 \times E^1$.

- The velocity of light is not the same in all inertial frames of reference.

- The $\widetilde{\text{velocity}}$ of light is the same in all inertial frames of reference.

- Time and distance are invariant, the reference frame independent concepts, $\widetilde{\text{time}}$ and $\widetilde{\text{distance}}$ are not.

- t-simultaneity is an invariant, frame-independent concept, while $\widetilde{t}$-simultaneity is not.

  ⋮

Finally, note that in an arbitrary inertial frame K' for every event A the tags $x_1^{K'}(A)$, $x_2^{K'}(A)$, $x_3^{K'}(A)$, $t^{K'}(A)$ can be expressed in terms of $\widetilde{x}_1^{K'}(A)$, $\widetilde{x}_2^{K'}(A)$, $\widetilde{x}_3^{K'}(A)$, $\widetilde{t}^{K'}(A)$ and vice versa. Consequently, we can express the laws of physics—as is done in special relativity—equally well in terms of the variables $\widetilde{x}_1, \widetilde{x}_2, \widetilde{x}_3, \widetilde{t}$ instead of the space and time tags x_1, x_2, x_3, t. On the other hand, we should emphasize that the one-to-one correspondence between $\widetilde{x}_1, \widetilde{x}_2, \widetilde{x}_3, \widetilde{t}$ and x_1, x_2, x_3, t also entails that *the (relativistic) laws of physics can be equally well expressed in terms of the (traditional) space and time tags* x_1, x_2, x_3, t *instead of the variables* $\widetilde{x}_1, \widetilde{x}_2, \widetilde{x}_3, \widetilde{t}$.