Deconstructing superposition

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1. Preliminary

- The terminology of quantum mechanics was heavily shaped by the specific intellectual climate of the Weimar Germany (Forman thesis).
- It is better to take its phraseology with a grain of salt.
2. Two suggestions

- Don’t use the word “system.” Rather decide whether you speak about an individual particle or an ensemble of particles.

- Don’t use the word “observable.” Rather decide whether you speak about a real-world physical measurement or its formal representation by an operator.
Contrary to the Copenhagen interpretation the notion of the state refers to an ensemble.

Specifically, the trace formula should be interpreted as the conditional probability of obtaining a certain outcome given a certain measurement is performed on an ensemble in a given state all represented by appropriate self-adjoint operators.

That states refer to ensembles is independent of the fact that in quantum mechanics “a measurement can change the state of the system” or whether there exist hidden elements of reality or not.
A state can be pure or mixed.

Mathematically, a state is pure if it is an extremal point of the state space.

Then there is a self-adjoint operator yielding dispersion-free measurement statistics in the state.

Whether a state is pure, cannot be decided by using only one operator; to this aim one needs a certain number of operators (called the quorum in quantum tomography).
Physically, a state is pure if it is a real-world physical measurement which is dispersion-free in that state. 

Namely, not all operators represent a measurement.
**Excursus: a toy-model**

- Consider a box filled with balls. Denote the preparation of the box by $s$.

- Three measurements:
  - $a$: Color measurement
  - $b$: Size measurement
  - $c$: Shape measurement

- Two possible outcomes:
  - $A^+$: Black
  - $A^-$: White
  - $B^+$: Large
  - $B^-$: Small
  - $C^+$: Round
  - $C^-$: Oval
Excursus: a toy-model

Conditional probabilities:

\[ p_a^{\pm} := p_s(A^{\pm}|a) \]
\[ p_b^{\pm} := p_s(B^{\pm}|b) \]
\[ p_c^{\pm} := p_s(C^{\pm}|c) \]
Excursus: a toy-model

Quantum representation of the model:

<table>
<thead>
<tr>
<th>System</th>
<th>$\mathcal{H}_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Color: $a$</td>
<td>$O_a = a\sigma$</td>
</tr>
<tr>
<td>Size: $b$</td>
<td>$O_b = b\sigma$</td>
</tr>
<tr>
<td>Shape: $c$</td>
<td>$O_c = c\sigma$</td>
</tr>
<tr>
<td>Black/White: $A^\pm$</td>
<td>$P_a^\pm = \frac{1}{2}(1 \pm a\sigma)$</td>
</tr>
<tr>
<td>Large/Small: $B^\pm$</td>
<td>$P_b^\pm = \frac{1}{2}(1 \pm b\sigma)$</td>
</tr>
<tr>
<td>Round/Cubic: $C^\pm$</td>
<td>$P_c^\pm = \frac{1}{2}(1 \pm c\sigma)$</td>
</tr>
<tr>
<td>State: $s$</td>
<td>$W_s = \frac{1}{2}(1 + s\sigma)$</td>
</tr>
</tbody>
</table>

where $a, b, c \in \mathbb{R}^3$, $|a| = |b| = |c| = 1$ and $\sigma = (\sigma_x, \sigma_y, \sigma_z)$. 
Excursus: a toy-model

Born rule:

\[ p^\pm_a = \frac{1}{2}(1 \pm sa) \]
\[ p^\pm_b = \frac{1}{2}(1 \pm sb) \]
\[ p^\pm_c = \frac{1}{2}(1 \pm sc) \]
Excursus: a toy-model
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- Measurement-operator assignment:
  \[ \{a, b, c\} \mapsto \{a, b, c\} \]

- Proposition:
  \[ p^\pm_a, p^\pm_b, p^\pm_c \quad \& \quad a, b, c \implies W_s \]

- Specifically:
  \[ s = \frac{(p^+_a - \frac{1}{2})(b \times c) + (p^+_b - \frac{1}{2})(c \times a) + (p^+_c - \frac{1}{2})(a \times b)}{a \cdot (b \times c)} \]
Excursus: a toy-model
Physically, a state is pure if it is a real-world physical measurement which is dispersion-free in that state.

Namely, not all operators represent a measurement.
Mathematically, a state is a superposition if it is not an eigenstate of the operator under investigation but there is another operator for which it is.

Physically, a state is a superposition if it is not dispersion-free for the actual measurement but there is another measurement for which it would be.

When in quantum mechanics we say that the system is in superposition then this means that in the actual statistical ensemble the actual measurement has dispersion; however, there is another measurement which would be dispersion-free if performed on the same ensemble.
7. Consequences

- With this reading of superposition many of the problems of quantum mechanics just disappear:

  - Measurement problem: due to linearity the joint object-apparatus system will be in a superposition state
  - Schrödinger’s cat: how can a macroscopic system be in a superposition?
  - Collapse: evolution is non-unitary during measurement
  - Wigner’s friend: the collapse is evoked by consciousness
  - Decoherence: superposition vanishes by the system’s interaction with the environment
  - Many-worlds, many-minds: unitary dynamics is ubiquitous; all components of the superposition are actual