Bell’s local causality for philosophers

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Bell’s notion of **local causality** presupposes a **framework** integrating probabilistic and spatiotemporal entities.

Our aim is to develop such a framework called **local physical theory**.
I. What is a local physical theory?
II. Bell’s local causality in a local physical theory
III. Local causality and the Bell inequalities
I. What is a local physical theory?
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Discretized Minkowski spacetime:
Definition. A local physical theory (LPT) is a net associating (von Neumann) algebras to spacetime regions which satisfies

1. isotony,
2. microcausality,
3. covariance.
I. What is a local physical theory?

- **Isotony:** if $V_1 \subset V_2$, then $\mathcal{N}(V_1)$ is a unital subalgebra of $\mathcal{N}(V_2)$
I. What is a local physical theory?

- Microcausality (Einstein causality): \([\mathcal{N}(V_A), \mathcal{N}(V_B)] = 0\)
I. What is a local physical theory?

**Covariance:** covariant group homomorphism on the net
II. Bell’s local causality in a LPT
“A theory will be said to be locally causal if the probabilities attached to values of local beables in a space-time region $V_A$ are unaltered by specification of values of local beables in a space-like separated region $V_B$, when what happens in the backward light cone of $V_A$ is already sufficiently specified, for example by a full specification of local beables in a space-time region $V_C$.” (Bell, 1990/2004, p. 239-240)
II. Bell’s local causality in a LPT

Basic terms:

1. “The beables of the theory are those entities in it which are, at least tentatively, to be taken seriously, as corresponding to something real.”

2. “there are things which do go faster than light. British sovereignty is the classical example. When the Queen dies in London (long may it be delayed) the Prince of Wales, lecturing on modern architecture in Australia, becomes instantaneously King.”

3. “Local beables are those which are definitely associated with particular space-time regions. The electric and magnetic fields of classical electromagnetism, \( E(t, x) \) and \( B(t, x) \) are again examples.”
II. Bell’s local causality in a LPT

Basic terms:

4. “It is important that region $V_C$ completely shields off from $V_A$ the overlap of the backward light cones of $V_A$ and $V_B$.”
Basic terms:

5. “And it is important that events in $V_C$ be specified completely. Otherwise the traces in region $V_B$ of causes of events in $V_A$ could well supplement whatever else was being used for calculating probabilities about $V_A$.”
II. Bell’s local causality in a LPT

Translation:

- “local beable” $\rightarrow$ element of a local (von Neumann) algebra
- “complete specification” $\rightarrow$ an atomic element of a local algebra
- “completely shielder-off region” $\rightarrow$
II. Bell’s local causality in a LPT

“completely shield-off region”:

(i) \( V_C \subset J_-(V_A) \)

(ii) \( V_A \subset V_C'' \)

(iii) \( V_C \subset V_B' \)
II. Bell’s local causality in a LPT

Definition. A LPT is called *(Bell) locally causal*, if

- for any *pair of projections* $A \in \mathcal{N}(V_A)$ and $B \in \mathcal{N}(V_B)$ supported in spacelike separated regions, and
- for every locally normal and faithful *state* $\phi$ establishing a correlation between $A$ and $B$, $\phi(AB) \neq \phi(A)\phi(B)$, and
- for any *spacetime region* $V_C$ satisfying Requirements (i)-(iii), and
- for any *atomic event* $C_k$ in $\mathcal{N}(V_C)$:

\[
\frac{\phi(C_kABC_k)}{\phi(C_k)} = \frac{\phi(C_kAC_k)}{\phi(C_k)} \cdot \frac{\phi(C_kBC_k)}{\phi(C_k)}
\]
II. Bell’s local causality in a LPT

Question:

- When is a LPT locally causal?
Local primitive causality: $\mathcal{N}(V) = \mathcal{N}(V'')$ for any $V$
Proposition:

Any atomic LPT satisfying local primitive causality is locally causal.
II. Bell’s local causality in a LPT

But... how can a LQT be locally causal if local causality implies the Bell inequalities which are violated for certain quantum correlations?
III. Local causality and the Bell inequalities
III. Local causality and the Bell inequalities

Local causality:
Common Cause Principle:
III. Local causality and the Bell inequalities

A nice **parallelism**: 

Local causality $\implies$ Bell inequalities

Common Cause Principle $\implies$ Bell inequalities
Proposition:

- Joint common cause $\iff$ Bell inequalities
- Joint common cause $+$ commutativity $\implies$ Bell inequalities
Proposition:

- Local causality $\iff$ Bell inequalities
- Local causality + **commutativity** $\implies$ Bell inequalities
Conclusions

- Bell’s notion of local causality presupposes a clear-cut framework integrating probabilistic and spatiotemporal entities. This goal can be met by introducing the notion of a LPT.

- In this general framework one can define Bell’s notion of local causality and show sufficient conditions on which a LPT will be locally causal.

- There is a nice parallelism between local causality and the CCPs: Bell inequalities cannot be derived from neither unless the LPT is classical or the common cause is commuting.
References