On Einstein’s Reality Criterion

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Reality Criterion (RC):

“If, without in any way disturbing a system, we can predict with certainty (i.e. with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.”
Main messages:

1. The EPR argument, making use of the RC, is devised to prove *incompleteness*, whereas Einstein’s latter arguments, not using the RC, are to prove *unsoundness*.

2. The RC is a special case of Reichenbach’s Common Cause Principle and also of Bell’s Local Causality Principle.
Project

I. Interpretations of QM in a systematic way
II. The EPR argument and Einstein’s latter arguments
III. What is Einstein’s Reality Criterion?
I. Interpretations of QM
I. Interpretations of QM

\[ A_1^5 \]
\[ a_1 \]

\[ A_2^6 \]
\[ a_2 \]

\[ A_1^4 \]
\[ a_1 \]
I. Interpretations of QM

What is a measurement?

- Measurement settings: \( a_m \)
- Measurement outcomes: \( A_m^i \)
- Probability = long run relative frequency:

\[
p(A_m^i|a_m) = \frac{\#(A_m^i \wedge a_m)}{\#(a_m)}
\]
I. Interpretations of QM

Minimal Interpretation (MI):

- Posited ontology: $a_m, A^i_m$
- Born rule:
  \[ \text{Tr}(WP^i) = p(A^i_m|a_m) \]
- Preparations: $W$
I. Interpretations of QM

Property Interpretation (PI):

- Posited ontology: $a_m$, $A^i_m$, $\alpha^i_m$

- Properties: $\alpha^i_m$

$$p(A^i_m|a_m \land \alpha^j_m) = \delta_{ij}$$

$$p(a_m \land \alpha^i_n) = p(a_m) p(\alpha^i_n)$$

- Consequently:

$$\text{Tr}(WP^i) = p(A^i_m|a_m) = p(\alpha^i_m)$$
Propensity Interpretation (Prl):

- Posited ontology: $a_m, A^i_m, \alpha^q_m$
- Propensities: $\alpha^q_m$

\[
p(A^i_m|a_m \land \alpha^q_m) = q^i
\]
\[
p(a_m \land \alpha^q_n) = p(a_m) p(\alpha^q_n)
\]

- Consequently:

\[
\text{Tr}(WP^i) = p(A^i_m|a_m) = \sum_q q^i p(\alpha^q_m)
\]
I. Interpretations of QM

Copenhagen Interpretation (CI):

- **Posited ontology:** \( a_m, A^i_m, \omega, \omega^i_m \)
- **Wave function:** \( \omega, \omega^i_m \)

\[
p(A^i_m | a_m \land \omega) = \text{Tr}(WP^i) \\
p(A^i_m | a_m \land \omega^i_n) = \text{Tr}(W^j_nP^i)
\]

where \( W^j_n = \frac{P^j_nWP^j_n}{\text{Tr}(P^j_nWP^j_n)} \)

- **Preparations and propensities are associated:** \( W \leftrightarrow \omega \)
I. Interpretations of QM

The ontology posited by the different interpretations:

<table>
<thead>
<tr>
<th>Interpretation</th>
<th>Ontology</th>
</tr>
</thead>
<tbody>
<tr>
<td>MI:</td>
<td>outcomes $A_m^i$ &amp; settings $a_m$</td>
</tr>
<tr>
<td>PI:</td>
<td>outcomes $A_m^i$ &amp; settings $a_m$ &amp; properties $\alpha_m^i$</td>
</tr>
<tr>
<td>PrI:</td>
<td>outcomes $A_m^i$ &amp; settings $a_m$ &amp; propensities $\alpha_m^q$</td>
</tr>
<tr>
<td>CI:</td>
<td>outcomes $A_m^i$ &amp; settings $a_m$ &amp; wave function $\omega, \omega_m^i$</td>
</tr>
</tbody>
</table>
II. The EPR and Einstein’s latter arguments
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Completeness and soundness:

<table>
<thead>
<tr>
<th>Ontology of the world</th>
<th>⊆</th>
<th>of the theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Completeness:</td>
<td></td>
<td>⊆</td>
</tr>
<tr>
<td>Soundness:</td>
<td></td>
<td>⊇</td>
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</table>

Ontology of the real world: posited by principles independent of the theory
Main message: The EPR argument is devised to show that certain interpretations of QM are *incomplete*; Einstein’s latter arguments are to show that the CI is *unsound*.

<table>
<thead>
<tr>
<th>EPR argument</th>
<th>Einstein’s latter arguments</th>
</tr>
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<tbody>
<tr>
<td>MI: incomplete</td>
<td>—</td>
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</table>
II/a. The EPR argument
II/a. The EPR argument

- $A_1^5$
  - $a_1$
- $A_2^6$
  - $a_2$
- $B_1^1$
  - $b_1$
- $B_2^6$
  - $b_2$
- $B_3^3$
  - $b_3$
II/a. The EPR argument

- $A_1^5$
  - $a_1$

- $A_2^6$
  - $a_2$
  - $C_2^6$

- $B_1^1$
  - $b_1$

- $B_2^6$
  - $b_2$

- $B_3^3$
  - $b_3$
II/a. The EPR argument

Elements of reality: \( C^i_m \)

\[
p(A^i_m | a_m \land C^j_m) = \delta_{ij}
\]

\[
p(C^i_m \land a_n \land b_n) = p(C^i_m) p(a_n \land b_n)
\]

- Elements of reality are properties
II/a. The EPR argument

Completeness: the elements of reality are contained in the ontology of the interpretation.
II/a. The EPR argument

**Completeness**: the elements of reality are contained in the ontology of the interpretation.

- MI is incomplete since it contains no surplus ontological structure
II/a. The EPR argument

Completeness: the elements of reality are contained in the ontology of the interpretation.

- \( \text{PrI} \) and \( \text{CI} \) are also incomplete since propensities do not satisfy

\[
p(A^i_m | a_m \land C^j_m) = \delta_{ij}
\]
**Completeness**: the elements of reality are contained in the ontology of the interpretation.

- **PI** is the only interpretation which is compatible with the EPR argument.
II/b. Einstein’s latter arguments
Einstein, 1936:

“Consider a mechanical system consisting of two partial systems A and B which interact with each other only during a limited time. Let the \( \psi \) function before their interaction be given. Then the Schrödinger equation will furnish the \( \psi \) function after the interaction has taken place. Let us now determine the physical state of the partial system A as completely as possible by measurements. Then quantum mechanics allows us to determine the \( \psi \) function of the partial system B from the measurements made, and from the \( \psi \) function of the total system. This determination, however, gives a result which depends upon which of the physical quantities (observables) of A have been measured (for instance, coordinates or momenta).”
Einstein, 1936:

“Since there can be only one physical state of B after the interaction which cannot reasonably be considered to depend on the particular measurement we perform on the system A separated from B it may be concluded that the $\psi$ function is not unambiguously coordinated to the physical state. This coordination of several $\psi$ functions to the same physical state of system B shows again that the $\psi$ function cannot be interpreted as a (complete) description of a physical state of a single system. Here also the coordination of the $\psi$ function to an ensemble of systems eliminates every difficulty.”
II/b. Einstein’s latter arguments

**Soundness:** the ontology of the interpretation supervenes on the ontology of the real world.
II/b. Einstein’s latter arguments

**Soundness:** the ontology of the interpretation supervenes on the ontology of the real world.

**MI:** The argument is neutral since it contains no surplus ontological structure.
II/b. Einstein’s latter arguments

**Soundness:** the ontology of the interpretation supervenes on the ontology of the real world.

- **PI** and **PrI:** The argument is neutral since properties and propensities are local (contrary to wave functions)
II/b. Einstein’s latter arguments

**Soundness:** the ontology of the interpretation supervenes on the ontology of the real world.

- **CI** is unsound since the wave functions are “not unambiguously coordinated to the physical state”
What do the arguments show?

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III. What is Einstein’s Reality Criterion?
Main message: The Reality Criterion is a special case of Reichenbach’s Common Cause Principle and also of Bell’s Local Causality Principle.
Prediction:

- A prediction is an event which stands in (ideally strong) correlation with another event.

- The predicted event is not causally relevant for the predicting event.
III. What is Einstein’s Reality Criterion?

Prediction:

- A prediction is an event which stands in (ideally strong) correlation with another event.
- The predicted event is not causally relevant for the predicting event.

RC adds:

- “without in any way disturbing a system”: neither the predicting event is causally relevant for the predicted event.
- “predict with certainty”: the correlation between the two events is perfect.
Common Cause Principle: If there is a correlation between two events and there is no direct causal connection between the correlating events, then there always exists a common cause of the correlation.
III/a. Common Cause Principle

Common Cause Principle:

- Conditional correlation between measurement outcomes:

\[ p(A_m^i \land B_n^j | a_m \land b_n) \neq p(A_m^i | a_m) p(B_n^j | b_n) \]

- Explanation: common cause: partition \{C_k\}

\[
\begin{align*}
  p(A_m^i \land B_n^j | a_m \land b_n \land C_k) &= p(A_m^i | a_m \land C_k) p(B_n^j | b_n \land C_k) \\
  p(a_m \land b_n \land C_k) &= p(a_m \land b_n) p(C_k)
\end{align*}
\]
Reality Criterion:

- Perfect correlation between the predicting and the predicted events

\[ p(A_m^i \land B_m^i|a_m \land b_m) = p(A_m^i|a_m \land b_m) = p(B_m^i|a_m \land b_m) \]

- Explanation: common cause: partition \( \{C_k\} \)

\[ p(A_m^i \land B_m^i|a_m \land b_m \land C_k) = p(A_m^i|a_m \land C_k) p(B_m^i|b_m \land C_k) \]
\[ p(a_m \land b_n \land C_k) = p(a_m \land b_n) p(C_k) \]
Reality Criterion:

- But in case of perfect correlation:

\[ p(A^i_m \land B^i_m | a_m \land b_m \land C_k) = \delta_{ik} \]
\[ p(a_m \land b_n \land C_k) = p(a_m \land b_n) p(C_k) \]

hence \( \{C_k\} \) is an element of reality (property).

- The RC is a special case of the Common Cause Principle when the correlation is perfect
**Local Causality Principle:** “A theory will be said to be locally causal if the probabilities attached to values of local beables in a space-time region $V_A$ are unaltered by specification of values of local beables in a space-like separated region $V_B$, when what happens in the backward light cone of $V_A$ is already sufficiently specified, for example by a full specification of local beables in a space-time region $V_C$.”
Local Causality Principle:

- Conditional correlation between measurement outcomes
- Screened-off by the (atomic) partition \( \{C_k\} \)
- In case of perfect correlation the partition is deterministic
III/b. Local causality

Reality Criterion:

- The RC is a special case of the Local Causality Principle when the correlation is perfect.
- Bonus: elements of reality are localized around the outcomes.
Conclusions

The RC is not an analytic truth but a general metaphysical principle.

- The EPR argument, using the RC, is devised to prove incompleteness, whereas Einstein’s latter arguments, not using the RC, are to prove unsoundness.

- The RC is a special case of the Common Cause Principle and also of Local Causality.
References


“How is the criterion analytic? Just consider the terms used in it. What is meant, in the first case, by “disturbing a physical system”? This requires changing the physical state of a system: if the physical state is not changed, then the system has not been disturbed. If I do something that does not disturb the physical state of a system, then after I am done the system is in the same physical state (or lack of physical state, if that makes sense) as it was before I did whatever I did. So suppose, as the criterion demands, that I can without in any way disturbing a system predict with certainty the value of a physical quantity (for example, predict with certainty how the system will react in some experiment). Then, first, there must be some physical fact about the system that determines it will act that way. That is just to say that the physical behavior of a system depends on its physical state: if a system is certain to do something physical, then something in its physical state entails that it will do it.”
“So determining that the system is certain to behave in some way is determining that some such physical state (element of reality) obtains. Second, if the means of determining this did not disturb the system, then the relevant element of reality obtained even before the determination was made, and indeed obtained independently of the determination being made. Because, as we have said, the means of determination did not (by hypothesis) disturb the system.

Now suppose, as the criterion postulates, I am even in a position to determine how the system will behave without disturbing it. That is, even if I don’t happen to make the determination, suppose that the means exist to do so (without disturbing the system). Then, by just the same argument, there must already be some element of reality pertaining to the system that determines how it will behave. For by assumption, my performing or not performing the experiment makes no difference to the system itself.”
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be some element of reality pertaining to the system that determines how it will
behave. For by assumption, my performing or not performing the experiment
makes no difference to the system itself.”
II/a. The EPR argument

Argument:

- “without in any way disturbing a system”
  - \( a_m \) and \( b_m \) are token-wise spacelike separated

- “we can predict with certainty”
  
  \[
p(A_m^i | a_m \wedge b_m \wedge B_m^i) = 1
  \]

- “there exists an element of physical reality”: \( C_m^i \)

  \[
p(A_m^i | a_m \wedge C_m^j) = \delta_{ij}
  \]

  \[
p(C_m^i \wedge a_n \wedge b_n) = p(C_m^i) p(a_n \wedge b_n)
  \]

- Elements of reality are properties