Understanding Emergent Social Phenomena

Methods, Tools, and Applications for Agent-Based Modeling

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6. Discrete Choices on Networks

Agent-based models explicitly model the interaction among the agents. This is important as system behavior may depend on the particular interaction structure. [11] In this Chapter I consider this problem by investigating how different interaction topologies affect the outcome of a decision theory model. I am interested in aggregate outcomes of individual choices in the presence of social influence. The chosen approach is that of discrete choice theory, which allows for the computation of individual choice probabilities given the heterogeneous agents' evaluation of the alternatives. To investigate the effect of social influence, I consider a scenario of repeated decisions, where the individual choices depend on the previous decisions of the community.

The motivation for this work is an actual policy making problem in the Netherlands[1], namely the infamous rush hour road congestion in the Western region of the country. A wide spectrum of policy measures has been put forward over the past decade to try to address the issue, but it is still largely unsolved. The fundamental cause of rush hour congestion is commuters traveling between their homes and workplaces. One way to approach this problem is to look at it as the aggregate effect of residential and transportation mode choices of individuals. People living in the town (district) of their employment, or opting to use public transportation contribute to the decrease of congestion, while those who cannot or would not make any of these choices may increase the problem. This approach is well supported by econometric discrete choice analysis, a standard tool in transportation demand modeling. [19] However, as Dugundji points out, that method is fundamentally grounded in individual choice, while it seems plausible to assume that social networks influence both residential and transportation mode choices. Thus, an outstanding challenge remains in the econometric treatment of the interdependence of various decision-makers' choices. [66] The particular model discussed here aims at laying the groundwork for such investigations, and is an extension of the discrete choice models of Aoki and Brock and Durlauf. [5][31]

[1] This Chapter stems in part from joint research with Elenna R. Dugundji in the framework of the AMADEUS program [4] sponsored by the Dutch National Science Organization (NWO).

1.1 The Model

In this Section I first provide a general formal framework of my approach, then I narrow the focus to the model this Chapter studies in detail.

1.1.1 The General Framework

Let's consider a set A of N agents, indexed by $a \in [1,N]$, each of which has to choose among M choices, $\Psi=\{\psi_1, \ldots, \psi_M\}$. The choices made by the agents at any fixed time t are denoted by $\sigma^t_1, \ldots, \sigma^t_N$. The probabilities of the choices for the agents are given by the rule set R, which consists of parameterized probability distributions for each agent:

$$R = \left\{ h_{aj}(v_1,...,v_{V_{aj}}) \text{ such that } a \in [1,N],\ j \in [1,M] \text{ and } \sum_{\mathrm{j=1}}^{\mathrm{M}} \mathrm{h}_{\mathrm{aj}}(v_1,...,v_{V_{aj}}) = 1 \right\}$$

Here $h_{a1}()$, …, $h_{aM} : \mathbb{R}^{Vaj} \rightarrow [0,1]$ are functions yielding the probability distribution for agent a, and $v_{aj}, \ldots, v_{Vaj} \in \mathbb{R}$ are parameters, $V_{aj} \in \mathbb{N}$. The social interactions are described as a set G of directed links among the agents. Formally, $G \subseteq A \times A$, and the pair of (A, G) forms a directed graph. The agents take the proportion of their neighbors' choices into account. That is, it is assumed that a subset of parameters $v_{aj}, \ldots, v_{Vaj}$ for each agent a is $v^*_{aj} = \frac{\sum_{g \in G(a)} \chi(\sigma^{\mathrm{t}}_{\mathrm{g}} = \psi_j)}{|G(a)|}$, for each j=[1, M], where $\chi(\cdot)$ is the usual membership function (see Chapter 2).

Finally, $\Delta: G \rightarrow G$ stands for the rules of dynamic changes to graph G. This completes the abstract formalization of the class of models considered, which are characterized by the (N, M, R, G, Δ) quintet.

1.1.2 Binary Logit Rules on Static Networks

In the following I will consider binary logit models on static networks. That is, the number of choices is fixed at M=2 (for example, car versus public transit); dynamic changes to the network are non-existent ($\Delta=Id$), and the rules of R are based on the probabilistic logit model as described below. [49]

Given M fixed at 2, I consider two complementary alternatives. For convenience, I introduce a notation: for any alternative j let $\bar{j}$ stand for the complementary choice. Moreover, let

$U_{aj}^{t} \in [0,1]$ stand for the so-called "systematic" utility that agent *a* associates with alternative *j* at time *t*. Then, according to the binary logit model, the probability that agent *a* chooses alternative *j* is given by:

$$(1) \qquad P_{aj}^{t} \equiv P_{a}^{t}\left(\psi_{j} \middle| \Psi\right) = \frac{e^{(U_{aj}^{t} - U_{a\bar{j}}^{t})}}{e^{(U_{aj}^{t} - U_{a\bar{j}}^{t})} + 1}.$$

Following the spirit of [5] and [66] social dynamics is introduced by allowing the difference $U_{aj}^{t} - U_{a\bar{j}}^{t}$ in systematic utility to be a linear-in-parameter β function of the proportion w_{aj}^{t} of agent *a*'s neighbors making each choice:

$$(2) \qquad w_{aj}^{t} = \frac{\sum_{g \in G(a)} \chi\left(\sigma_{g}^{t} = \psi_{j}\right)}{\left|G(a)\right|} \text{ for } j \in [1,2],$$

where $\chi(\cdot)$ is the above membership function. For convenience, let's define w^{t} as $w^{t} = w_{aj}^{t} - w_{a\bar{j}}^{t}$. Then the decision-making rules of the model take the following general form:

$$(3) \qquad P_{aj}^{t} \equiv h_{aj}^{t}\left(\beta, w_{aj}^{t}, w_{a\bar{j}}^{t}\right) = \frac{e^{\beta \cdot w^{t}}}{e^{\beta \cdot w^{t}} + 1}.$$

The parameter β indicates the level of certainty in the model. If it is fairly certain that the utility of alternative ψ_1 is greater than the utility of alternative ψ_2, then $\beta >> 0$, and we have an effectively deterministic choice. If there is uncertainty as to which alternative has higher utility, then $\beta \sim 0$, and we have effectively a "fair coin toss" between the two alternatives:

$$(4) \qquad P_{aj}^{t} = \frac{e^{\beta \cdot w^{t}}}{e^{\beta \cdot w^{t}} + 1} \sim 1, \text{ for } \beta >> 0, \text{ and}$$

$$(5) \qquad P_{aj}^{t} = \frac{e^{0}}{e^{0} + 1} \sim 0.5, \text{ for } \beta \sim 0.$$

This completes the definition of the model, except for the network (graph) component *G*. I will deal with it after providing an agent-based formulation.

1.1.3 An Agent-Based Formulation

The agent-based formulation of the above models is as follows. A population of *N* agents is created, each with a randomly assigned initial choice. The random choices are drawn from an uniform distribution. The agents are also given a set of neighbors, according to their position

in graph G. At the beginning of each time step the agents look at the choices of their neighbors, calculate the percentage of neighbors selecting each alternative, then they make up their minds about their respective next choices. The probability of choosing alternative j is given by Equation (3). When all of them have made their decisions, the agents update their choices. This version was implemented using the RePast agent-based simulation framework. [179] (See Appendix D.) My results are summarized in the following Sections.

1.2 Classic Models of Social Networks

In the transportation demand problem the structure of the actual social network in the studied region may have a pivotal effect. However, it is nearly impossible to gain exact knowledge about the network underlying residential and transportation mode choices. On one hand, it is difficult to define precisely what social links are important for influencing such decisions. On the other, the empirical mapping of such links raises many practical problems, including financial and organizational difficulties, and privacy concerns. Therefore, I study classes of networks that are known to have certain resemblances to real-world social networks. Such network classes can be generated by well-defined mathematical models borrowed from the rapidly growing field of social networks theory.

1.2.1 The Small-World Property and the Erdős-Rényi Model

Perhaps, the most widely known property of real-world social networks is the so-called *small-world effect* implying that the average path length in a network is small relative to the system's size. Following Stanley Milgram's 1967 experiment [151], this phenomenon is often popularized as the 'six degrees of separation'. [133] The first and simplest abstract model to reproduce this property is due to two Hungarian mathematicians, Paul Erdős and Alfréd Rényi. [72] Their random density (random graph) model defines an undirected graph G, with N nodes, $a \in [1, N]$. The parameter N is also referred to as the system's size. The nodes are connected randomly, so that each of the $N(N-1)/2$ possible links are independently present with the same p^{ER} probability. (The parameter p^{ER} is also referred to as 'network density').

The distance between pairs of nodes is defined as the number of edges forming the shortest path between them. Let $d_G(a,b)$ denote the distance between nodes a and b. By definition, $d_G(a,a)=0$ for any $a \in [1, N]$. The average distance (also called the average path length) in an undirected network is then defined as

$$(6) \qquad \ell = \frac{\sum_{a \geq b} d_G(a,b)}{\frac{1}{2}N(N+1)}$$

Notice that zero distances from each node to itself are included as the inclusion of zero distances only divides the average distance by *N*, which is often negligible for practical purposes. Also notice that the above definition of average path length is problematic for non-connected networks. Conventionally infinite length is assigned to non-existing paths, but then the value of ℓ also becomes infinite. To avoid the problem, ℓ is usually defined on such networks as the average distance between pairs of nodes that have a connecting path.

Given the definition of the average path-length, the network is said to have the small-world property, if ℓ scales logarithmically or slower with network size for a fixed mean degree (*z*). Erdős-Rényi graphs have the small-world property. [2][163][164]

1.2.2 Clustering and The Watts-Strogatz Model

In the early 90's Duncan Watts' work has drawn special attention to another important feature of complex networks. He observed that real-world networks usually exhibit a high-level of *clustering*, that is, nodes form groups that are densely connected internally, but have only a few links to other groups. There are various approaches to quantify this effect [164], among which I here work with the one proposed by Watts and Steven Strogatz. [204][205] This measure counts the frequency at which two nodes, both linked to a third node, are linked themselves. The definition of the *individual clustering coefficient* is the following:

$$(7) \qquad C_a = \frac{\textit{number of triangles connected to node } a}{\textit{number of triples centered on node } a}.$$

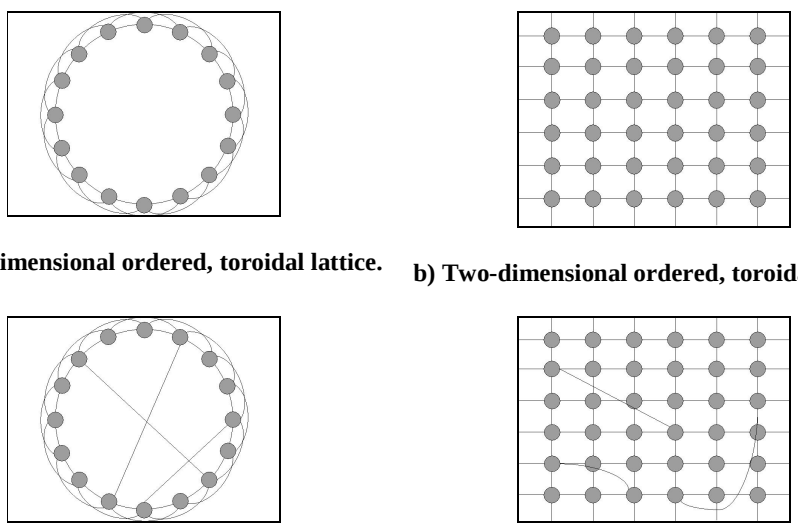
By triangle I mean a connected circle of length three, and 'connected triples' are ordered *(b, a, c)* triples of nodes, so that *b* is linked to *a*, and *a* is linked to *c*. For nodes with degree 0 or 1, $C_a=0$, by definition, and $0 \leq C_a \leq 1$ for all *a*. The *global clustering coefficient* is defined as the average of all local clustering measures:[2]

[2] Since low-degree nodes have smaller denominators in (7), C^* tends to give more weight to low-degree nodes.

$$(8) \qquad C^* = \frac{\sum_{a \in [1,N]} C_a}{N}$$

Real-world networks appear to have high-level of clustering. [164][204] However, Watts pointed out that the Erdős-Rényi model yields graphs with low clustering values, and thus provides a poor match to reality. For example, the physics co-authorship network studied in [163] has *52909* nodes with an average degree of $z=9.27$. Corresponding random graphs[3] have $C^*_{rand}=0.0017$, which is significantly lower than the empirical value of $C^*_{emp}=0.452$.

Watts also observed that abstract, ordered graphs, like regular lattices have high clustering values. Since these lattices have long peer-to-peer distances, Watts and Strogatz proposed a combination model that generates a class of networks that have a high clustering value and are small-worlds at the same time. [162][204] The Watts-Strogatz model is a regular lattice, in which a number of links are randomly rewired to connect distant regions of the lattice. [205] (See Figure 6-1 for an illustration of ordered lattices and Watts-Strogatz graphs in one- and two dimensions.)



a) One-dimensional ordered, toroidal lattice. **b) Two-dimensional ordered, toroidal lattice.**

c) One-dimensional Watts-Strogatz network. **d) Two-dimensional Watts-Strogatz network.**

Figure 6-1: Examples of ordered lattices and instances of Watts-Strogatz networks in one and two dimensions.

[3] The expected value of C^* can be derived analytically for Erdős-Rényi networks. Considering a node in a random graph and its immediate neighbors, the probability that two of these neighbors are also connected is the same as that of a link between two random nodes. That is, $C_{a,rand}= p^{ER}= z/N$. It immediately follows from (8) that $C^*_{rand}= p^{ER}$, too. [2][164]

Let $D=1, 2,\ldots$ stand for the dimensionality of the regular lattice. While the Watts-Strogatz model can be defined for lattices with arbitrary dimensions, usually $D=1$ or 2 is considered. The one-dimensional model is by far the most studied case. In higher dimensions, hypercubic lattices are considered with side length $\Lambda = N^{1/D}$. Each node on the lattice is connected to each of its neighbors not more than k lattice spacings away. In case of the one-dimensional lattice (ring), this means that each node has $s=2k$ links. In two dimensions, nodes are connected to nodes in their k-sized Moore neighborhood. (See Appendix D.) This yields $s=(2k+1)^2-1$ neighbors.

The process of rewiring involves going through each link and, with probability p^{rew}, moving one end of that link to a new location chosen uniformly from among the nodes, except that no double edges or loops (i.e., self-edges) are allowed.

This formulation, however, can easily yield unconnected graphs. Also, it is somewhat hard to study analytically. Therefore, a more tractable model was proposed by Monasson [156] and by Newman and Watts [165]. In this variant, no edges are rewired, but "shortcuts" are added connecting randomly chosen pairs of nodes. Here parameter p^{rew} becomes p^{short}, the density of these added links, among the allowed links (i.e., no double links or loops) not present in the original lattice.

The Watts-Strogatz model has been the subject of numerous studies recently, both analytical and numerical. While exact formulas are surprisingly hard to derive, it has been found that only a few rewirings (or shortcuts) are enough to collapse the average distance and to yield the small-world property. On the other hand these changes do little damage to the clustering property of the undisturbed lattices.

1.2.3 The Studied Classes of Networks

In the following Sections I study the above discrete choice model on Erdős-Rényi and Watts-Strogatz networks. Preceding these, however, I look at fully connected graphs to introduce my method of investigation and to gauge my model by replicating the general results of Aoki, and Brock and Durlauf. [5][25][31]

1.3 Discrete Choices on Fully Connected Networks

Perhaps the simplest possible meaningful network from the point of view of our decision theory model is the fully connected graph G^F. In this network each node is connected to each other node, i.e., $(a, b) \in G^F$ for all $a \neq b \in [1, N]$. Notice that this graph belongs both to the class of Erdős-Rényi (with $p^{ER}=1$) and of Watts-Strogatz networks (e.g., with $k=N/2$). In terms of the agent-based formulation this means that each agent has information about the previous choice of each other agent. That is, the agents know the actual state of the *entire* network. Thus, w^t_{aj} is the ratio of agents choosing alternative j at time $t-1$ for all a. This is why the well-studied case of the fully connected network is mentioned as the 'mean-field' case in the literature.

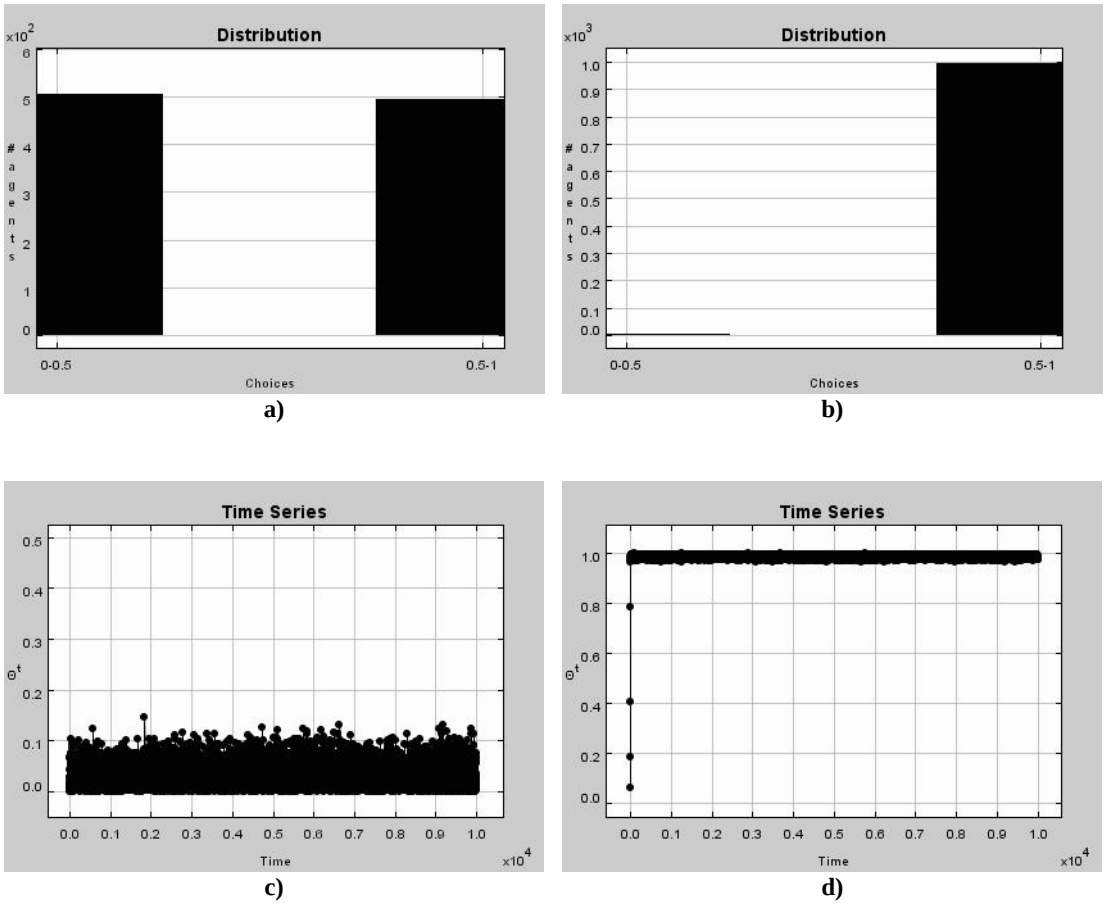
Aoki derives a general solution for a broader class of decision-making rules, of which equation (3) is a special case. [5] Dugundji has made the results for this special case more precise. [65][66] Her results imply that the system has two distinct regimes. When β is small the population is roughly evenly split between the two alternatives. However, when β is large the system converges to a situation when almost all agents pick the same alternative. The alternatives are symmetric, i.e., this latter case involves two equally likely outcomes: all agents selecting alternative 1 or all agents selecting alternative 2. To formulate these findings, let's introduce the following notation. Let W^t_j denote the ratio of agents choosing alternative j at time t:

$$(9) \qquad W_j^t = \frac{\sum_{a=1}^{n} \chi\left(\sigma_a^t = \psi_j\right)}{N},$$

where $\chi(\cdot)$ is the membership function of Chapter 2. Moreover, let Θ^t stand for the difference between the choice ratios: $\Theta^t = \left|W_1^t - W_2^t\right|$.

According to Dugundji's results, Θ^t remains at 0 for small β's, while it converges to 1 for large values.[4] Figure 6-2 shows how Θ^t changes during a simulation with different values of parameter β.

[4] The initial random assignment of choices implies that Θ^0 is close to 0.



a) b) c) d)

Figure 6-2: The distribution and time evolution of choices among 1000 agents after 10000 iterations with β=1 (a) and c)), and β=5 (b) and d)). Figure a) and b) show the number of agents choosing alternative 0 and 1, while c) and d) depict Θ^t versus time.

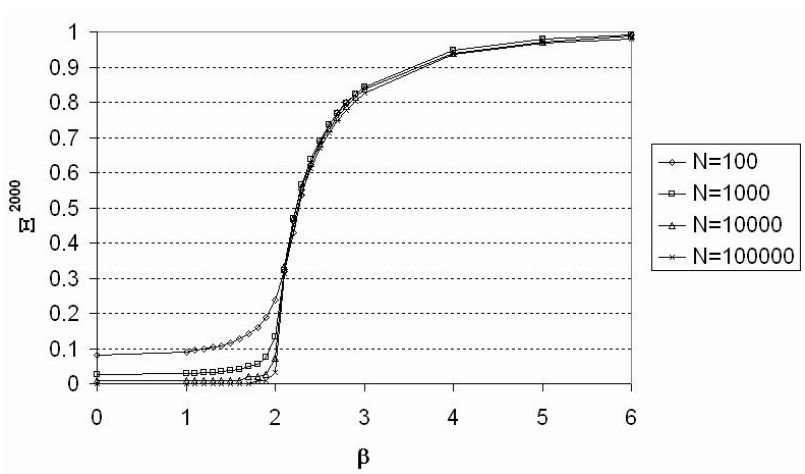


Figure 6-3: Phase transition at β=2. The system's state after 2000 iterations with various values of β and *N*. (The individual points represent the average values for 10 runs with different RNG seeds.)

Dugundji has also found that the transition between the system's two regimes is sharp around β=2. Now I turn towards replicating this result. For this, we need a way to compare different runs with different values of β. In order to capture the process of convergence during the runs

and the possible differences in their respective speeds, a more sophisticated measure is needed. Let's start by taking the average of the values of Θ^t for the length of the simulation. However, the values after the system settles down might overweight the values produced during the convergence process. Therefore, let's introduce a slight bias for early values, by taking the average over the averaged values calculated at the different time steps:

$$\Xi^t = \frac{\sum_{i=0}^{t} \frac{\sum_{j=0}^{i} \Theta^j}{i+1}}{t+1}. \tag{10}$$

Figure 6-3 summarizes the results of my numerical simulations. These are consistent with the findings of Aoki and thus with a phase transition at β=2. Notice how the transition becomes sharper for larger systems. This approximates the discontinuity found in the theoretical results.

Let us now turn toward investigating the influence special network structures have on the behavior of the discrete choice model. We have seen that on fully connected networks the system has two regimes, depending on the value of the β parameter, and with a phase transition between them. Since in the low-β regime the agents' uncertainty is shown to overweigh social influence, the expectation is that the effect of special network structures will manifest in the high-β region. Therefore, I fix the value of β at 5, well above the phase transition threshold. Hereinafter, I will always work with this value.

1.4 Discrete Choices on Erdős-Rényi Networks

In this Section, I study the discrete choice model on random density networks. I am interested in how the outcome changes, depending on the different instances of Erdős-Rényi networks. In particular, I am interested in the influence of parameter p^{ER}. For this I apply the measures developed in the previous Section. However, the following problem must be dealt with. Fully connected networks were unambiguous for a given value of N. In contrast, the Erdős-Rényi model defines a class of equally probable networks for a fixed pair of N and p^{ER} values. Therefore, I experiment with a *sample* from this class for each parameter combination. The results reported here are based on simulations with 10 randomly chosen networks in each network class.

Many properties of random density graphs are known in the limit of large system size. Typically, this limit is taken holding the average number of links per node $z=p^{ER}(N-1)$ constant. Consequently, investigation of Erdős-Rényi networks is usually carried out by varying p^{ER} over multiples of $1/N$. This is the practice what I also follow. Figure 6-4 summarizes my findings. The system exhibits the two distinct regimes that we have seen in the mean-field case, only this time depending on the value of p^{ER}. The transition occurs at around $1.5 \cdot 1/N$.

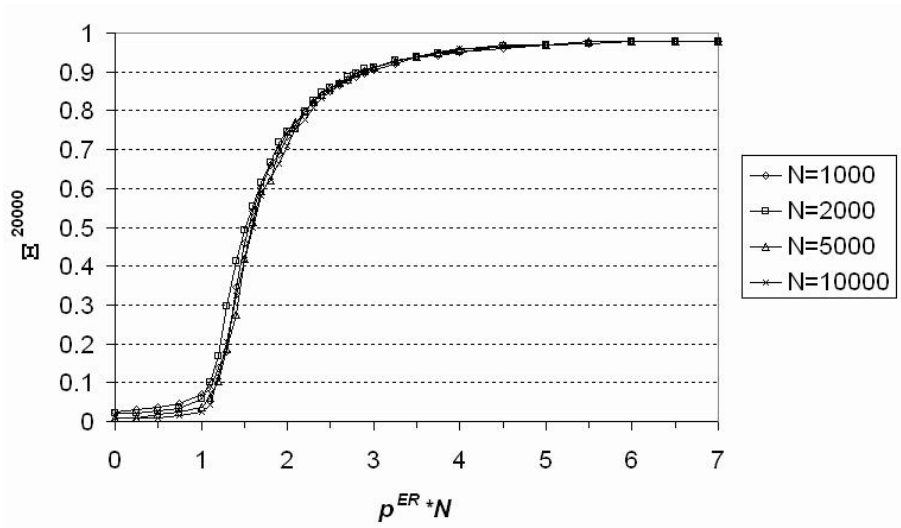


Figure 6-4: Transition at $p^{ER} \approx 1.5 \cdot (1/N)$ for $\beta=5$. The system's state after 20000 iterations with various values of p^{ER} and N. (The individual points represent the average values over 10 runs with different RNG seeds and over 10 different networks with the same generating parameters.)

Phase transitions are not alien to the model of Erdős and Rényi. In fact, most global properties of random density networks appear suddenly. [2][164] Perhaps, most famous of them all is the result derived by Erdős and Rényi themselves about the connectedness of the network. More precisely, they calculated the size of the graphs' *giant component*. [155][164] The links join nodes to form components, i.e., (maximal) subsets of nodes that are connected by paths through the network. At a certain point the largest of these components suddenly assumes almost the entire network, its size becoming $\mathbf{O}(N)$. This transition occurs at $z=1$, or $p^{ER} \approx 1/N$ [164], which vaguely corresponds to my results. (Figure 6-5 shows the average size of the giant component for the studied graphs.)

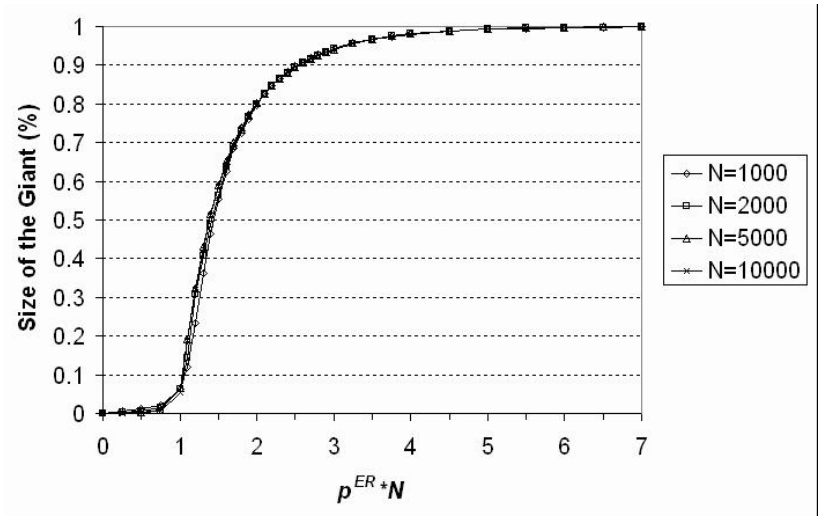


Figure 6-5: The emergence of the giant component. (The individual points represent the average values over 10 runs with different RNG seeds and over 10 different networks with the same generating parameters.)

It may seem that nothing more was found than that non-connected networks imply no social influence. Let's recall, however, that I introduced the Erdős-Rényi model as the first approximation of social networks, because it exhibits the small-world property. This means that the findings cannot be simply ascribed to the connectedness of the network. There might well be that for connected networks that are not small-worlds, the discrete choice model stays in the 'low-β' regime. To investigate this hypothesis I turn towards the network class proposed by Watts and Strogatz. [205]

1.5 Discrete Choices on Watts-Strogatz Networks

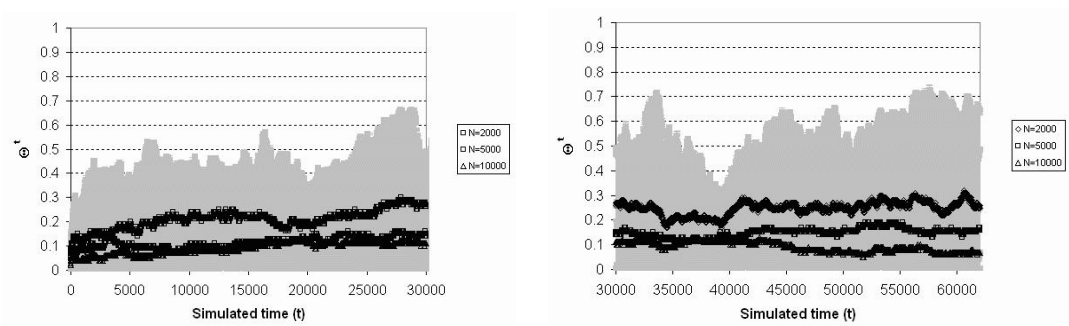


Figure 6-6: Large-worlds do not lead to convergence to a 100% outcome. The evolution of the system through simulated time for various system sizes with p^{rew}=0 and k=2. The individual points represent average values over 10 runs with different random number generator seeds. The gray error bars show minimum and maximum values.

Let's first make an elementary test of the hypothesis that 'large-worlds' do not converge to about 100% of the population choosing the same alternative. With p^{rew}=0, the Watts-Strogatz model keeps the initial regular lattice, in which the average path length scales with $\mathbf{O}(N^{-D}/2k)$. In one dimension, this means $\mathbf{O}(N/2k)$. [164][165] I first experiment with such networks of various sizes. Notice, however, that testing for non-convergence via numerical simulation is somewhat problematic. Theoretically, any simulated non-convergence result might, in fact, hide a very slow convergence. In order to avoid such traps one has to study the system's trajectory through simulated time, and look for possible trends that might be the sign of convergence. This is what the experiments on Figure 6-6 summarize. The results convincingly show that on regular lattices the system stays in the regime where the opinions are roughly evenly split between the alternatives. This is an initial confirmation for the viability of the above hypothesis.

I now turn towards exploring how decreasing average path length affects the outcome of the discrete choice model. I do this by building on a very important property of the model of Watts and Strogatz. Namely, that it can be tuned between 'large-' and small-worlds by varying the p^{rew} (p^{short}) parameter. It is known from the literature that the transition between 'large-' and small-worlds is sudden and sharp in p^{rew} (p^{short}): a phase transition. [164] I am interested in learning whether this effect carries through to the outcome of the discrete choice model. Keeping with the common practice, in the following I deal with one-dimensional Watts-Strogatz networks (D=1), with parameter k fixed at 2. This latter choice ensures that, on average, each agent has 4 social links.

1.5.1 Watts-Strogatz Networks with Rewiring

As discussed in Section 1.2.2, the Watts-Strogatz model has two variants. Let's first look at the original version where small-worlds are generated by rewiring random links on a regular lattice. It is known from the literature that only a few rewired links are enough to make the average path length short in the network, except for the fact that rewiring may make the graph disconnected. It is also known that the change occurs suddenly as system size and the value of p^{rew} increases, producing a sharp phase transition between large- and small-worlds. [164]

As shown on Figure 6-7, my model replicates the phase transition. This confirms my hypothesis that on small-worlds the system converges to a 'high-β' outcome, while it stays in the non-decisive regime for networks with long average path lengths.

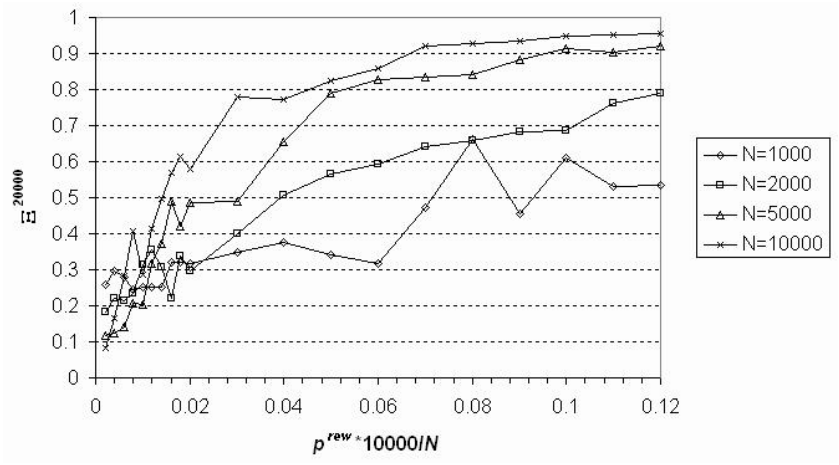


Figure 6-7: Transition at 0.01 for β=5. The system's state after 20000 iterations with various values of p^{rew} and *N*. (The individual points represent the average values over 10 runs with different RNG seeds and over 10 different networks with the same generating parameters.)

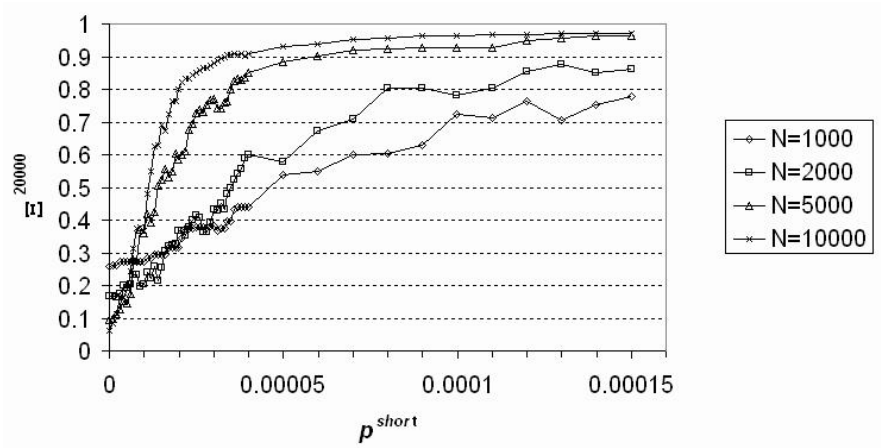


Figure 6-8: Transition at p^{WS}≈0.00001 for β=5. The system's state after 20000 iterations with various values of p^{short} and *N*. (The individual points represent the average values over 10 runs with different RNG seeds and over 10 different networks with the same generating parameters.)

1.5.2 Watts-Strogatz Networks with Shortcuts

Let's continue by looking at the version of the Watts-Strogatz network with shortcuts. These networks are always connected, thus eliminating many problems of the version with rewiring. This is the reason why most theoretical and analytical work on Watts-Strogatz networks has been done using this variant. [2][164] The properties of these networks are mostly similar to that of the rewiring version. There is a phase transition between long average path lengths and small-worlds, depending on system size and the value of p^{short}. Despite the many attempts, no exact results are available about this transition, but a number of partially exact results are

known. [164] Among these, the most important for my purposes here is that the transition occurs when p^{short} is around $c \cdot 1/N$, where $c \in \mathbb{R}$ is a constant.

The results, summarized on Figure 6-8, confirm that the phase transition carries through to the behavior of the networked discrete choice model. On 'large-worlds', the decisions of the agents are roughly evenly split between the two alternatives. However, when average path-length is small, almost all of them converge to one of the choices. This result further confirms our hypothesis.

1.6 Related Work

The field of discrete choice modeling has flourished in the past 30 years, ultimately extending the basic random utility model to incorporate cognitive and behavioral processes, flexible error structures and different types of data in so-called hybrid choice models [20][21]. However, as Dugundji points out, discrete choice theory is fundamentally grounded in individual choice, and thus an outstanding challenge remains in the treatment of the interdependence of various decision-makers' choices, be that via global or local interactions. The formulation of the nature of the interaction, in turn, raises the issues of networks. When considering the problem domain of transportation demand modeling with an activity-based approach, not only spatial land use and transportation networks but also social networks may be relevant. [64]

The topic of social networks has been a popular subject of research in recent years. For example, in a series of studies, the group lead by Vicsek investigated topological phase transitions in networks using methods from statistical mechanics. [53][74][169] However, these works include no dynamics *on* networks. On the other hand, the group also studied synchronization among agents in various systems (including the occurrence of 'Mexican waves'), a topic closely related to my work. [75][76] Yet, these latter works considered regular lattices or the so-called 'mean field' case only as the interaction topology of the agents. Similarly, in case of discrete choice models, the effect of social interactions was considered only by informing each agent about the global average choice of the population (i.e., the 'mean field' case). [5][25][31][33] My work extends this approach by investigating how various network structures affect the emergent population-level choice.

The outstanding popularity of the 'mean field case' is due to its plausible physical interpretation. Physical interpretations are important, because physicists for long have extensively dealt with a model similar to the discrete choice framework discussed in this Chapter. The so-called 'spin-glass' model of Ising, defined on a regular lattice, was originally proposed as a model of magnetization, where the use of the 'field' metaphor is evident. [34][123][190] Recently, a number of authors studied Ising-type models on small-world networks and come to conclusions similar to mine. [18][27][39] [52][56][88][116][209]

In Section 1.1.1 I presented a general model framework. The particular constraints, introduced in Section 1.1.2, indeed make the studied model essentially an Ising-type model. However, instead of taking a global, top-down approach, my work focuses on the agents and builds the dynamics from the point of view of the agents. Moreover, building on the model of Aoki, and Brock and Durlauf, my approach allows for the inclusion of more realistic, non-binary choices, and possible personal-level biases based on statistical estimations, as discussed in Section 1.8. No previous attempt has been made to study the effect of specific network structures using such a discrete choices formulation.

1.7 Summary

In this Chapter I have studied the effect various classes of interaction networks have on a model where agents make repeated decisions. The particular Ising-type model was motivated by a transportation mode choice application in the Netherlands. I showed that the phase transition in the underlying network model carries through to the aggregate choice made by the population both in the case of Erdős-Rényi and Watts-Strogatz networks. An important aspect of this result is that small average distances lead to *immediate* convergence; do not simply increase the ratio of the dominant opinion. Moreover, these findings have an important practical consequence: they extend the domain of applicability for discrete choice analysis. While it may be problematic to obtain knowledge about the exact social interaction network involved in a particular problem; it is generally possible and, according to my results, sufficient to test empirically for the presence of the small-world property.

Naturally, the results of this Chapter are but a first step in the solution of the actual transportation mode choice problem. Since most social networks are known to be 'small-worlds', further network classes are to be studied, to collect other empirically testable

properties that may have major influences on the system's behavior. Part of this work has already been done: these will be summarized in the next Section.

The primary purpose of this Chapter within the context of this dissertation, however, was to demonstrate the importance of interaction topologies as a major and independent component of agent-based models. I have showed what extreme differences they may cause in the outcome of an otherwise unchanged model. This was evident not only in case of the different network classes analyzed (i.e., the full, Erdős-Rényi, or Watts-Strogatz networks), but also within the classes themselves. For example, in case of the Watts-Strogatz class, the introduction of a handful of additional 'shortcuts' could move the system from one behavioral regime to another. Therefore, in the course of developing an agent-based model, one has to consider and test the agents' interaction topology as an independently variable model parameter.

1.8 Extensions and Future Work

In this last Section I turn back to the application problem motivating the discrete choice model presented in this Chapter. In addition to the work reported here, several extensions have been made, in order to make the model more applicable to the actual problem. All these extensions fit within the general model framework introduced in Section 1.1.1.

1.8.1 Multinomial and Nested Multinomial Choices

A natural extension involves the number of choices. In the above discussion, I fixed M at 2, and worked with binary choices only. Multinomial extensions to the binary choice model of Aoki can be found in [32] and [33]. I have studied the effects of various network classes on this model. The results are summarized in [59], [63] and [102].

Multinomial choices, however, are not always independent. For example, in case of the transportation problem, not all transportation modes are selectable for all residential locations. That is, if an agent opts to live in the city, it may choose between car, bicycle, or public transportation. However, if it moves out to the suburbs, bicycle may no longer be an option, while train often replaces public transportation. The nested multinomial logit model describes such choice structures. [20] In this formulation choices are arranged in a tree, and selections made higher on the tree enable/disable choices on lower levels. Results on network effects in case of this variant are reported in [61] and [62].

1.8.2 Empirical Applications

Another important direction for extension involves the rule set R of the general framework. Recall that the probability of the choices for agent *a* at time *t* were defined by functions $h_{a1}()$, …, $h_{aM}()$: $\mathbb{R}^{Vaj} \rightarrow [0,1]$, depending on parameters v_{aj}, …, $v_{Vaj} \in \mathbb{R}$. The general framework required that a subset of these parameters describe the choice of the agent's neighbors. In the model instance discussed in this Chapter these were the only parameters I dealt with, the rest was omitted. As a consequence, the agents were unbiased regarding the alternatives. Naturally, this is not true in the modeled system. Certain people, for example depending on their age, sex, or education, may be more likely to select car versus bicycle, or vice versa. Based on empirical survey data about the studied region, statistical estimations of various such parameters have been made and incorporated into the model.[5] The effect of various network structures on the empirical binary, multinomial and nested multinomial cases were reported in [60], [61] and [63].

1.8.3 Dynamic Networks

Component Δ in the general framework describes how the network changes in time. However, so far, I have dealt with static networks only. Therefore, my last set of extensions was aimed at discrete choices on dynamic networks. I considered two distinct types of dynamics. In one case, the network dynamics was independent of the choices made by the agents. This 'exogenous dynamics' was intended as a base line case for further experiments. Therefore, it was modeled as random choices to the underlying network. In contrast, in the real world the dynamics is affected by the agents' choices. Assuming that people's choices are influenced by their day-to-day acquaintances, moving from the city center to the suburbs is likely to partially change the influences one receives. I have carried out extensive experiments with this 'endogenous dynamics' version. My results will be reported in [96]. However, my work on dynamic networks so far has been constrained on the non-biased (i.e., non-empirical), binary case. In the future, I intend to continue this line of work and study also multinomial, nested multinomial choices, and biased agents on dynamic networks.

[5] The statistical estimations are due to Elenna R. Dugundji of the University of Amsterdam.

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Appendices

Appendix D: Notes

Ascape is a Java-based agent simulation environment, developed at the Brookings Institute, Washington D.C. While the package is modeled after Swarm, its design is geared towards spatial models, i.e., towards systems that contain a physical space. Models like cellular automata belong to this category. [36][67][153][172]

Double buffering is a standard programming technique to implement interacting agents in a sequential system. The aim is to avoid artifacts caused by updating an agent's state before another agent used that state as its input. To prevent this, the agents' new states are buffered until all parallel updates are executed, and only committed afterwards.

Mason is the latest agent-based simulation package, developed by a joint effort of the Evolutionary Computation Laboratory and Center for Social Complexity, both at the George Mason University. This Java-based system is modeled after both Swarm and RePast. [36][67][140][153]

The Moore neighborhood is one of the two standard neighborhoods defined on a regular lattice, often used by cellular automata. If *x(a)* and *y(a)* stand for cell *a*'s horizontal and vertical coordinates, respectively, then its *s*-sized Moore neighborhood $M^s(a)$ is the following:

$$(11) \qquad M^s(a) = \{b \neq a, \text{such that } |x(a) - x(b)| \leq s \wedge |y(a) - y(b)| \leq s\}.$$

The von Neumann-neighborhood is one of the two standard neighborhoods defined on a regular lattice, often used by cellular automata. If *x(a)* and *y(a)* stand for cell *a*'s horizontal and vertical coordinates, respectively, then its *s*-sized von Neumann neighborhood $N^s(a)$ is the following:

$$(12) \qquad N^s(a) = \{b \neq a, \text{such that } |x(a) - x(b)| + |y(a) - y(b)| \leq s\}$$

NetLogo is an agent-based simulation package, developed at the Center for Connected Learning and Computer-Based Modeling, Northwestern University. This Java-based system provides extensive support for cellular automata-like modeling tasks. [67][206]

Swarm was the first dedicated software toolkit for agent-based simulations, originally developed at the Santa Fe Institute. It aimed to provide a standardized set of well-engineered software tools usable on a wide variety of systems. Nowadays, Swarm is widely considered outdated, but it still maintains one of the strongest user communities. Partly, this is because

Swarm is “the father of all ABM toolkits”, and thus, the Swarm User Group acts as an international forum for scientists involved in agent-based modeling and simulation. [36][153]

RePast is a successor of Swarm, implemented in Java, developed by the Social Science Research Computing Center, University of Chicago. Although an independent project, it generally follows Swarm’s approach, and it is widely considered as an updated and advanced version of the previous package. Some also think it is the most suitable package for modeling complex social systems. [36][67][153][179][196]