TABLE 2.3

State Description	Weight	Structure Description	Weight
1. $Fa.Fb.Fc$	1/20	All F	1/20
2. $Fa.Fb.{\sim}Fc$	3/20		
3. $Fa.{\sim}Fb.Fc$	3/20	2 F, 1 $\sim F$	9/20
4. ${\sim}Fa.Fb.Fc$	3/20		
5. $Fa.{\sim}Fb.{\sim}Fc$	3/20		
6. ${\sim}Fa.Fb.{\sim}Fc$	3/20	1 F, 2 $\sim F$	9/20
7. ${\sim}Fa.{\sim}Fb.Fc$	3/20		
8. ${\sim}Fa.{\sim}Fb.{\sim}Fc$	1/20	No F	1/20

(The idea of a confirmation function of this type was given in Burks 1953; the philosophical issues are further discussed in Burks 1977, Chapter 3.) This method of weighting, which may be designated $m^{\diamond}$, yields a confirmation function $C^{\diamond}$, which is a sort of counterinductive method. Whereas m^* places higher weights on the first and last state descriptions, which are state descriptions for universes with a great deal of uniformity (either every object has the property, or none has it), $m^{\diamond}$ places lower weights on descriptions of uniform universes. Like c^*, $c^{\diamond}$ allows for "learning from experience," but it is a funny kind of anti-inductive "learning." Before we reject $m^{\diamond}$ out of hand, however, we should ask ourselves if we have any a priori guarantee that our universe is uniform. Can we select a suitable confirmation function without being totally arbitrary about it? This is the basic problem with the logical interpretation of probability.

Part IV: Confirmation and Probability

2.9 THE BAYESIAN ANALYSIS OF CONFIRMATION

We now turn to the task of illustrating how the probabilistic apparatus developed above can be used to illuminate various issues concerning the confirmation of scientific statements. Bayes's theorem (Rule 9) will appear again and again in these illustrations, justifying the appellation of Bayesian confirmation theory.

Various ways are available to connect the probabilistic concept of confirmation back to the qualitative concept, but perhaps the most widely followed route utilizes an incremental notion of confirmation: E confirms H relative to the background knowledge K just in case the addition of E to K raises the probability of H, that is, $Pr(H|E.K) >) Pr(H|K)$.[13] Hempel's study of instance confirmation in terms of a

[13] Sometimes, when we say that a hypothesis has been confirmed, we mean that it has been rendered highly probable by the evidence. This is a *high probability* or *absolute* concept of confirmation, and it should be carefully distinguished from the *incremental* concept now under discussion (see Carnap 1962, Salmon 1973, and Salmon 1975). Salmon (1973) is the most elementary discussion.

two-place relation can be taken to be directed at the special case where K contains no information. Alternatively, we can suppose that K has been absorbed into the probability function in the sense that $Pr(K) = 1$,[14] in which case the condition for incremental confirmation reduces to $Pr(H|E) > Pr(H)$. (The unconditional probability $Pr(H)$ can be understood as the conditional probability $Pr(H|T)$, where T is a vacuous statement, for example, a tautology. The axioms of Section 2.7 apply only to conditional probabilities.)

It is easy to see that on the incremental version of confirmation, Hempel's consistency condition is violated as is

Conjunction condition: If E confirms H and also H' then E confirms $H.H'$.

It takes a bit more work to construct a counterexample to the special consequence condition. (This example is taken from Carnap 1950 and Salmon 1975, the latter of which contains a detailed discussion of Hempel's adequacy conditions in the light of the incremental notion of confirmation.) Towards this end take the background knowledge to contain the following information. Ten players participate in a chess tournament in Pittsburgh; some are locals, some are from out of town; some are juniors, some are seniors; and some are men (M), some are women (W). Their distribution is given by

TABLE 2.4

	Locals	Out-of-towners
Juniors	M, W, W	M, M
Seniors	M, M	W, W, W

And finally, each player initially has an equal chance of winning. Now consider the hypotheses H: an out-of-towner wins, and H': a senior wins, and the evidence E: a woman wins. We find that

$$Pr(H|E) = 3/5 > Pr(H) = 1/2$$

so E confirms H. But

$$Pr(H \vee H'|E) = 3/5 <(Pr(H \vee H') = 7/10.$$

So E does not confirm $H \vee H'$; in fact E confirms $\sim(H \vee H')$ and so disconfirms $H \vee H'$ even though $H \vee H'$ is a consequence of H.

The upshot is that on the incremental conception of confirmation, Hempel's adequacy conditions and, hence, his definition of qualitative confirmation, are inadequate. However, his adequacy conditions fare better on the high probability conception of confirmation according to which E confirms H relative to K just in case $Pr(H|E.K) > r$, where r is some number greater than 0.5. But this notion of

[14] As would be the case if learning from experience is modeled as change of probability function through conditionalization; that is, when K is learned, Pr_{old} is placed by $Pr_{new}(\ ) = Pr_{old}(\ |\ K)$. From this point of view, Bayes's theorem (Rule 9) describes how probability changes when a new fact is learned.

confirmation cannot be what Hempel has in mind; for he wants to say that the observation of a single black raven (E) confirms the hypothesis that all ravens are black (H), although for typical K, $Pr(H|E.K)$ will surely not be as great as 0.5. Thus, in what follows we continue to work with the incremental concept.

The probabilistic approach to confirmation coupled with a simple application of Bayes's theorem also serves to reveal a kernel of truth in the H-D method. Suppose that the following conditions hold:

$$\text{(i) } H, K \vdash E; \text{ (ii) } 1 > Pr(H|K) > 0; \text{ and (iii) } 1 > Pr(E|K) > 0.$$

Condition (i) is the basic H-D condition. Conditions (ii) and (iii) say that neither H nor E is known on the basis of the background information K to be almost surely false or almost surely true. Then on the incremental conception it follows, as the H-D methodology would have it, that E confirms H on the basis of K. By Bayes's theorem

$$Pr(H|E.K) = \frac{Pr(H|K)}{Pr(E|K)}$$

since by (i),

$$Pr(E|H.K) = 1.$$

It then follows from (ii) and (iii) that

$$Pr(H|E.K) > Pr(H|K).$$

Notice also that the smaller $Pr(E|K)$ is, the greater the incremental confirmation afforded by E. This helps to ground the intuition that "surprising" evidence gives better confirmational value. However, this observation is really double-edged as will be seen in Section 2.10.

The Bayesian analysis also affords a means of handling a disquieting feature of the H-D method, sometimes called the problem of irrelevant conjunction. If the H-D condition (i) holds for H, then it also holds for $H.X$ where X is anything you like, including conjuncts to which E is intuitively irrelevant. In one sense the problem is mirrored in the Bayesian approach, for assuming that $1 > Pr(H.X|K) > 0$, it follows that E incrementally confirms $H.X$. But since the special consequence condition does not hold in the Bayesian approach, we cannot infer that E confirms the consequence X of $H.X$. Moreover, under the H-D condition (i), the incremental confirmation of a hypothesis is directly proportional to its prior probability. Since $Pr(H|K) \geq Pr(H.X|K)$, with strict inequality holding in typical cases, the incremental confirmation for H will be greater than for $H.X$.

Bayesian methods are flexible enough to overcome various of the shortcomings of Hempel's account. Nothing, for example, prevents the explication of confirmation in terms of a Pr-function which allows observational evidence to boost the probability of theoretical hypotheses. In addition the Bayesian approach illuminates the paradoxes of the ravens and Goodman's paradox.

In the case of the ravens paradox we may grant that the evidence that the individual a is a piece of white chalk can confirm the hypothesis that "All ravens are black" since, to put it crudely, this evidence exhausts part of the content of the hypothesis. Nevertheless, as Suppes (1966) has noted, if we are interested in subjecting the hypothesis to a sharp test, it may be preferable to do outdoor ornithology and sample from the class of ravens rather than sampling from the class of nonblack things. Let a denote a randomly chosen object and let

$$Pr(Ra.Ba) = p_1, \qquad Pr(Ra.{\sim}Ba) = p_2$$
$$Pr({\sim}Ra.Ba) = p_3, \qquad Pr({\sim}Ra.{\sim}Ba) = p_4.$$

Then

$$Pr({\sim}Ba|Ra) = p_2 \div (p_1 + p_2)$$
$$Pr(Ra|{\sim}Ba) = p_2 \div (p_2 + p_4)$$

Thus, $Pr({\sim}Ba|Ra) > Pr(Ra|{\sim}Ba)$ just in case $p_4 > p_1$. In our world it certainly seems true that $p_4 > p_1$. Thus, Suppes concludes that sampling ravens is more likely to produce a counterinstance to the ravens hypothesis than is sampling the class of nonblack things.

There are two problems here. The first is that it is not clear how the last statement follows since a was supposed to be an object drawn at random from the universe at large. With that understanding, how does it follow that $Pr({\sim}Ba|Ra)$ is the probability that an object drawn at random from the class of ravens is nonblack? Second, it is the anti-inductivists such as Popper (see item 4 in Section 2.8 above and 2.10 below) who are concerned with attempts to falsify hypotheses. It would seem that the Bayesian should concentrate on strategies that enhance absolute and incremental probabilities. An approach due to Gaifman (1979) and Horwich (1982) combines both of these points.

Let us make it part of the background information K that a is an object drawn at random from the class of ravens while b is an object drawn at random from the class of nonblack things. Then an application of Bayes's theorem shows that

$$Pr(H|Ra.Ba.K) > Pr(H|{\sim}Rb.{\sim}Bb.K)$$

just in case

$$1 > Pr({\sim}Rb|K) > Pr(Ba|K).$$

To explore the meaning of the latter inequality, use the principle of total probability to find that

$$Pr(Ba|K) = Pr(Ba|H.K) \cdot Pr(H|K) + Pr(Ba|{\sim}H.K) \cdot Pr({\sim}H|K)$$
$$= Pr(H|K) + Pr(Ba|{\sim}H.K) \cdot Pr({\sim}H|K)$$

and that

$$Pr({\sim}Rb|K) = Pr(H|K) + Pr({\sim}Rb|{\sim}H.K) \cdot Pr({\sim}H|K).$$

So the inequality in question holds just in case

$$1 > Pr({\sim}Rb|{\sim}H.K) > Pr(Ba|{\sim}H.K),$$

or

$$Pr({\sim}Ba|{\sim}H.K) > Pr(Rb|{\sim}H.K) > 0,$$

which is presumably true in our universe. For supposing that some ravens are nonblack, a random sample from the class of ravens is more apt to produce such a bird than is a random sample from the class of nonblack things since the class of nonblack things is much larger than the class of ravens. Thus, under the assumption of the stated sampling procedures, the evidence $Ra.Ba$ does raise the probability of the ravens hypothesis more than the evidence $\sim Rb.\sim Bb$ does. The reason for this is precisely the differential propensities of the two sampling procedures to produce counterexamples, as Suppes originally suggested.

The Bayesian analysis also casts light on the problems of induction, old and new, Humean and Goodmanian. Russell (1948) formulated two categories of induction by enumeration:

> Induction by simple enumeration is the following principle: "Given a number n of α's which have been found to be β's, and no α which has been found to be not a β, then the two statements: (a) 'the next α will be a β,' (b) 'all α's are β's,' both have a probability which increases as n increases, and approaches certainty as a limit as n approaches infinity."
>
> I shall call (a) "particular induction" and (b) "general induction." (1948, 401)

Between Russell's "particular induction" and his "general induction" we can interpolate another type, as the following definitions show (note that Russell's "α" and "β" refer to properties, not to individual things):

Def. Relative to K, the predicate "P" is *weakly projectible* over the sequence of individuals $a_1, a_2, \ldots$ just in case[15]

$$\lim_{n \to \infty} Pr(Pa_{n+1}|Pa_1. \ldots .Pa_n.K) = 1.$$

Def. Relative to K, "P" is *strongly projectible* over $a_1, a_2, \ldots$ just in case

$$\lim_{n,\, m \to \infty} Pr(Pa_{n+1}. \ldots .Pa_{n+m}| Pa_1. \ldots .Pa_n.K) = 1.$$

(The notation $\lim_{m,\, n \to \infty}$ indicates the limit as m and n both tend to infinity in any manner you like.) A sufficient condition for both weak and strong probability is that the general hypothesis H: $(i)Pa_i$ receives a nonzero prior probability. To see that it is sufficient for weak projectibility, we follow Jeffreys's (1957) proof. By Bayes's theorem

$$Pr(H|Pa_1. \ldots .Pa_{n+1} . K) = \frac{Pr(Pa_1. \ldots .Pa_{n+1}|H.K) \cdot Pr(H|K)}{Pr(Pa_1. \ldots .Pa_{n+1}|K)}$$

$$= \frac{Pr(H|K)}{Pr(Pa_1|K) \cdot Pr(Pa_2|Pa_1.K) \cdot \ldots \cdot Pr(Pa_{n+1}|Pa_1. \ldots .Pa_n \cdot K)}$$

[15] Equation $\lim_{n\to\infty} x_n = L$ means that, for any real number $\epsilon > 0$, there is an integer $N > 0$ such that, for all $n > N$, $| x_n - L | < \epsilon$.

Unless $Pr(Pa_{n+1}|Pa_1. \dots .Pa_n.K)$ goes to 1 as $n \rightarrow \infty$, the denominator on the right-hand side of the second equality will eventually become less than $Pr(H|K)$, contradicting the truth of probability that the left-hand side is no greater than 1.

The posit that

$$\text{(P)}\ Pr([(i)Pa_i|K] > 0$$

is not necessary for weak projectibility. Carnap's systems of inductive logic (see item 6 in Section 2.8 above) are relevant examples since in these systems (P) fails in a universe with an infinite number of individuals although weak projectibility can hold in these systems.[16] But if we impose the requirement of countable additivity

$$\text{(CA)}\quad \lim_{n \rightarrow \infty} Pr(Pa_i. \dots .Pa_n|K) = Pr[(i)\ Pa_i|K]$$

then (P) is necessary as well as sufficient for strong projectibility.

Also assuming (CA), (P) is sufficient to generate a version of Russell's "general induction," namely

$$\text{(G)}\quad \lim_{n \rightarrow \infty} Pr[(i)Pa_i|Pa_1. \dots .Pa_n.K] = 1.$$

(Russell 1948 lays down a number of empirical postulates he thought were necessary for induction to work. From the present point of view these postulates can be interpreted as being directed to the question of which universal hypotheses should be given nonzero priors.)

Humean skeptics who regiment their beliefs according to the axioms of probability cannot remain skeptical about the next instance or the universal generalization in the face of ever-increasing positive instances (and no negative instances) unless they assign a zero prior to the universal generalization. But

$$Pr[(i)Pa_i|K] = 0$$

implies that

$$Pr[(\exists i) \sim Pa_i|K] = 1,$$

which says that there is certainty that a counterinstance exists, which does not seem like a very skeptical attitude.

[16] A nonzero prior for the general hypothesis is a necessary condition for strong projectibility but not for weak projectibility. The point can be illustrated by using de Finetti's representation theorem, which says that if P is exchangeable over $a_1, a_2, \dots$ (which means roughly that the probability does not depend on the order) then:

$$Pr(Pa_1.Pa_2. \dots .Pa_n \mid K) = \int_0^1 \theta^n\, d\mu(\theta)$$

where $\mu(\theta)$ is a uniquely determined measure on the unit interval $0 \leq \theta \leq 1$. For the uniform measure $d\mu(\theta) = d(\theta)$ we have

$$Pr\ (Pa_{n+1}|Pa_1. \dots .Pa_n. K) = n + 1/n + 2$$

and

$$Pr\ (Pa_{n+1}. \dots .Pa_{n+m}|Pa_1. \dots .Pa_n.K) = m + 1/n + m + 1.$$

Note also that the above results on instance induction hold whether "P" is a normal or a Goodmanized predicate—for example, they hold just as well for P^*a_i which is defined as

$$[(i \leq 2000).Pa_i] \vee [(i > 2000).\sim Pa_i)],$$

where Pa_i means that a_i is purple. But this fact just goes to show how weak the results are; in particular, they hold only in the limit as $n \rightarrow \infty$ and they give no information about how rapidly the limit is approached.

Another way to bring out the weakness is to note that (P) does not guarantee even a weak form of Hume projectibility.

> *Def.* Relative to K, "P" is *weakly Hume projectible* over the doubly infinite sequence $\ldots, a_{-2}, a_{-1}, a_0, a_1, a_2, \ldots$ just in case for any n,
> $\lim_{k \rightarrow \infty} Pr(Pa_n | Pa_{n-1}. \ldots .Pa_{n-k} . K) = 1.$

(To illustrate the difference between the Humean and non-Humean versions of projectibility, let Pa_n mean that the sun rises on day n. The non-Humean form of projectibility requires that if you see the sun rise on day 1, on day 2, and so on, then for any $\varepsilon > 0$ there will come a day N when your probability that the sun will rise on day $N + 1$ will be at least $1 - \varepsilon$. By contrast, Hume projectibility requires that if you saw the sun rise yesterday, the day before yesterday, and so on into the past, then eventually your confidence that the sun will rise tomorrow approaches certainty.)

If (P) were sufficient for Hume projectibility we could assign nonzero priors to both $(i)Pa_i$ and $(i)P^*a_i$, with the result that as the past instances accumulate, the probabilities for Pa_{2001} and for P^*a_{2001} both approach 1, which is a contradiction.

A sufficient condition for Hume projectibility is exchangeability.

> *Def.* Relative to K, "P" is *exchangeable* for Pr over the a_is just in case for any n and m
>
> $$Pr(\pm Pa_n. \ldots .\pm Pa_{n+m} | K) = Pr(\pm Pa_{n'}. \ldots .\pm Pa_{n'+m'} | K)$$

where $\pm$ indicates that either P or its negation may be chosen and $[a_{i'}]$ is any permutation of the a_is in which all but a finite number are left fixed. Should we then use a Pr-function for which the predicate "purple" is exchangeable rather than the Goodmanized version of "purple"? Bayesianism per se does not give the answer anymore than it gives the answer to who will win the presidential election in the year 2000. But it does permit us to identify the assumptions needed to guarantee the validity of one form or another of induction.

Having touted the virtues of the Bayesian approach to confirmation, it is now only fair to acknowledge that it is subject to some serious challenges. If it can rise to these challenges, it becomes all the more attractive.

2.10 CHALLENGES TO BAYESIANISM

1. Nonzero priors. Popper (1959) claims that "in an infinite universe . . . the probability of any (non-tautological) universal law will be zero." If Popper were right and universal generalizations could not be probabilified, then Bayesianism would be worthless as applied to theories of the advanced sciences, and we would presumably have to resort to Popper's method of corroboration (see item 4 in Section 2.8 above).

To establish Popper's main negative claim it would suffice to show that the prior probability of a universal generalization must be zero. Consider again H: $(i)Pa_i$. Since for any n

$$H \vdash Pa_1 . Pa_2 . \ldots .Pa_n,$$
$$Pr(H|K) \leq \lim_{n \to \infty} Pr(Pa_1 . \ldots .Pa_n|K).$$

Now suppose that

(I) For all n, $Pr(Pa_1 . \ldots .Pa_n|K) = Pr(Pa_1|K) \cdot \ldots \cdot Pr(Pa_n|K)$

and that

(E) For any m and n, $Pr(Pa_m|K) = Pr(Pa_n|K)$.

Then except for the uninteresting case that $Pr(Pa_n|K) = 1$ for each n, it follows that

$$\lim_{n \to \infty} Pr(Pa_1 . \ldots .Pa_n|K) = 0$$

and thus that $Pr(H|K) = 0$.

Popper's argument can be attacked in various places. Condition (E) is a form of exchangeability, and we have seen above that it cannot be expected to hold for all predicates. But Popper can respond that if (E) does fail then so will various forms of inductivism (e.g., Hume projectibility). The main place the inductivist will attack is at the assumption (I) of the independence of instances. Popper's response is that the rejection of (I) amounts to the postulation of something like a causal connection between instances. But this a red herring since the inductivist can postulate a probabilistic dependence among instances without presupposing that the instances are cemented together by some sort of causal glue.

In another attempt to show that probabilistic methods are ensnared in inconsistencies, Popper cites Jeffreys's proof sketched above that a non-zero prior for $(i)Pa_i$ guarantees that

$$\lim_{n \to \infty} Pr(Pa_{n+1}|Pa_1 . \ldots .Pa_n . K) = 1.$$

But, Popper urges, what is sauce for the goose is sauce for the gander. For we can do the same for a Goodmanized P^*, and from the limit statements we can conclude that for some $r > 0.5$ there is a sufficiently large N such that for any $N' > N$, the probabilities for $P_{a_{N'}}$ and for $P^*_{a_{N'}}$ are both greater than r, which is a contradiction for appropriately chosen P^*. But the reasoning here is fallacious and there is in fact no contradiction lurking in Jeffreys's limit theorem since the convergence is not supposed to be uniform over different predicates—indeed, Popper's reasoning shows that it cannot be.

Of course, none of this helps with the difficult questions of which hypotheses should be assigned nonzero priors and how large the priors should be. The example from item 5 in Section 2.8 above suggests that the latter question can be ignored to some extent since the accumulation of evidence tends to swamp differences in priors and force merger of posterior opinion. Some powerful results from advanced probability theory show that such merger takes place in a very general setting (on this matter see Gaifman and Snir 1982).

2. Probabilification vs. inductive support. Popper and Miller (1983) have argued that even if it is conceded that universal hypotheses may have nonzero priors and thus can be probabilified further and further by the accumulation of positive evidence, the increase in probability cannot be equated with genuine inductive support. This contention is based on the application of two lemmas from the probability calculus:

Lemma 1. $Pr(\sim H|E.K) \times Pr(\sim E|K) = Pr(H \vee \sim E|K) - Pr(H \vee \sim E|E.K)$.

Lemma 1 leads easily to

Lemma 2. If $Pr(H|E.K) < 1$ and $Pr(E|K) < 1$ then

$$Pr(H \vee \sim E|E.K) < Pr(H \vee \sim E|K).$$

Let us apply Lemma 2 to the case discussed above where Bayesianism was used to show that under certain conditions the H-D method does lead to incremental confirmation. Recall that we assumed that

$$H, K \vdash E;\ 1 > Pr(E|K) > 0;\ \text{and}\ 1 > Pr(H|K) > 0$$

and then showed that

$$Pr(H|E.K) > Pr(H|K),$$

which the inductivists want to interpret as saying that E inductively supports H on the basis of K. Against this interpretation, Popper and Miller note that H is logically equivalent to $(H \vee E).(H \vee \sim E)$. The first conjunct is deductively implied by E, leading Popper and Miller to identify the second conjunct as the part of H that goes beyond the evidence. But by Lemma 2 this part is *countersupported* by E, except in the uninteresting case that $E.K$ makes H probabilistically certain.

Jeffrey (1984) has objected to the identification of $H \vee \sim E$ as the part of H that goes beyond the evidence. To see the basis of his objection, take the case where

$$H{:}\ (i)Pa_i \text{ and } E{:}\ Pa_1. \ldots .Pa_n.$$

Intuitively, the part of H that goes beyond this evidence is $(i)\ [\ (i > n) \ldots Pa_i]$ and not the Popper-Miller $(i)Pa_1 \vee \sim(Pa_1. \ldots .Pa_n)$.

Gillies (1986) restated the Popper-Miller argument using a measure of inductive support based on the incremental model of confirmation: (leaving aside K) the support given by E to H is $S(H, E) = Pr(H|E) - \Pr(H)$. We can then show that

Lemma 3. $S(H, E) = S(H \vee E, E) + S(H \vee \sim E, E)$.

Gillies suggested that $S(H \vee EE,)$ be identified as the deductive support given H by E and $S(H \vee \sim E, E)$ as the inductive support. And as we have already seen, in the interesting cases the latter is negative. Dunn and Hellman (1986) responded by dualizing. Hypothesis H is logically equivalent to $(H.E) \vee (H.\sim E)$ and $S(H, E) = S(H.E, E) + S(H.\sim E, E)$. Identify the second component as the deductive countersupport. Since this is negative, any positive support must be contributed by the first component which is a measure of the nondeductive support.

3. The problem of old evidence. In the Bayesian identification of the valid kernel of the H-D method we assumed that $Pr(E|K) < 1$, that is, there was some surprise to the evidence E. But this is often not the case in important historical examples. When Einstein proposed his general theory of relativity (H) at the close of 1915 the anomalous advance of the perihelion of Mercury (E) was old news, that is, $Pr(E|K) = 1$. Thus, $Pr(H|E.K) = Pr(H|K)$, and so on the incremental conception of confirmation, Mercury's perihelion does not confirm Einstein's theory, a result that flies in the face of the fact that the resolution of the perihelion problem was widely regarded as one of the major triumphs of general relativity. Of course, one could seek to explain the triumph in nonconfirmational terms, but that would be a desperate move.

Garber (1983) and Jeffrey (1983) have suggested that Bayesianism be given a more human face. Actual Bayesian agents are not logically omniscient, and Einstein for all his genius was no exception. When he proposed his general theory he did not initially know that it did in fact resolve the perihelion anomaly, and he had to go through an elaborate derivation to show that it did indeed entail the missing 43″ of arc per century. Actual flesh and blood scientists learn not only empirical facts but logicomathematical facts as well, and if we take the new evidence to consist in such facts we can hope to preserve the incremental model of confirmation. To illustrate, let us make the following assumptions about Einstein's degrees of belief in 1915:

(a) $Pr(H|K) > 0$ (Einstein assigned a nonzero prior to his general theory.)
(b) $Pr(E|K) = 1$ (The perihelion advance was old evidence.)
(c) $Pr(H \vdash E|K) < 1$ (Einstein was not logically omniscient and did not invent his theory so as to guarantee that it entailed the 43″.)
(d) $Pr[(H \vdash E) \vee (H \vdash \sim E)|K] = 1$ (Einstein knew that his theory entailed a definite result for the perihhelion motion.)
(e) $Pr[H.(H \vdash \sim E)|K] = Pr[H.(H \vdash \sim E).\sim E|K]$ (Constraint on interpreting $\vdash$ as logical implication.)

From (a)–(e) it can be shown that $Pr[H|(H \vdash E).K]. > Pr(H|K)$. So learning that his theory entailed the happy result served to increase Einstein's confidence in the theory.

Although the Garber-Jeffrey approach does have the virtue of making Bayesian agents more human and, therefore, more realistic, it avoids the question of whether the perihelion phenomena did in fact confirm the general theory of relativity in favor of focusing on Einstein's personal psychology. Nor is it adequate to dismiss this concern with the remark that the personalist form of Bayesianism is concerned precisely with psychology of particular agents, for even if we are concerned principally with Einstein himself, the above calculations seem to miss the mark. We now believe that for Einstein in 1915 the perihelion phenomena provided a strong confirmation of his general theory. And contrary to what the Garber-Jeffrey approach would suggest, we would not change our minds if historians of science discovered a manuscript showing that as Einstein was writing down his field equations he saw in a flash of mathematical insight that $H \vdash E$ or alternatively that he consciously constructed his field equations so as to guarantee that they entailed E. "Did E confirm H for Einstein?" and "Did learning that $H \vdash E$ increase Einstein's confidence in H?" are two distinct questions with possibly different answers. (In addition, the fact that agents are allowed to assign $Pr\ (H \vdash E|K) < 1$ means that the Dutch book justification for the probability axioms has to be abandoned. This is anathema for orthodox Bayesian personalists who identify with the betting quotient definition of probability.)

A different approach to the problem of old evidence is to apply the incremental model of confirmation to the counterfactual degrees of belief that would have obtained had E not been known. Readers are invited to explore the prospects and problems of this approach for themselves. (For further discussion of the problem of old evidence, see Howson 1985, Eells 1985, and van Fraassen 1988.)

2.11 CONCLUSION

The topic of this chapter has been *the logic of science*. We have been trying to characterize and understand the patterns of inference that are considered legitimate in establishing scientific results—in particular, in providing support for the hypotheses that become part of the corpus of one science or another. We began by examining some extremely simple and basic modes of reasoning—the hypothetico-deductive method, instance confirmation, and induction by enumeration. Certainly (pace Popper) all of them are frequently employed in actual scientific work.

We find—both in contemporary science and in the history of science—that scientists do advance hypotheses from which (with the aid of initial conditions and auxiliary hypotheses) they deduce observational predictions. The test of Einstein's theory of relativity in terms of the bending of starlight passing close to the sun during a total solar eclipse is an oft-cited example. Others were given in this chapter. Whether the example is as complex as general relativity or as simple as Boyle's law, the logical problems are the same. Although the H-D method contains a valid kernel—as shown by Bayes's rule—it must be considered a serious oversimplification of what actually is involved in scientific confirmation. Indeed, Bayes's rule itself seems to offer a schema far more adequate than the H-D method. But—as we have seen—it, too, is open to serious objections (such as the problem of old evidence).

When we looked at Hempel's theory of instance confirmation, we discussed an example that has been widely cited in the philosophical literature—namely, the generalization "All ravens are black." If this is a scientific generalization, it is certainly at a low level, but it is not scientifically irrelevant. More complex examples raise the same logical problems. At present, practicing scientists are concerned with—and